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On $\hat{g}^*$s –connected and $\hat{g}^*$s-compact spaces

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Abstract: *In this paper, we introduce $\hat{g}^*$s –connected spaces and $\hat{g}^*$s-compact spaces.Their basic characteristics are studied.*
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I. Introduction

The compactness and connectedness are fundamental notions of general topology. Basic properties of compactness and connectedness have been investigated by many researchers. Generalisation of closed sets leads to attempts by Mathematicians to generalize these notions of compactness and connectedness. In 1974, P.Das [2,3] defined the concept of semi-connectedness in topology and investigated its characteristics. In 1981, Dorsett [4] introduced and studied the concept of semi-compactness. In 1991, Balachandran.K, Sundaram.P and Maki, J [1] introduced a class of compact spaces called GO – compact spaces. In the course of these attempts many stronger and weaker forms of compactness and connectedness have been introduced. The authors [6] introduced $\hat{g}^*$s –closed sets in 2014. In this chapter, we make an attempt to generalize compactness and connectedness using $\hat{g}^*$s –closed sets to obtain a weaker form of compactness and connectedness and study the basic characteristics.

II. Preliminaries

Definition 2.1: [5] Two subsets A and B of a space X is called separated if cl(A) ∩ B= ∅ and A ∩ cl(B) = ∅.
Definition 2.2: [2,3] Two subsets A and B of a space X is called semi separated if scl(A) ∩ B= ∅ and A ∩ scl(B) = ∅.

Definition 2.3: A topological space (X,τ) is called connected if X cannot be expressed as the union of two non empty disjoint open sets. Or equivalently, a topological space (X,τ) is called connected if X cannot be expressed as the union of two non empty disjoint sets A and B satisfying (A ∩ cl(B)) ∪ (cl(A) ∩ B) = ∅.Suppose X can be so expressed, then X is called disconnected and we write X = A⊖B and A⊖B is called separation of X.

Definition 2.4: [2,3] A topological space (X,τ) is called semi connected if X cannot be expressed as the union of two non empty disjoint semi open sets. Or equivalently, a topological space (X,τ) is called semi connected if X cannot be expressed as the union of two non empty disjoint sets A and B satisfying (A ∩ scl(B)) ∪ (scl(A) ∩ B) = ∅.Suppose X can be so expressed, then X is called semi -disconnected and we write X = A⊖B and A⊖B is called semi-separation of X.
Definition 2.5: [6] A subset A of a topological space (X,τ) is called a $\hat{g}^*$s –closed set if scl(A) ⊆ U whenever A ⊆ U and U is $\hat{g}$-open.
Definition 2.6: [7] A function f: (X,τ) → (Y,σ) is $\hat{g}^*$s – continuous if f^{-1}(U) is $\hat{g}^*$s –closed in X for each closed set U in Y.

III. $\hat{g}^*$S –Connected Spaces

This section deals with the basic characteristics of $\hat{g}^*$s –connected spaces.
Definition 3.1: Two subsets A and B of a space X is called $\hat{g}^*$s -separated if $\hat{g}^*$scl(A) ∩ B= ∅ and A ∩ $\hat{g}^*$scl(B) = ∅.
Proposition 3.2: If A and B are separated, then they are $\hat{g}^*$s – separated.
Proof: Let A and B be separated in (X,τ). Then (A ∩ cl(B)) ∪ (cl(A) ∩ B) = ∅. Since every closed set is $\hat{g}^*$s –closed, we have $\hat{g}^*$scl(A) ⊆ scl(A) ⊆ cl(A) and $\hat{g}^*$scl(B) ⊆ scl(B) ⊆ cl(B). Therefore (A ∩ $\hat{g}^*$s cl(B)) ∪ ($\hat{g}^*$s cl(A) ∩ B) = ∅ and hence, A and B are $\hat{g}^*$s –separated.

Proposition 3.3: If A and B are semi separated, then they are $\hat{g}^*$s –separated.
Proof: Proof similar to previous Proposition.

Remark 3.4: The following example shows that $\hat{g}^*$s –separatedness need not imply separatedness and $\hat{g}^*$s –separatedness need not imply semi separatedness.

