IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 12, Issue 5 Ver. I (Sep. - Oct.2016), PP 70-71
www.iosrjournals.org

Classes of Regular Semiring

M.Amala*[1], N.Sulochana[2] and T.Vasanthi[3]
[1,3]*Dept. of Applied Mathematics, Yogi Vemana University, Kadapa, Andhra Pradesh, India*
[2]*Asst. Prof., K.S.R.M College of Engineering, Kadapa, Andhra Pradesh, India*

Abstract: *In this paper, it was proved that,* if S is a Right regular semiring, then (S, +) is a band under the following cases.
1. If S is multiplicatively subidempotent semiring.
2. If S is an almost idempotent semiring.
Keywords: *Periodic, rectangular band, Singular semigroup.*
AMS Mathematics Subject Classification (2010): *20M10, 16Y60.*

I. Introduction

In this paper we introduce the notion of Right regular semiring as a generalization of regular semiring. Sen, Ghosh & Mukhopadhyay studied the congruences on inverse semirings with the commutative additive reduct and Maity improved this to the regular semirings with the set of all additive idempotents a bi-semilattice. The study of regular semigroups has yielded many interesting results. These results have applications in other branches of algebra and analysis. We will see the interrelations between different semirings. We characterize almost idempotent semiring. A semiring S is a Right regular semiring, if S satisfies the identity a + xa + a = a for all a, x in S.

Definition 1.1:
A semigroup (S, •) is left (right) singular if it satisfies the identity ax = a (ax = x) for all a, x in S.

Definition 1.2:
A semigroup (S, +) is rectangular band if a = a + x + a for all a, x in S.

Definition 1.3:
A semiring S is said to be multiplicatively subidempotent semiring if $a + a^2 = a$ for all a in S.

Definition 1.4:
A semiring S is almost idempotent if $a + a^2 = a^2$ for all a in S.

Definition 1.5:
A semigroup (S, +) is band if a + a = a for all a in S.

Definition 1.6.:
An element a in a semigroup (S, +) is periodic if ma = na where m and n are positive integers. A semigroup (S, +) is periodic if every one of its elements is periodic.

Definition 1.7:
In a totally ordered semiring (S, +, •, ≤)
(i) (S, +, ≤) is positively totally ordered (p.t.o), if a + x ≥ a, x for all a, x in S
(ii) (S, •, ≤) is positively totally ordered (p.t.o), if ax ≥ a, x for all a, x in S.
(iii) (S, +, ≤) is negatively totally ordered (n.t.o), if a + x ≤ a, x for all a, x in S
(iv) (S, •, ≤) is positively totally ordered (n.t.o), if ax ≤ a, x for all a, x in S.

II. Classes of Right Regular Semiring

Theorem 2.1: If S is a semiring in which (S, •) is left singular semigroup, then S is a Right regular semiring if and only if (S, +) is a rectangular band.

Proof: Since (S, •) is left singular semigroup xa = x for all a, x in S
By hypothesis we have a + xa + a = a for all a, x in S
Which implies a + x + a = a for all a, x in S

Therefore (S, +) is rectangular band
Conversely let us assume $xa = x \Rightarrow a + xa = a + x$
Which leads to $a + xa + a = a + x + a$
Also we have (S, +) is a rectangular band then $a + x + a = a$
This implies $a + xa + a = a$ for all a, x in S
Hence S is a Right regular semiring

Proposition 2.2: If S is a Right regular semiring, then (S, +) is a band under the following cases.

(i) If S is multiplicatively subidempotent semiring.
(ii) If S is an almost idempotent semiring.

Proof: (i) Since S is multiplicatively subidempotent semiring. $a + a^2 = a$

$$\Rightarrow a + a^2 + a = a + a \rightarrow \textbf{(1)}$$

By hypothesis S is Right regular then $a + a^2 + a = a$ for all a in S
Therefore equation (1) becomes $a = a + a$ for all a in S
Hence (S, +) is a band

(ii) Since S is an almost idempotent semiring then $a^2 + a = a^2$

$$\Rightarrow a + a^2 + a = a + a^2$$

Then above equation becomes $a = a + a^2$
Adding 'a' on both sides we obtain $a + a = a + a^2 + a$
It takes the form $a + a = a$ for all a in S
Thus (S, +) is a band

Proposition 2.3: Let S be a Right regular semiring and (S, •) be a right singular semigroup. Then (S, +) is periodic.

Proof: By hypothesis (S, •) is a right singular, $xa = a$ for all a, x in S
Since S is right regular, $a + xa + a = a$ for all a, x in S
Which implies $a + a + a = a$
$\Rightarrow 3a = a$ for all a in S
Therefore (S, +) is periodic

Theorem 2.4: Suppose S is a totally ordered Right regular semiring and (S, +) is positively totally ordered (negatively totally ordered). Then (S, •) is negatively totally ordered (positively totally ordered).

Proof: By Right regular semiring we have $a + xa + a = a \rightarrow$ **(1)**
Since (S, +) is positively totally ordered, $a + x \geq a, x$ for all a, x in S
Then equation (1) becomes $a = a + xa + a \geq xa \Rightarrow a \geq xa$
Suppose let us consider $xa > x$
This implies $a + xa + a \geq a + x + a$
Which is again equal to $a + xa + a \geq a + x \Rightarrow a \geq a + x$
Which contradicts the hypothesis that (S, +) is p.t.o
Therefore $xa \leq x$ and $xa \leq a$
Hence (S, •) is negatively totally ordered
In a similar manner (S, •) is positively totally ordered if (S, +) is n.t.o

References

[1]. Kanchan Jana, "Quasi *K*-Ideals In *K*-Regular And Intra *K*-Regular Semirings", Pu. M.A.Vol. 22 (2011), No.1, PP. 65-74.
[2]. N.Kehayopulu, "On a characterization of regular duo le Semigroups", Maths. Balkonica 7 (1977), 181-186.
[3]. K.S.S.Nambooripad, "Structure of regular semigroups I fundamental regular semigroups", Semigroup Forum, Vol.9 (1975), 354-363.
[4]. M.Satyanarayana, "On the additive semigroup structure of semirings", Semigroup Forum, Vol.23 (1981), 7-14.
[5]. M.Satyanarayana, "On the additive semigroup of ordered semirings". Semigroup Forum, 31 (1985), 193-199.
[6]. M.K.Sen and S.K.Maity, "Semirings Embedded in a Completely Regular Semiring", Acta Univ. Palacki. Olomuc, Fac. rer. nat., Mathematica 43 (2004) 141–146.
[7]. Haixuan Yang and S.Ponizovskii, "Identities of Regular Semigroup Rings", Semigroup Forum Vol. 56 (1998) 293- 295 Springer-Verlag New York Inc.