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Hall Current Effects on Free Convective Flow of Stratified Fluid over an Infinite vertical porous plate

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Abstract: *In this paper, we have studied the effects of heat and mass transfer on the free convective flow of stratified fluid through a porous medium past a vertical porous isothermal plate fluctuating with time dependent suction velocity. An analytical solution is obtained for the velocity field, temperature field and concentration using regular perturbation method. The effects of various physical quantities on the Velocity field, Temperature field, Concentration, Nusselt number and Skin friction have been discussed through graphs in detail.*
Keywords: *MHD flows, Heat and mass transfer, stratified flows, Vertical plates*

I. Introduction

Industries based on engineering disciplines, preferably chemical engineering sectors encounter fluid, heat, and mass flow induced by continuous stretching heated surfaces. Some examples are the extrusion process, wire and fiber coating, polymer processing, foodstuff processing, and design of various heat exchangers. As stretching brings a unidirectional orientation to the extrudate, the quality of the final product considerably depends on the flow and heat and mass transfer mechanism. Steady heat flow on a moving continuous flat surface was first considered by Sakiadis [1] who developed a numerical solution using a similarity transformation. Crane [2] motivated by the processes of polymer extrusion, in which the extrudate emerges from a narrow slit, examined semi-infinite fluid flow driven by a linearly stretching surface. Many researchers and academicians have advanced their studies relating to problems involving MHD in stretching sheet considering various parameters: Sharidan [3], Carraagher et al. [4], Gupta and Gupta [5], Ishak et al. [6–8], Aziz et al. [9], Abel et al. [10], Mukhopadhyay and Mondal [11], Krishnendu [12], Swati Mukhopadhyay et al. [13], Laha et al. [14], Afzal [15], Prasad et al. [16], Wang [17] among others. Rashad et al. [18] considered MHD free convective heat and mass transfer of a chemically-reacting fluid from radiate stretching surface embedded in a saturated porous medium. Theuri et al. [19] studied unsteady double diffusive magnetohydrodynamic boundary layer flow of a chemically reacting fluid over a flat permeable surface. The influence of a uniform transverse magnetic field on the motion of an electrically conducting fluid past a stretching sheet was investigated by Pavlov [20]. The effect of chemical reaction on free-convective flow and mass transfer of a viscous, incompressible and electrically conducting fluid over a stretching sheet was investigated by Afify [21] in the presence of transverse magnetic field. Das et al. [22] investigated MHD Boundary layer slip flow and heat transfer of nanofluid past a vertical stretching sheet with non-uniform heat generation/ absorption. Ibrahim and Makinde [23] studied the doublediffusive mixed convection and MHD Stagnation point flow of nanofluid over a stretching sheet. Rudraswamy et al. [24] considered the effects of magnetic field and chemical reaction on stagnation-point flow and heat transfer of a nanofluid over an inclined stretching sheet. In most of the investigations, the electrical conductivity of the fluid was assumed to be uniform and the magnetic field intensity was assumed to be low. However, in an ionized fluid where the density is low and thereby magnetic field intensity is very strong, the conductivity normal to the magnetic field is reduced due to the spiraling of electrons and ions about the magnetic lines of force before collisions take place and a current induced in a direction normal to both the electric and magnetic fields. This phenomenon is known as Hall Effect. Hall currents are important and they have a marked effect on the magnitude and direction of the current density and consequently on the magnetic force term. The problem of MHD free convection flow with Hall currents has many important engineering applications such as in power generators, MHD accelerators, refrigeration coils, transmission lines, electric transformers, and heating elements, and Watanabe and Pop [25], Abo-Eldahab and Salem [26], Rana et al. [27], Singh and Gorla [28], and Shit [29] among others have advanced studies on Hall effect on MHD past stretching sheet. Shateyi and Motsa [30] considered boundary layer flow and double diffusion over an unsteady stretching surface with Hall effect. Chamkha et al. [31] analyzed the unsteady MHD free convective heat and mass transfer from a vertical porous plate with Hall current, thermal radiation and chemical reaction effects. The effect of radiation on MHD flow and heat transfer problem has become more important industrially. At high operating temperature, radiation effect can be quite significant. Many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. Several authors [32–

38] are among various names who studied the problems relating to MHD with radiation effect. Makinde [39] analyzed chemically reacting hydromagnetic unsteady flow of a radiating fluid past a Nomenclature B ~ magnetic induction vector Bo strength of applied magnetic field C dimensionless concentration variable Cf local skin-friction coefficient Cs concentration susceptibility factor Cp specific heat at constant pressure D thermal molecular diffusivity Dm solution diffusivity of the medium E ~ intensity vector of the electric field E charge of the electron G Grashof number go acceleration due to gravity J ~ electric current density vector K mean absorption coefficient Kf thermal conductivity k0 measures the rate of reaction KT thermal diffusion ratio M Hartmann number m Hall current parameter N Buoyancy ratio Nr thermal radiation parameter Nu local Nusselt number ne number of density of the electron Pr Prandtl number Q1 radiation absorption Sc Schmidt number Sr Soret parameter Sh local Sherwood number T dimensional temperature variable Tw fluid temperature at walls T1 free stream temperature u,v,w velocity components x,y,z direction components le magnetic permeability V~ velocity vector Greek symbols a thermal diffusivity bc coefficient of species concentration bT coefficient of thermal expansion g similarity variable q1 fluid density l co-efficient of dynamic viscosity r electrical conductivity of the fluid h non-dimensional temperature hr viscosity parameter / non-dimensional concentration k Heat generation/absorption parameter c non-dimensional chemical reaction parameter sx, sz shear stress components m kinematic viscosity Subscripts c cold wall w warm side wall 384 G. Sreedevi et al. vertical plate with constant heat flux. Chamkha et al. [40] further analyzed the unsteady coupled heat and mass transfer by mixed convection flow of a micropolar fluid near the stagnation point on a vertical surface in the presence of radiation and chemical reaction. To accurately predict the flow and heat transfer rates, it is necessary to take into account the temperature-dependent viscosity of the fluid. The effect of temperature-dependent viscosity on heat and mass transfer laminar boundary layer flow has been discussed by many authors [41–46] in various situations. They showed that when this effect was included, the flow characteristics might change substantially compared with the constant viscosity assumption. Salem [47] investigated variable viscosity and thermal conductivity effects on MHD flow and heat transfer in viscoelastic fluid over a stretching sheet. Devi and Ganga [48] have considered the viscous dissipation effects on MHD flows past stretching porous surfaces in porous media. Recently, Mohamed El-Aziz [49] analyzed unsteady mixed convection heat transfer along a vertical stretching surface with variable viscosity and viscous dissipation. Tian et al. [50] investigated the 2D boundary layer flow and heat transfer in variable viscosity MHD flow over a stretching plate. Studies related to double diffusive hydromagnetic boundary layer flow with heat and mass transfer over flat surfaces are extremely important and have many applications in engineering and industrial processes [51,52]. For instance, heat and mass transfer occur in processes, such as drying, evaporation at the surface of a water body, and energy transfer in a wet cooling tower, cooling of nuclear reactors, MHD power generators, MHD pump, chemical vapor deposition on surfaces, formation and dispersion of fog, and distribution of temperature and moisture over agriculture fields. Moreover, when heat and mass transfer occur simultaneously between the fluxes, the driving potentials are of more intricate nature [53–55]. An energy flux can be generated not only by temperature gradients but by composition gradients. The energy flux caused by a composition is called Dufour or diffusion-thermo effect. Temperature gradients can also create mass fluxes, and this is the Soret or thermal-diffusion effect. The concept of Soret and Dufour effects on heat and mass transfer has been developed from the kinetic theory of gases by Chapman and Cowling [56] and Hirshfelder et al. [57]. They explained the phenomena and derived the necessary formulae to calculate the thermal diffusion coefficient and the thermal-diffusion factor for monatomic gases or for polyatomic gas mixtures. The thermal-diffusion effect, for instance, has been utilized for isotope separation and in mixture between gases with very light molecular weight (Hydrogen–Helium) and of medium molecular weight (Nitrogen–air) and the diffusion-thermo effect was found to be of a magnitude such that it cannot be neglected Kafoussias and Williams [58]. Alam et al. [59] studied Dufour and Soret effects on steady free convection and mass transfer flow past a semi-infinite vertical porous plate in a porous medium. Makinde [60] studied numerically the influence of a magnetic field on heat and mass transfer by mixed convection from vertical surfaces in porous media considering Soret and Dufour effects. Makinde and Olanrewaju [61] have investigated the Dufour and Soret effects on unsteady mixed convection flow past a porous plate moving through a binary mixture of chemically reacting fluid. The thermal diffusion and diffusion thermo effects on chemically reacting hydromagnetic boundary layer flow of heat and mass transfer past a moving vertical plate with suction/injection have been addressed by Olanrewaju and Makinde [62]. Zhang et al. [63] 2013 considered the unsteady heat and mass transfer in MHD over an oscillatory stretching surface with Soret and Dufour effects. Recently, Rashidi et al. [64] analyzed heat and mass transfer for MHD viscoelastic fluid flow over a vertical stretching sheet considering Soret and Dufour effects.