Example 3.5: Let (X,τ) be a topological space where X= {a, b, c, d} with τ = { ∅, X, {a}, {a,b,c},{a,d}} and τ^c= {∅, X,{b,c,d},{d},{b,c}}. Then SC(X,τ) = {∅, X, {b, c, d},{c, d},{ b, d},{ b, c},{d},{ c},{ b}} and $\hat{g}^*$s C(X,τ) = {∅, X, {b, c, d},{a, c, d} , {a, b, d}, {c, d},{ b, d},{ b, c},{d},{ c},{ b}}.

Let A = {a, b,d} and B = {c}. Then $\hat{g}^*$scl(A) = {a, b, d} and $\hat{g}^*$scl(B) ={ c}. As A ∩ $\hat{g}^*$scl(B) =∅ and $\hat{g}^*$scl(A)∩B = ∅, A and B are $\hat{g}^*$s –separated. But scl(A) = X and scl(B) = { c}. As A ∩ scl(B) =∅ and scl(A)∩B ≠ ∅, A and B are not semi separated. And also cl(A) = X and cl(B) = {b, c}.As A ∩ cl(B) ≠ ∅ and cl(A)∩B ≠ ∅, A and B are not separated. Thus $\hat{g}^*$s –separatedness need not imply separatedness and $\hat{g}^*$s –separatedness need not imply semi separatedness.

Definition 3.6: A topological space (X,τ) is called $\hat{g}^*$s -connected if X cannot be expressed as the union of two non empty disjoint open sets. Or equivalently, a topological space (X,τ) is called $\hat{g}^*$s -connected if X cannot be expressed as the union of two non empty disjoint sets A and B satisfying (A ∩ $\hat{g}^*$s cl(B)) ∪ ($\hat{g}^*$s cl(A) ∩ B) = ∅.Suppose X can be so expressed, then X is called $\hat{g}^*$s -disconnected and we write X = A⊖B and A⊖B is called $\hat{g}^*$s -separation of X.

Remark 3.7: A subset A of X is $\hat{g}^*$s –connected if it is $\hat{g}^*$s –connected as a subspaces.

Proposition 3.8: (i) Every $\hat{g}^*$s –connected space is connected.

(ii) Every $\hat{g}^*$s –connected space is semi connected.

Proof: Follow from Definitions 2.3, 2.4 and 3.6

Remark 3.9: The topological space given below is a $\hat{g}^*$s –disconnected one.

Example 3.10: Let (X,τ) be a topological space where X = X= {a, b, c} with τ = { ∅, X, {a}, {b,c}} and $\hat{g}^*$s C(X,τ) = {∅, X, {b, c },{a}}.The (X,τ) is $\hat{g}^*$s –disconnected.

Remark 3.11: The topological space given below is a $\hat{g}^*$s – connected one.

Example 3.12: Let (X,τ) be a topological space where X = X= {a, b, c} with τ = { ∅, X, {a,b}} and $\hat{g}^*$s C(X,τ) = {∅, X, {b, c },{a,c},{c}}.The (X,τ) is $\hat{g}^*$s –connected.

Remark 3.13: The following two examples show that

1. Semi-connectedness need not imply $\hat{g}^*$s – connectedness
2. Connectedness need not imply $\hat{g}^*$s – connectedness
3. Connectedness need imply semi– connectedness

Example 3.14: Let (X,τ) be a topological space where X = {a, b, c, d} with τ = {∅, X, {a}, {a, b}, {a, b, c}} and τ^c = {∅, X, {b, c, d}, {c, d}, {d}}. Then SC(X,τ) = { ∅, X, {b, c, d}, {c, d}, {b, d}, {b, c}, { d}, {c}, {b }};$\hat{g}$C(X,τ) = { ∅, X, {b, c, d}, {c, d}, { d}} and $\hat{g}^*$s C(X,τ) = { ∅, X, {b, c, d}, {a, c, d}, {a, b, d}, {c, d}, {b, d}, {b, c}, {a, d }, { d}, {c}, {b }}. This topological space (X,τ) is connected as well as semi connected but not $\hat{g}^*$s – connected.