In view of above studies, in this chapter we studied the effects of Hall currents and mass transfer on the free convective flow of stratified fluid through a porous medium past a vertical porous isothermal plate fluctuating with time dependent suction velocity. An analytical solution is obtained for the velocity field, temperature field and concentration using regular perturbation method. The effects of various physical quantities on the Velocity field, Temperature field, Concentration, Nusselt number and Skin friction are discussed through graphs in detail.

Nomenclature

n	: Frequency parameter
B	: Magnetic flux density vector
u	: Velocity in the direction of x-axis
T	: Temperature of the liquid
T_∞	: Temperature of the fluid far from the plate
C_p	: Specific heat at constant pressure
Gr	: Grashoff number
S	: Stratification factor
So	: Sorret number
Sc	: Schmidt number
Gc	: Modified Grashof number
Pr	: Prandtl number
M	: Magnetic field parameter
Nu	: Nusselt number
m	: Hall Parameter
D_1	: Coefficient of thermal diffusivity
D	: Chemical Molecular diffusivity
C_∞	: Concentration in the fluid for away from the plate
C_w	: Concentration of the plate
ρ	: Fluid density
K_T	: Thermal conductivity
β	: Volumetric coefficient expansion
β^*	: Coefficient of Species coefficient
k	: Permeability
T'_∞	: Temperature of the fluid far away from the plate
T'_w	: Temperature of the plate
t	: Dimensionless time
μ	: Viscosity
υ	: Kinematic viscosity
g	: Acceleration due to gravity

II. Formulation and Solution of the Problem:

We consider the free convective flow of an incompressible, electrically conducting viscous, stratified fluid through a porous medium past a vertical, porous, isothermal infinite plate with time dependent suction at the plate. We choose a Cartesian coordinate system with the x -axis vertically upward along the plate and y-axis normal to the plate. A uniform magnetic field with magnetic flux density vector $B=\left(B_0,0,0\right)$ is applied (which is assumed to be the total magnetic field, as the induced magnetic field is neglected by taking a very small magnetic Reynolds number). The Hall current effect is taken into account. Further we assume the Density (ρ), Viscosity (μ), Thermal conductivity (K_T), Volumetric coefficient expansion (β) and coefficient of Species coefficient (β^*) satisfy exponential law. Namely,

$\rho=\rho_0 e^{-\alpha y}$, $\mu=\mu_0 e^{-\alpha y}$, $K_T=K_{T0}e^{-\alpha y}$, $\beta=\beta_0 e^{-\alpha y}$ and $\beta^*=\beta_0^* e^{-\alpha y}$

where α is the stratification factor.

In addition, the magnetic field $B=B_0 e^{-\alpha y}$ is applied perpendicular to the flow region and the suction velocity $v=-v_0\left\{1+\varepsilon f(t)\right\}$ is a function of time and the pressure gradient is negligible. In addition to above considerations, the present analysis is made on the basis of following assumptions:

All the fluid properties are assumed to be constant except the influence of the density variation with temperature only in body force term.

In the influence of density variation, other terms of the momentum equation and variation of expansion coefficient with temperature are considered negligible.

The free convection currents are in existence due to temperature difference ($T - T_\infty$).

Hence, using the above assumptions, the governing equations for the flow are:

Momentum equation

$$\rho\left(\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \frac{\sigma B_0^2}{\left(1+m^2\right)}u - \frac{\mu}{k}u + \rho g\beta(T - T_\infty) + \rho g\beta^*(C - C_\infty) \quad (1)$$

Temperature equation

$$\rho Cp\left(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y}\right) = K_T\,\frac{\partial^2 T}{\partial y^2} \quad (2)$$

Concentration equation

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D^2\,\frac{\partial^2 C}{\partial y^2} + D_1\,\frac{\partial^2 T}{\partial y^2} \quad (3)$$

Using the values of ρ , μ , K_T , β , β^* and B_0 into the equations (1)-(3), we get

$$\frac{\partial u}{\partial t} - v_0\{1+\varepsilon f(t)\}\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \alpha\upsilon\frac{\partial u}{\partial y} - \frac{\sigma B_0^2 u}{\rho(1+m^2)} - \frac{\upsilon}{k}u + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) \quad (4)$$

$$\frac{\partial T}{\partial t} - v_0\{1+\varepsilon f(t)\}\frac{\partial T}{\partial y} = \frac{K_T}{\rho C_p}\frac{\partial^2 T}{\partial y^2} \quad (5)$$

$$\frac{\partial C}{\partial t} - v_0\{1+\varepsilon f(t)\}\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + D_1\frac{\partial^2 T}{\partial y^2} \quad (6)$$

The boundary conditions are

$$\begin{aligned}
&u = v_0\left[1+\varepsilon f(t)\right] && \text{at } y = 0\\
&\mathrm{T} = \mathrm{T_w}\left[1+\varepsilon f(t)\right] && \text{at } y = 0\\
&\mathrm{C} = \mathrm{C_w}\left[1+\varepsilon f(t)\right] && \text{at } y = 0 \qquad (7)\\
&\mathrm{u} \to 0, \quad \mathrm{T} \to 0 \text{ and } \mathrm{C} \to 0 \text{ as } \mathrm{y} \to \infty
\end{aligned}$$

We introduce the following non-dimensional variables:

$$\bar{y} = \frac{y v_0}{\upsilon},\ \bar{u} = \frac{u}{v_0},\ \bar{t} = \frac{t v_0^2}{\upsilon},\ \bar{T} = \left(\frac{T - T_\infty}{T_\omega - T_\infty}\right),\ \bar{C} = \frac{C - C_\infty}{C_\omega - C_\infty},\ \bar{K} = \frac{K v_0^2}{\upsilon^2}.$$

using the above non-dimensional variables into the equations (4) – (6), we get

$$\frac{\partial u}{\partial t} - \{1+\varepsilon f(t)\}\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - S\frac{\partial u}{\partial y} - \left(\frac{M^2}{1+m^2} + \frac{1}{K}\right)u + GrT + G_C C \quad (8)$$

$$\frac{\partial T}{\partial t} - \{1+\varepsilon f(t)\}\frac{\partial T}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} \quad (9)$$

$$\frac{\partial C}{\partial t} - \{1+\varepsilon f(t)\}\frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} + S_0\frac{\partial^2 T}{\partial y^2}, \quad (10)$$

where

$$S = \frac{\upsilon\alpha}{V_0},\ Gr = \frac{g\beta\upsilon(T_\omega - T_\infty)}{V_o^3},\ M^2 = \frac{\sigma\beta_0^2\upsilon}{\rho V_0^2},\ Gc = \frac{g\beta^*\upsilon(C_\omega - C_\infty)}{V_0^3}\quad Pr = \frac{\mu Cp}{K_T},\ Sc = \frac{\upsilon}{D},\ \mathrm{S}_0 = \frac{D_1}{\upsilon}\left(\frac{T_w - T_\infty}{C_w - C_\infty}\right)$$

The corresponding non-dimensional boundary conditions are

$$\begin{aligned}
&u = 1+\varepsilon f(t),\ T = 1+\varepsilon f(t) \text{ and } C = 1+\varepsilon f(t) \qquad \text{at } y = 0\\
&u \to 0, \quad T \to 0 \text{ and } C \to 0 \text{ as } y \to \infty \qquad (11)
\end{aligned}$$

To obtain the solution of the problem, we consider the case when the suction velocity is exponentially decreasing function of the time. Hence we assume

$$f(t) = e^{-nt} \quad (12)$$

Substituting equation (12) in to the equations (8) – (10), we get

$$\frac{\partial u}{\partial t}-\{1+\varepsilon e^{-nt}\}\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-S\frac{\partial u}{\partial y}-\left(\frac{M^2}{1+m^2}+\frac{1}{K}\right)u+GrT+G_C C \tag{13}$$

$$\frac{\partial T}{\partial t}-\{1+\varepsilon e^{-nt}\}\frac{\partial T}{\partial y}=\frac{1}{Pr}\frac{\partial^2 T}{\partial y^2} \tag{14}$$

$$\frac{\partial C}{\partial t}-\{1+\varepsilon e^{-nt}\}\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}+S_0\frac{\partial^2 T}{\partial y^2} \tag{15}$$

The boundary conditions are transformed to

$$u=1+\varepsilon e^{-nt},\ T=1+\varepsilon e^{-nt},\ C=1+e^{-nt} \quad \text{at } y=0 \tag{16}$$

$$u\to 0,\ \ T\to 0,\ \text{and C}\to 0\ \text{ as y}\to\infty \tag{17}$$

Let us assume the velocity field, the temperature field and the concentration in the form

$$\begin{aligned} u(y,t)&=u_0(y)+\varepsilon u_1(y)e^{-nt}\\ T(y,t)&=T_0(y)+\varepsilon T_1(y)e^{-nt}\\ C(y,t)&=C_0(y)+\varepsilon C_1(y)e^{-nt}\end{aligned} \tag{18}$$

Substituting equation (18) in to the equations (13) – (15) and equating the harmonic and non-harmonic terms, we obtain

$$\frac{\partial^2 u_0}{\partial y^2}+(1-S)\frac{\partial u_0}{\partial y}-\left(\frac{M^2}{1+m^2}+\frac{1}{K}\right)u_0=-GrT_0-G_C C_0 \tag{19}$$

$$\frac{\partial^2 u_1}{\partial y^2}+(1-S)\frac{\partial u_1}{\partial y}-\left(\frac{M^2}{1+m^2}+\frac{1}{K}-n\right)u_1=-\frac{\partial u_0}{\partial y}-GrT_1-G_C C_1 \tag{20}$$

$$\frac{\partial^2 T_0}{\partial y^2}+Pr\frac{\partial T_0}{\partial y}=0 \tag{21}$$

$$\frac{\partial^2 T_1}{\partial y^2}+Pr\frac{\partial T_1}{\partial y}+nPrT_1=-Pr\frac{\partial T_0}{\partial y} \tag{22}$$

$$\frac{\partial^2 C_0}{\partial y^2}+Sc\frac{\partial C_0}{\partial y}=-Sc\ \text{So}\frac{\partial^2 T_0}{\partial y^2} \tag{23}$$

$$\frac{\partial^2 C_1}{\partial y^2}+Sc\frac{\partial C_1}{\partial y}+nSc\ \text{C}_1=-Sc\ \frac{\partial C_0}{\partial y}-SoSc\frac{\partial^2 T_1}{\partial y^2} \tag{24}$$

The corresponding boundary conditions are

$$\begin{aligned} &u_0=1,\ u_1=1,\ T_0=1,\ T_1=0,\ \ C_0=1\ \text{ and }\ C_1=1 \quad \text{at}\ \ y=0\\ &u_0\to 0,\ u_1\to 0,\ T_0\to 0,\ T_1\to 0,\ \ C_0\to 0\ \text{and}\ \ C_1\to 0\ \ \text{ as }\ y\to\infty\end{aligned} \tag{25}$$

The solution of the above coupled equations, using the corresponding boundary conditions (25) is obtained as:

$$T_0=e^{-Pr\,y} \tag{26}$$

$$T_1=(1-\frac{Pr}{n})e^{-by}+\frac{Pr}{n}e^{-Pr\,y} \tag{27}$$

$$C_0=\left(1+\frac{S_0ScPr}{Pr-Sc}\right)e^{-Sc\,y}-\left(\frac{S_0ScPr}{Pr-Sc}\right)e^{-Pr\,y} \tag{28}$$

$$C_1=(1-A_1+A_2+A_3)e^{-dy}+A_1e^{-Sc\,y}-A_2e^{-by}-A_3e^{-Pr} \tag{29}$$

$$u_0=(1-F+F_{21})e^{r_2 y}+F_1e^{-Pr\,y}-F_2e^{-Sc\,y} \tag{30}$$

$$u_1=F_7e^{-S_2 y}+\frac{r_2}{n}(1-F_1+F_2)e^{-r_2 y}+F_3e^{-Pr\,y}-F_4e^{-Sc\,y}F_5e^{-by}-F_6e^{-dy} \tag{31}$$