Example 3.15: Let (X,τ) be a topological space where X = {a, b, c, d} with τ = {∅, X, {a}, {b}, {a, b}} and τ^c = {∅, X, {b, c, d}, {a,c, d}, {c, d}}. Then SC(X,τ) = { ∅, X, {b, c, d}, {a, c, d}, {c, d}, {b, d }, { b, c }, {a, d }, {a, c } , { d}, {c}, {b }} and $\hat{g}^*$s C(X,τ) = { ∅, X, {b, c, d}, {a, c, d}, {a, b, d}, {a, b, c}, {c, d}, {b, d}, {b, c}, {a, d }, {a, c }, { d}, {c}, {b },{a } }. This topological space (X,τ) is connected but neither semi connected nor $\hat{g}^*$s – connected.

Proposition 3.16: The following statements are equivalent in a topological space (X,τ).

1. X is $\hat{g}^*$s – connected.
2. X cannot be expressed as the union of two nonempty disjoint sets A and B such that A is $\hat{g}^*$s-open and b is $\hat{g}^*$s-open.
3. X contains no non empty proper subset which is both $\hat{g}^*$s-open and b is $\hat{g}^*$s-closed. (i.e., X and ∅ are the only subsets of X which are both $\hat{g}^*$s-open and is $\hat{g}^*$s-closed.)
4. Each $\hat{g}^*$s –continuous map of X into a discrete space Y with atleast two points is a constant map.

Proof: (i) ⇒(ii)

Given, X is $\hat{g}^*$s – connected. Suppose that X can be expressed as the union of two non empty disjoint sets A and B such that A and B are $\hat{g}^*$s-open. Then A ⊆ B^c and so $\hat{g}^*$scl(A) ⊆ $\hat{g}^*$scl(B^c) = B^c. Therefore $\hat{g}^*$scl(A) ∩ B = ∅. Similarly A ∩ $\hat{g}^*$scl(B) = ∅. Hence (A ∩ $\hat{g}^*$scl(B)) ∪ ($\hat{g}^*$scl(A) ∩ B)) = ∅ which is a contradiction to the fact that X is $\hat{g}^*$s – connected. Hence X cannot be expressed as the union of two nonempty disjoint sets A and B such that A is $\hat{g}^*$s-open and $\hat{g}^*$s-closed.

(ii) ⇒(iii)

Suppose, X cannot be expressed as the union of two nonempty disjoint $\hat{g}^*$s-open sets A and B. Suppose that X contains a nonempty proper subset which is both $\hat{g}^*$s-open and b is $\hat{g}^*$s-closed. Then X = A ∪ A^c where

A is $\hat{g}^*$s-open and A^c is $\hat{g}^*$s-open and are disjoint which is a contradiction to our assumption. Therefore X contains no nonempty proper subset which is both $\hat{g}^*$s-open and $\hat{g}^*$s-closed.

(iii) ⇒(i)

Given, X contains no nonempty proper subset which is both $\hat{g}^*$s-open and b is $\hat{g}^*$s-closed. Suppose that X is not $\hat{g}^*$s – connected. Then X can be expressed as the union of two nonempty disjoint sets A and B such that $(A \cap \hat{g}^*scl(B)) \cup (\hat{g}^*scl(A) \cap B)) = \emptyset$. Then $A = B^c$ and $B = A^c$ and also $\hat{g}^*scl(A) \cap B = \emptyset$ and $A \cap \hat{g}^*scl(B) = \emptyset$. Since $\hat{g}^*scl(A) \cap B = \emptyset$, we have $\hat{g}^*scl(A) \subseteq B^c$. Hence $\hat{g}^*scl(A) \subseteq A$. Therefore A is $\hat{g}^*$s-closed. Similarly B is $\hat{g}^*$s-closed. Since $A = B^c$, A is $\hat{g}^*$s-open. Thus there exists a nonempty proper set A which is both $\hat{g}^*$s-open and A is $\hat{g}^*$s-closed which is a contradiction to (iii). Hence X is $\hat{g}^*$s – connected.

(iii) ⇒(iv)

Let f: X → Y be a $\hat{g}^*$s –continuous map [7]. Then X is covered by $\hat{g}^*$s-open and $\hat{g}^*$s-closed covering $\{\{f^{-1}(y)\} : y \in Y\}$ is a covering of X such that $\{f^{-1}(y)\}$ is both $\hat{g}^*$s-open and $\hat{g}^*$s-closed. By assumption, $\{f^{-1}(y)\} = \emptyset$ or X for each $y \in Y$. If $\{f^{-1}(y)\} = \emptyset$ for each $y \in Y$, then f fails to be a map. Then there exists only one point $y \in Y$ such that $\{f^{-1}(y)\} = X$. This shows that f is a constant map.