Substituting the values of $T_0(y),\ T_1(y),\ C_0(y),\ C_1(y),\ u_0(y)$ and $u_1(y)$ in to the equation (18), we obtain

$$T = e^{-Pr\, y} + \varepsilon \left[(1 - \frac{Pr}{n})\, e^{-by} + \frac{Pr}{n}\, e^{-Pr\, y} \right] e^{-nt} \tag{32}$$

$$C = \left(1 + \frac{S_0 Sc Pr}{Pr - Sc}\right) e^{-Sc\, y} - \left(\frac{S_0 Sc Pr}{Pr - Sc}\right) e^{-Pr\, y} + \varepsilon \left[(1 - A_1 + A_2 + A_3) e^{-dy} + A_1 e^{-Sc\, y} - A_2 e^{-by} - A_3 e^{-Pr} \right] e^{-nt} \tag{33}$$

$$u = (1 - F_1 + F_2)\, e^{r_2 y} + F_1 e^{-Pr\, y} - F_2 e^{-Sc\, y} + \varepsilon \left[F_7 e^{-S_2 y} + \frac{r_2}{n} (1 - F_1 + F_2) e^{-r_2 y} + F_3 e^{-Pr\, y} - F_4 e^{-Sc\, y} F_5 e^{-by} - F_6 e^{-dy} \right] \tag{34}$$

Where the constants $b, d, A_1, A_2, A_3, F_1, F_2, F_3, F_4, F_5, F_6, F_7, S_2, r_2$........... used above are functions of the physical parameters involved in the problem expressed in Appendix.

Rate of Heat Transfer:

The rate of hear transfer in terms of Nusselt number (Nu) at the vertical plate at y=0 is:

$$Nu = -\left(\frac{\partial T}{\partial y}\right)_{y=0} = Pr + \varepsilon \mathrm{e}^{-nt} \left[b\left(1 - \frac{Pr}{n}\right) + \frac{Pr^2}{n} \right] \tag{35}$$

Skin-Friction:

The Skin friction coefficient (τ) at the plate at y=0 is:

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\partial u_0}{\partial y} + \varepsilon e^{-nt} \frac{\partial u_1}{\partial y}\bigg|_{y=0}$$

$$\frac{\partial u_0}{\partial y}\bigg|_{y=0} = -r_2 (1 - F_1 + F_2) - F_1 Pr + F_2 Sc = G_0 (Say)$$

$$\frac{\partial u_1}{\partial y}\bigg|_{y=0} = -S_2 F_7 - \frac{r_2}{n}(1 - F_1 + F_2) - F_3 Pr + F_4 Sc - bF_5 + dF_6 = G_1 (say)$$

$$\tau = G_0 + \varepsilon\, e^{-nt} G_1 \tag{36}$$

III. Results and Discussion

The velocity, temperature, Nusselt number and skin friction derived so for, have been evaluated for various values of Prandtl number ‘ Pr ’, Sorret number ‘ So ’, Schmidt number ‘ Sc ’, Hartmann number ‘M’, Hall parameter m, Grashoff number ‘Gr’, Modified Grashoff number ‘Gc’, Nusselt number ‘ Nu ’, and frequency parameter ‘n’ through graphically.

Fig.1 gives the effect of Prandtl number Pr on the temperature field T. It is observed that, the temperature decreases with increasing Prandtl number Pr .

Fig.2. shows the Concentration profiles. It was observed that the Concentration ‘C’ increases with increase in Prandtl number ‘ Pr ’, Sorret number ‘ So ’ and decreases with increasing Schmidt number ‘Sc’.

Figs. 5 – 12 show the effects Pr, Sc, So, M, m, Gr, Gc and S on the velocity field. Fig. 4.5 shows the effect of Prandtl number ‘ Pr ’ on the velocity field ‘ u ’.It was observed that, the velocity is decreasing with increasing ‘ Pr ’. Fig. 6 depicts the effect of ‘ Sc ’ on the velocity field u . It is found that, the velocity decreases with increasing ‘ Sc ’. Fig. 7 gives the effect of ‘ So ’on the velocity field u . It is noticed that, the velocity increases with increasing ‘ So ’. Fig.8 shows the effect of ‘M’ on the velocity field u . It is found that, the velocity decreases with increasing ‘M’. Fig. 9 shows that the velocity u increases with increasing Hall parameter m. Fig.10 shows the effect of ‘Gr’on the velocity field u , and found that, the velocity increases with increasing ‘Gr’. Fig. 11 depicts the effect of ‘Gc’on the velocity field u . It is observed that, the velocity increases with increasing ‘Gc’. The effect of ‘S’on the velocity field u shows in Fig.12. It is observed that, the velocity increases with increasing ‘S’.

Fig.[13-14] gives the effect of ‘ n ’on the Nusselt number ‘ Nu ’. It is found that the Nusselt number ‘ Nu ’ decreases with increase in time ‘t’, and the frequency parameter ‘n’, and increases with increasing ‘ Pr ’.

Figs. 15 – 23 show the effects of Prandtl number Pr, Schmidt number ' Sc ', Sorret number ' So ', Hartmann number 'M', Hall parameter m, Grashoff number 'Gr', Modified Grashoff number 'Gc', frequency parameter 'n' and Stratification factor 'S' on the skin friction τ. From these figures it is clear that the skin friction ' τ ' decreases with the increase in Prandtl number Pr, Schmidt number ' Sc ' and Hartmann number 'M', and increases with the increase in Sorret number ' So ', Hall parameter m, Grashoff number 'Gr', Modified Grashoff number 'Gc', and Stratification factor 'S', and finally the skin friction ' τ ' first increases and then decreases with increase in 'n'.

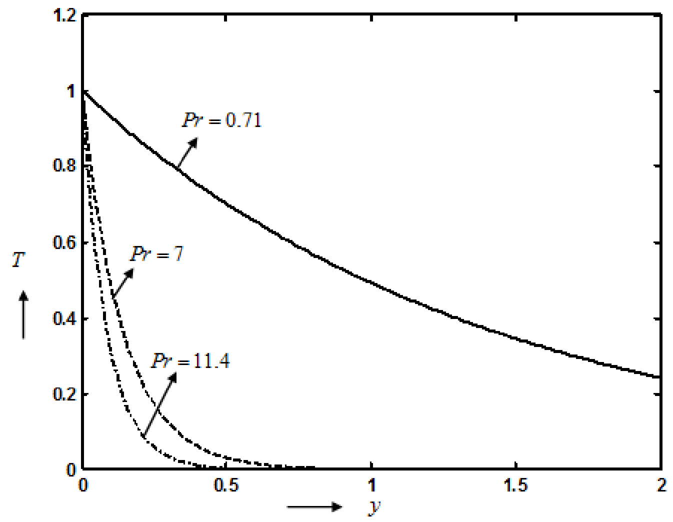

Fig.1 The effect of Pr on the temperature field T with $n = 2$

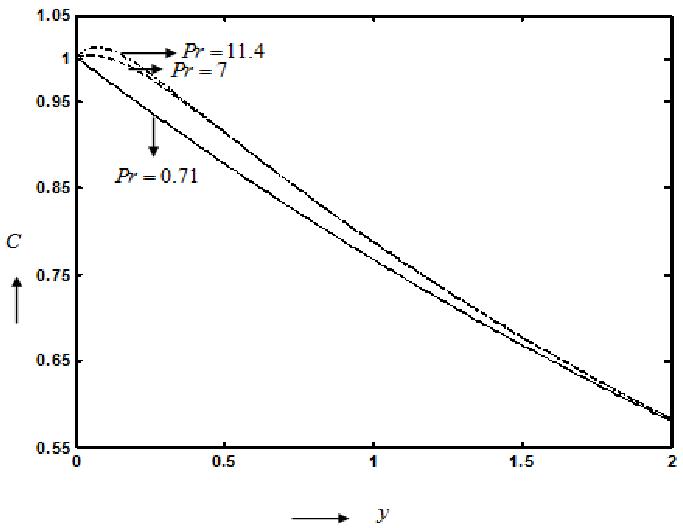

Fig.2 The effect of Pr on the concentration C
With $n = 2, So = 0.2$ and $Sc = 0.3$.

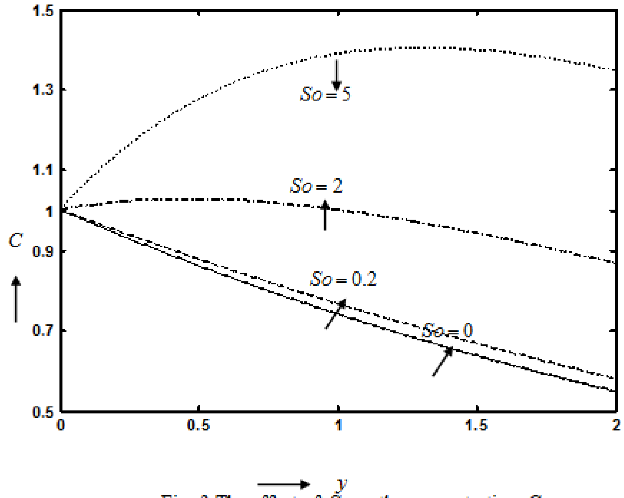

Fig. 3 The effect of So on the concentration C
with $n=2, Pr=0.71$ and $Sc=0.3$.

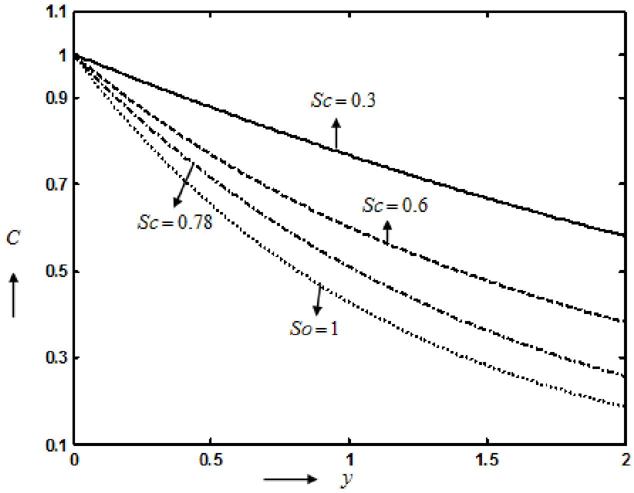

Fig. 4 The effect of Sc on the concentration C
With $n=2, Pr=0.71$ and $So=0.2$.

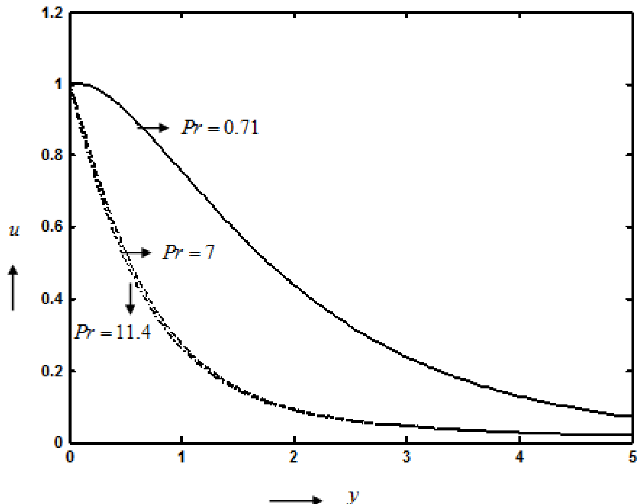

Fig. 5 The effect of Pr on the velocity field u with
$n=2, Gr=5, Gc=0.3, S=2, So=0.2, m=0.2, M=2, K=5$ and $Sc=0.3$

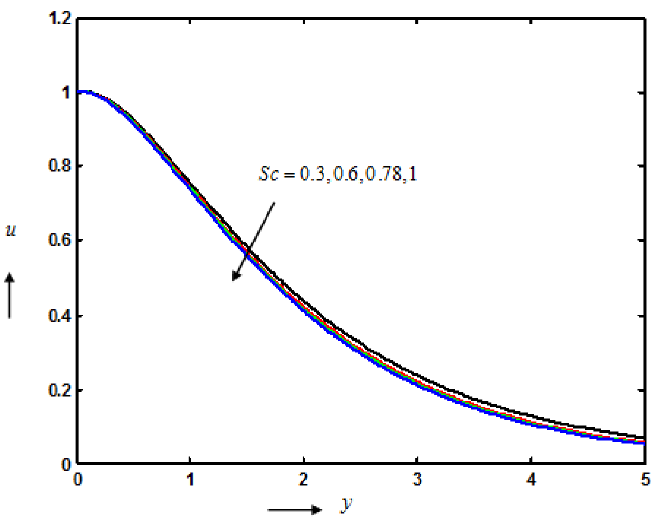

Fig. 6 The effect of Sc on the velocity field u with Gr=5, Gc=0.2, S=2, So=0.2, n=2, m=0.2, M=2, K=5 and Pr=0.71

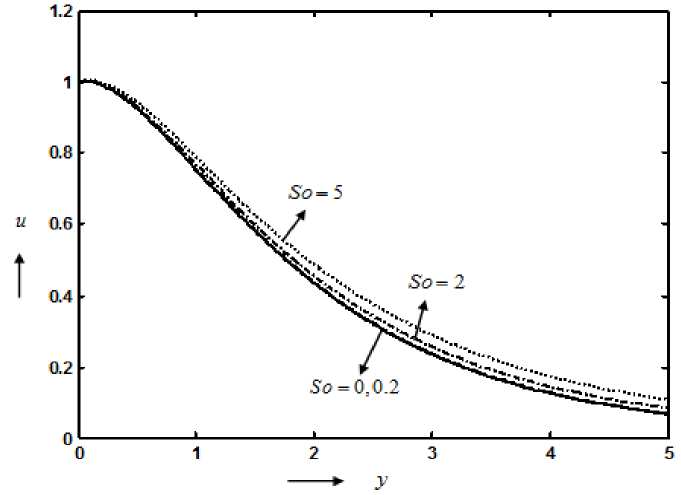

Fig. 7 The effect of So on the velocity field u with Gr=5, Gc=0.2, S=2, Sc=0.3, n=2, m=0.2, M=2, K=5 and Pr=0.71