(iv) ⇒(iii)

Let O be both $\hat{g}^*$s-open and $\hat{g}^*$s-closed in X. Suppose $O \neq \emptyset$. Let f:X → Y be a $\hat{g}^*$s –continuous map defined by $f(O) = \{y\}$ and $f(O^c) = \{w\}$ for some distinct points y and w in Y. By assumption f is constant. Therefore we have $y = w$ and so O = X.

Proposition 3.17: If A is $\hat{g}^*$s – connected subset of a topological space (X,τ) which has the $\hat{g}^*$s –separation $X = C \ominus D$, then $A \subseteq C$ or $A \subseteq D$.

Proof: Suppose that X has the $\hat{g}^*$s –separation $X = C \ominus D$. Then $X = C \cup D$ where C and D are two nonempty disjoint sets such that $(C \cap \hat{g}^*scl(D)) \cup (\hat{g}^*scl(C) \cap D)) = \emptyset$. Since $X = C \cup D$ and $C \cap D = \emptyset$, we have $C = D^c$ and $D = C^c$. Now, we have $((C \cap A) \cap \hat{g}^*scl(D\cap A)) \cup (\hat{g}^*scl(C \cap A) \cap (D\cap A)) \subseteq (C \cap \hat{g}^*scl(D)) \cup (\hat{g}^*scl(C) \cap D)) = \emptyset$. Also $A = (C \cap A) \cup (D\cap A)$ and $(C \cap A) \cap (D\cap A) = \emptyset$. Hence $A = (C \cap A) \ominus (D\cap A)$ is a $\hat{g}^*$s – separation of A. Since A is $\hat{g}^*$s – connected, either $C \cap A = \emptyset$ and $D\cap A = \emptyset$. Consequently, $A \subseteq C^c$ or $A \subseteq D^c$. Hence $A \subseteq D$ or $A \subseteq C$.

Proposition 3.18: If A is $\hat{g}^*$s – connected and $A \subseteq B \subseteq \hat{g}^*scl(A)$, then B is $\hat{g}^*$s – connected.

Proof: Suppose that B is not $\hat{g}^*$s – connected. Then $B = C \cup D$ where C and D are two nonempty disjoint sets such that $(C \cap \hat{g}^*scl(D)) \cup (\hat{g}^*scl(C) \cap D)) = \emptyset$. By 3.17, we have $A \subseteq C$ or $A \subseteq D$, since A is $\hat{g}^*$s – connected and $A \subseteq B$. Suppose $A \subseteq C$. Then $D = D \cap B \subseteq (D \cap \hat{g}^*scl(A)) \subseteq (D \cap \hat{g}^*scl(C)) = \emptyset$. Thus $D = \emptyset$. Similarly we can prove $C = \emptyset$ if $A \subseteq D$ which is a contradiction to the fact that C and D are nonempty. Hence B is $\hat{g}^*$s – connected.

Proposition 3.19: If A is a $\hat{g}^*$s – connected subset of a topological space (X,τ) which has the separation $X = C \ominus D$, then $A \subseteq C$ or $A \subseteq D$.

Proof: Suppose that X has the $\hat{g}^*$s –separation $X = C \ominus D$. Then $X = C \cup D$ where C and D are two nonempty disjoint sets such that $(C \cap cl(D)) \cup (cl(C) \cap D)) = \emptyset$. Since $X = C \cup D$ and $C \cap D = \emptyset$, we have $C = D^c$ and $D = C^c$. We know that $\hat{g}^*scl(D) \subseteq cl(D)$ and $\hat{g}^*scl(C) \subseteq cl(C)$. Then $C \cap \hat{g}^*scl(D) \subseteq C \cap cl(D)$ and $D \cap \hat{g}^*scl(C) \subseteq D \cap cl(C)$. Therefore, $(C \cap \hat{g}^*scl(D)) \cup (\hat{g}^*scl(C) \cap D)) \subseteq (C \cap cl(D)) \cup (cl(C) \cap D)) = \emptyset$.Now, we have $((C \cap A) \cap \hat{g}^*scl(D\cap A)) \cup (\hat{g}^*scl(C \cap A) \cap (D\cap A)) \subseteq (C \cap \hat{g}^*scl(D)) \cup (\hat{g}^*scl(C) \cap D)) = \emptyset$. Also $A = (C \cap A) \cup (D\cap A)$ and $(C \cap A) \cap (D\cap A) = \emptyset$. Hence $A = (C \cap A) \ominus (D\cap A)$ is a $\hat{g}^*$s –separation of A. Since A is $\hat{g}^*$s – connected, either $C \cap A = \emptyset$ and $D\cap A = \emptyset$. Consequently, $A \subseteq C^c$ or $A \subseteq D^c$. Hence $A \subseteq D$ or $A \subseteq C$.