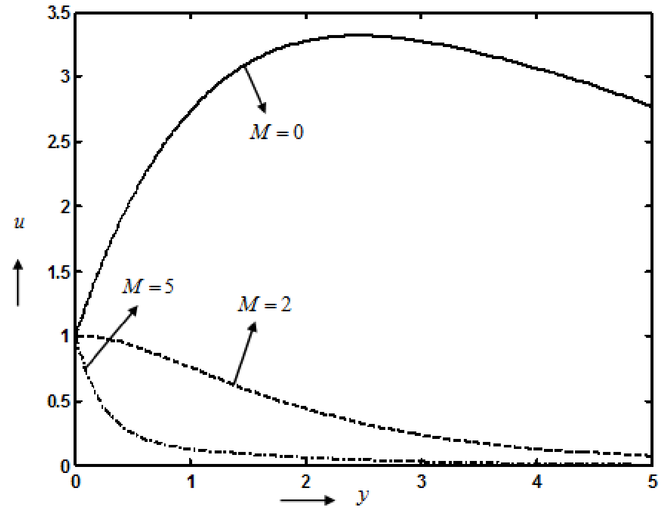

Fig.8 The effect of M on the velocity field u with Gr=5, Gc=0.2, S=2, Sc=0.3, n=2, m=0.2, So=0.2, K=5 and Pr=0.71

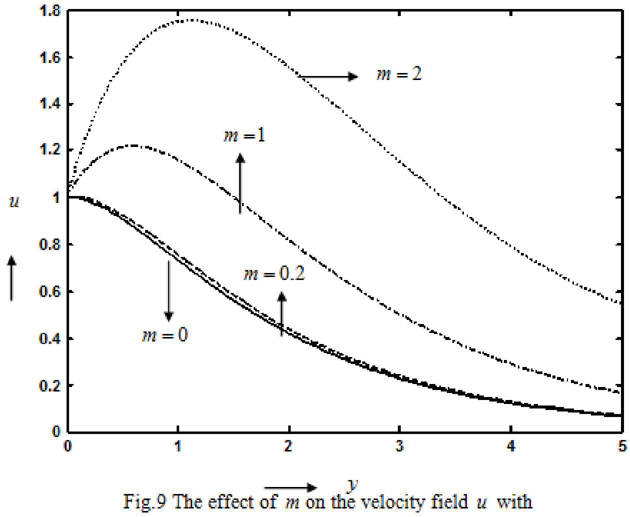

Fig.9 The effect of m on the velocity field u with Gr=5, Gc=0.2, S=2, Sc=0.3, n=2, M=2, So=0.2, K=5 and Pr=0.71

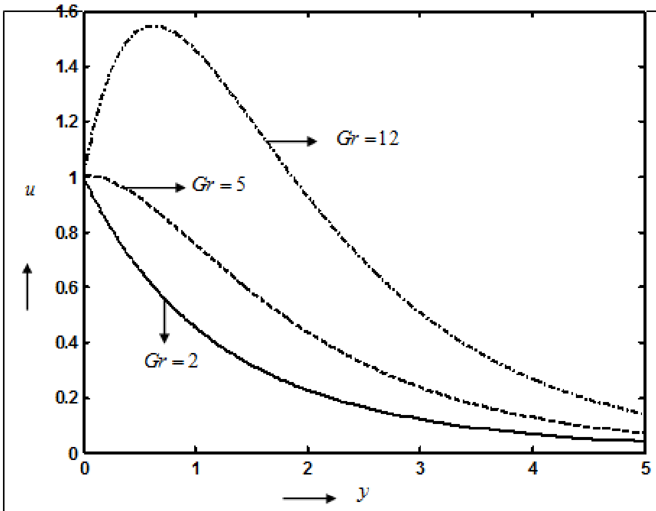

Fig.10 The effect of Gr on the velocity field u with m=0.2, Gc=0.2, S=2, Sc=0.3, n=2, M=2, So=0.2, K=5 and Pr=0.71

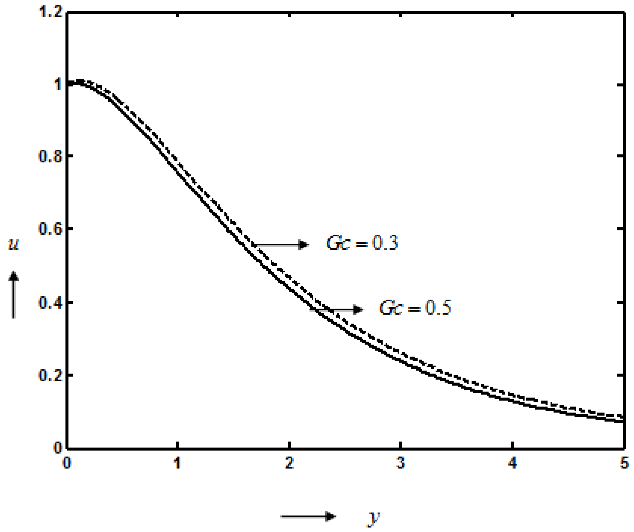

Fig.11 The effect of Gc on the velocity field u with Gr=5, Sc=0.3, S=2, So=0.2, n=2, m=0.2, M=2, K=5 and Pr=0.71

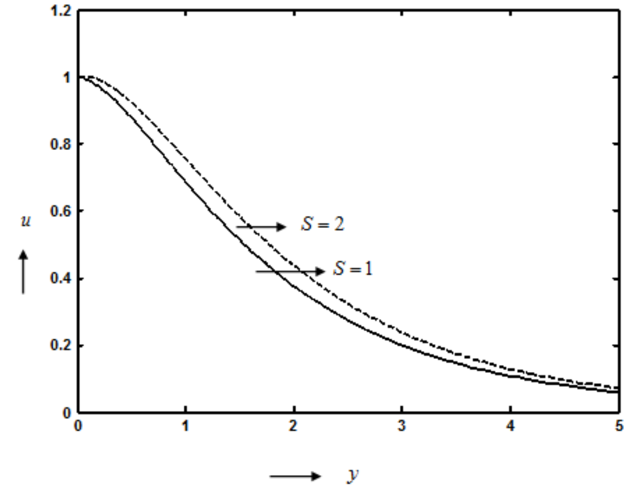

Fig.12 The effect of S on the velocity field u with Gr=5, Sc=0.3, Gc=0.3, So=0.2, n=2, m=0.2, M=2, K=5 and Pr=0.71

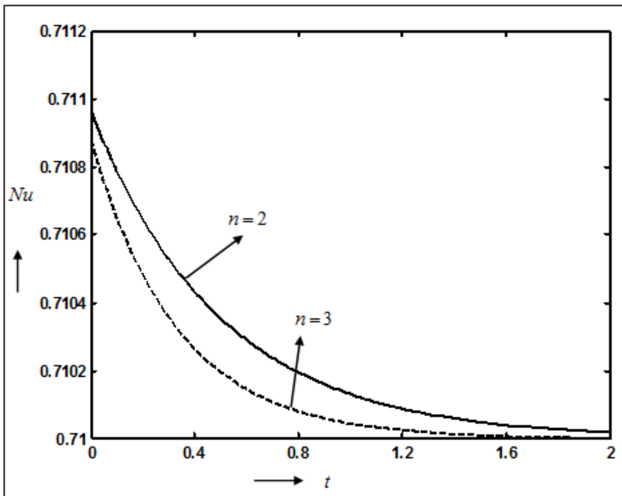

Fig.13 The effect of n on the Nusselt number with Pr=0.71

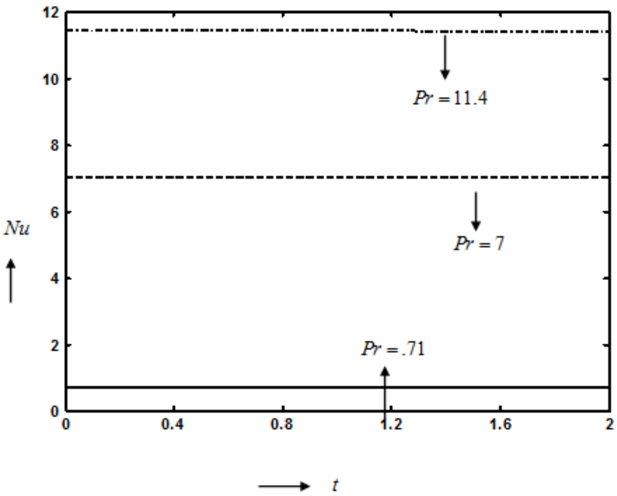

Fig.14 The effect of Pr on the Nusselt number with n=2

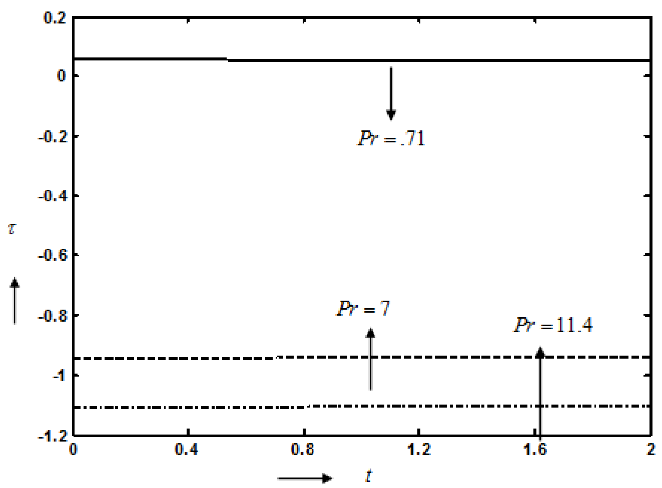

Fig.15 The effect of Pr on the Skin friction with Gr =5, Gc=0.3, S=0.01, Sc=0.3, So=0.2, n=2, m=0.2, M=2 and K=5.

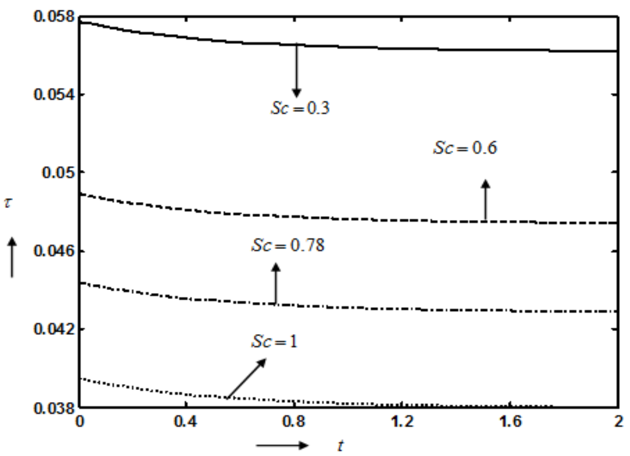

Fig.16 The effect of Sc on the Skin friction with Gr=5, Gc=0.2, S=2, So=0.2, n=2, m=0.2, M=2, K=5 and Pr=0.71

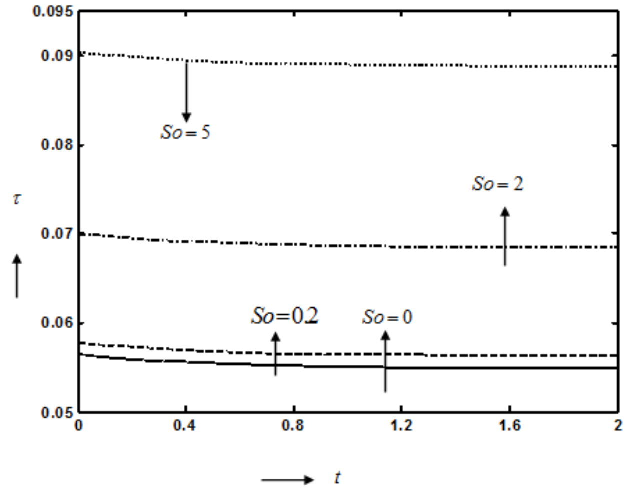

Fig.17 The effect of So on the Skin friction with Gr=5, Gc=0.2, S=2, Sc=0.3, n=2, m=0.2, M=2, K=5 and Pr=0.71

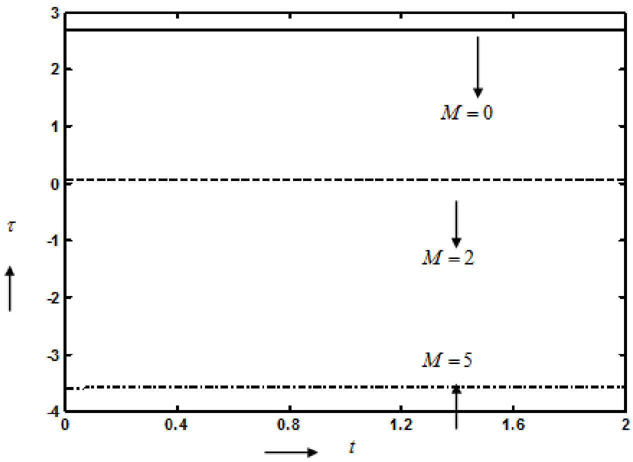

Fig.18 The effect of M on the Skin friction with Gr=5, Gc=0.2, S=2, Sc=0.3, n=2, m=0.2, So=0.2, K=5 and Pr=0.71

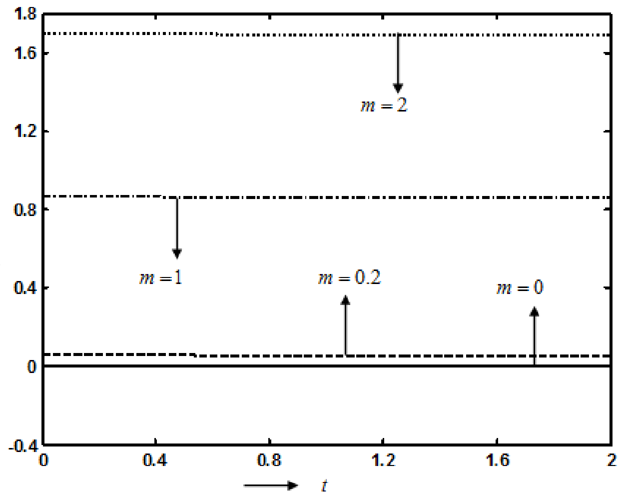

Fig.19 The effect of m on the Skin friction with Gr=5, Gc=0.2, S=2, Sc=0.3, n=2, M=2, So=0.2, K=5 and Pr=0.71