Proposition 3.20: If A is $\hat{g}^*$s – connected and $A \subseteq B \subseteq cl(A)$, then B is connected.

Proof: Suppose that B is not connected. Then $B = C \cup D$ where C and D are two nonempty disjoint sets such that $(C \cap cl(D)) \cup (cl(C) \cap D)) = \emptyset$. i.e., $C \ominus D$ is a separation for B. By 3.19, we have $A \subseteq C$ or $A \subseteq D$, since A is $\hat{g}^*$s – connected and $A \subseteq B$. Suppose $A \subseteq C$. Then $D = D \cap B \subseteq (D \cap cl(A)) \subseteq (D \cap cl(C)) = \emptyset$. Thus $D = \emptyset$. Similarly we can prove $C = \emptyset$ if $A \subseteq D$ which is a contradiction to the fact that C and D are nonempty. Hence B is connected.

Corollary 3.21: The closure of a $\hat{g}^*$s – connected set is a $\hat{g}^*$s – connected set.

Proof: Let A be a $\hat{g}^*$s – connected set. Let B = cl(A) in Proposition 3.20, we have B = cl(A) is $\hat{g}^*$s – connected.

Corollary 3.22: If B is dense subset of a topological space X and if B is $\hat{g}^*$s – connected, then X is $\hat{g}^*$s – connected.

Proof: Since B is dense in X, cl(B) = X. By Proposition 3.21, X is $\hat{g}^*$s – connected.

Proposition 3.23: The union of any family of $\hat{g}^*$s – connected sets having a non empty intersection is $\hat{g}^*$s – connected.

Proof: Let ∇ be an index set and $\alpha \in \nabla$. Let $\{ A_\alpha : \alpha \in \nabla \}$ be a family of $\hat{g}^*$s – connected subsets of a topological space X. Let A = $\cup A_\alpha$ where each A_α is $\hat{g}^*$s – connected with $\cap A_\alpha \neq \emptyset$. So, let p ∈ $\cap A_\alpha$. Suppose that A is not $\hat{g}^*$s – connected. Then A = C ⊖ D is a $\hat{g}^*$s – separation of A. i.e., A = C ∪ D and C∩ D = ∅ with C ∩ $\hat{g}^*$scl(D) = ∅ and $\hat{g}^*$scl(C) ∩ D) = ∅, by definition 3.6. Then p is in one of the sets C or D. Suppose p ∈ C. Each A_α is $\hat{g}^*$s – connected and $A_\alpha \subseteq$ C ∪ D. Then by 3.19, either $A_\alpha \subseteq$ C or $A_\alpha \subseteq$ C ∪ D for each α. So A_α can not lie in D because A_α p of C. Hence $A_\alpha \subseteq$ C for every α, so that $\cup A_\alpha \subseteq$ C, contradicting the fact that D is non empty.

Proposition 3.24: Let X be a topological space such that each pair of points in X is contained in a $\hat{g}^*$s – connected subset of X. Then X is g^*s – connected.

Proof: Let *x* be a given point of X and $y \neq x$ be an arbitrary point in X. By hupothesis, there exists a $\hat{g}^*$s – connected subset C_y containing *x* and *y*. Also X = $\cup\{C_y : y \in X\}$ and $\cap\{C_y : y \in X\} \neq \emptyset$. Since each C_y is ĝ*s – connected, by 3.23, it follows that X is ĝ*s – connected.

Proposition 3.25: (i) If f: X → Y is a ĝ*s- continuous surjection and X is ĝ*s – connected, then Y is connected.

(ii) If f: X → Y is a ĝ*s- irresolute surjection and X is ĝ*s – connected, then Y is ĝ*s – connected.