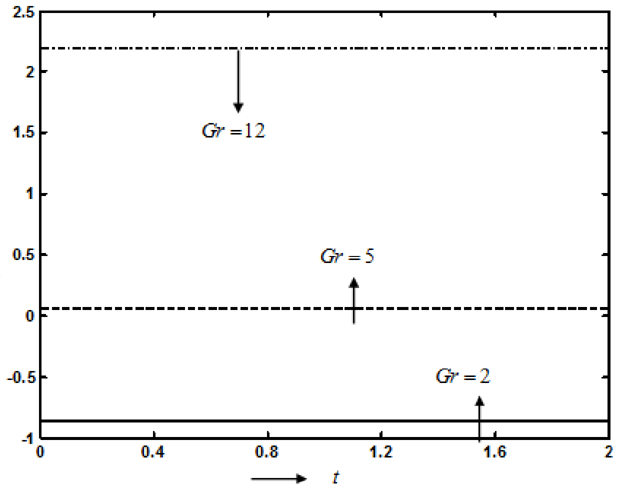

Fig.20 The effect of Gr on the Skin friction with m=0.2, Gc=0.2, S=2, Sc=0.3, n=2, M=2, So=0.2, K=5 and Pr=0.71

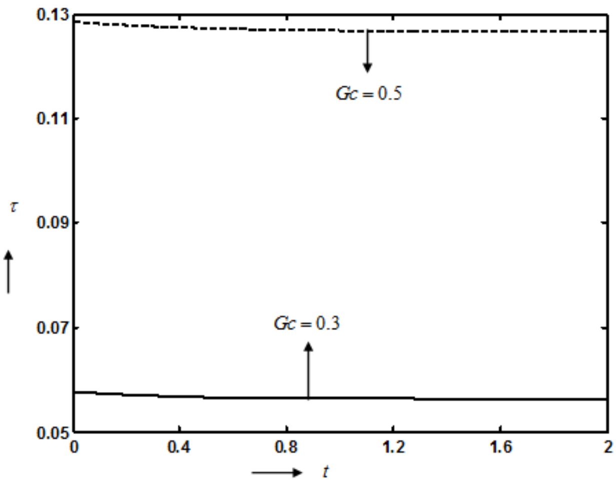

Fig.21 The effect of Gc on the Skin friction with Gr=5, Sc=0.3, S=2, So=0.2, n=2, m=0.2, M=2, K=5 and Pr=0.71

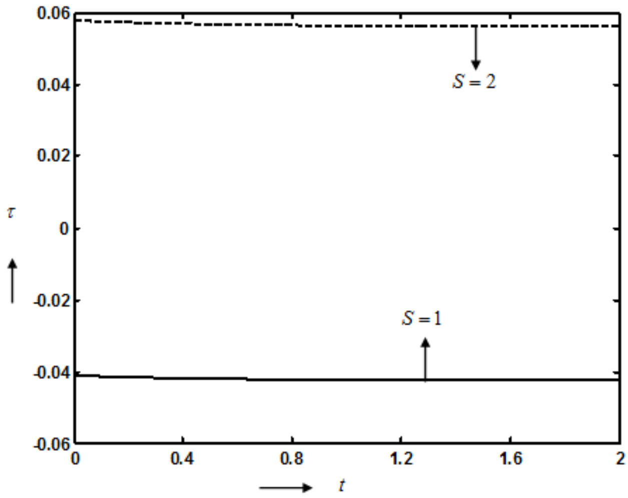

Fig.22 The effect of S on the Skin friction with Gr=5, Sc=0.3, Gc=0.3, So=0.2, n=2, m=0.2, M=2, K=5 and Pr=0.71

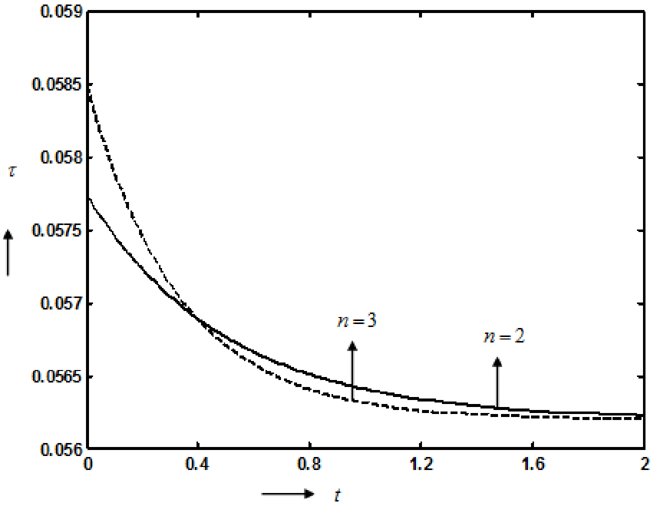

Fig.23 The effect of n on the Skin friction with Gr=5, Sc=0.3,

IV. Conclusions

We concluded that the temperature and velocity decreases with increase in Prandtl number Pr, Schmidt number Sc and Hartmann number M. The velocity increases with increase in Sorret number So, Hall parameter *m*, Grashoff number Gr, modified Grashof number Gc and Stratification factor S. The Concentration C increases with increase in Pr, So and decreases with increase in Sc. The skin friction in general decreases with increase in the parameters.

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Appendix

$$b = \frac{Pr + \sqrt{Pr^2 - 4nPr}}{2}\ , \ d = \frac{Sc + \sqrt{Sc^2 - 4nSc}}{2}, \ , \ A_1 = \frac{Sc}{n}\left[1 + \frac{S_0 ScPr}{Pr - Sc}\right]$$

$$A_2 = \frac{S_0 Scb^2\left(1 - \frac{Pr}{n}\right)}{b^2 - bSc + nSc}, \ A_3 = \frac{S_0 ScPr^2}{Pr^2 - ScPr + nSc}\left(\frac{Sc}{Pr - Sc} + \frac{Pr}{n}\right)$$

$$F_1 = \frac{GcB_2 - Gr}{Pr^2 - (1-S)Pr - \left(\frac{M^2}{1+m^2} + \frac{1}{K}\right)}, \quad F_2 = \frac{GcB_1}{Sc^2 - (1-S)Sc - \left(\frac{M^2}{1+m^2} + \frac{1}{K}\right)},$$

$$F_3 = \frac{PrF_1 - Pr\left(\frac{Gr}{n}\right) + GcA_3}{Pr^2 - (1-S)Pr - \left(\frac{M^2}{1+m^2} + \frac{1}{K} - n\right)} \quad , \quad F_4 = \frac{F_2 Sc + A_1 Gc}{Sc^2 - (1-S)Sc - \left(\frac{M^2}{1+m^2} + \frac{1}{K} - n\right)}$$

$$F_5 = \frac{GcA_2 - Gr(1 - \frac{Pr}{n})}{b^2 - (1-S)b - \left(\frac{M^2}{1+m^2} + \frac{1}{K} - n\right)} \ , \ F_6 = \frac{Gc(1 - A_1 + A_2 + A_3)}{d^2 - (1-S)d - \left(\frac{M^2}{1+m^2} + \frac{1}{K} - n\right)}$$

$$F_7 = 1 - \frac{r_2}{n}(1 - F_1 + F_2) - F_3 + F_4 - F_5 + F_6$$

$$s_2 = \frac{(1-S) - \sqrt{(1-S)^2 + 4\left(\frac{M^2}{1+m^2} + \frac{1}{K} - n\right)}}{2} \ , \ r_2 = \frac{(1-S) + \sqrt{(1-S)^2 + 4\left(\frac{M^2}{1+m^2} + \frac{1}{K}\right)}}{2}$$