Proof:(i) Suppose that Y is not connected. Let Y = A ∪ B where A and B are disjoint nonempty open sets in Y. Since f is ĝ*s- continuous and onto, X = f^{-1}(A) ∪ f^{-1}(B) where f^{-1}(A) and f^{-1}(B) are disjoint non empty ĝ*s-open in X which is a contradiction to the fact that X is ĝ*s – connected. Hence Y is connected.

(ii) Suppose that Y is not ĝ*s- connected. Let Y = A ∪ B where A and B are disjoint non empty ĝ*s – open sets in Y. Since f is ĝ*s-irresolute and onto, X = f^{-1}(A) ∪ f^{-1}(B) where f^{-1}(A) and f^{-1}(B) are disjoint non emptyĝ*s-open in X which is a contradiction to the fact that X is ĝ*s – connected. Hence Y is ĝ*s – connected.

IV. ĝ*S –Compact Spaces

This section deals with the basic characteristics of ĝ*s –compact spaces.

Definition 4.1: A non empty collection $\{ A_\alpha : \alpha \in \nabla \}$ of ĝ*s –open sets in a topological space (X,τ) is called a ĝ*s –open cover of a subset B of X if B ⊆ $\cup\{ A_\alpha : \alpha \in \nabla \}$ holds.

Definition 4.2: A topological space (X,τ) is ĝ*s –compact if every ĝ*s –open cover of X has a finite subcover.

Definition 4.3: A subset B of a topological space (X,τ) is a ĝ*s –compact relative to X, if for every collection $\{ A_\alpha : \alpha \in \nabla \}$ of ĝ*s –open subsets of X such that B ⊆ $\cup\{ A_\alpha : \alpha \in \nabla \}$, there exists a finite subset ∇_0 of ∇ such that B ⊆ $\cup\{ A_\alpha : \alpha \in \nabla_0 \}$.

Or equivalently, a subset B of a topological space (X,τ) is ĝ*s –compact relative to X, if every ĝ*s –open cover of B has a finite subcover as a subspace.

Definition 4.4: A subset B of a topological space X is said to be ĝ*s –compact if B is ĝ*s –compact as a subspace of X.

Proposition 4.5: Every ĝ*s –compact space is compact.

Proof: Let (X,τ) be a ĝ*s –compact space. Let $\{ A_\alpha : \alpha \in \nabla \}$ be an open cover of X. Then X =$\cup\{ A_\alpha : \alpha \in \nabla \}$. Since every open set is ĝ*s –open, $\{ A_\alpha : \alpha \in \nabla \}$ is a ĝ*s –open cover of X. Since X is ĝ*s –compact, it ha s a finite sub cover, say $\{A_1, A_2, A_3, ……., A_n\}$. Hence, X is compact.

Proposition 4.6: Every ĝ*s –closed subset of a ĝ*s –compact space is ĝ*s –compact relative to X.

Proof: Let A be a ĝ*s –closed subset of a ĝ*s –compact space (X,τ). Then A^c is ĝ*s –open in (X,τ). Let M = $\{ G_\alpha : \alpha \in \nabla \}$ be a cover of A by ĝ*s –open subsets of X. Then M* = M ∪ A^c is a ĝ*s –open cover of X. i.e., X = ($\cup\{ G_\alpha : \alpha \in \nabla \}$) ∪ Ac. Since X is ĝ*s –compact, M* is reducible to a finite cover of X, say X = $G_{\alpha_1} \cup G_{\alpha_2} \cup G_{\alpha_3} \cup$ ……………………… $\cup G_{\alpha_m} \cup A^c$ where $G_{\alpha_k} \in$ M. Since A and A^c are disjoint, we have A ⊆ $G_{\alpha_1} \cup G_{\alpha_2} \cup G_{\alpha_3} \cup$ ……………………… $\cup G_{\alpha_m}$ where $G_{\alpha_k} \in$ M. Thus any ĝ*s –open cover M of A contains a finite sub cover. Hence, A is ĝ*s –compact relative to X.

Proposition 4.7: Every closed subset of a ĝ*s –compact space is ĝ*s –compact relative to X.

Proof: Let A be a closed subset of a ĝ*s –compact space (X,τ). Then A^c is open in (X,τ). Let M = $\{ G_\alpha : \alpha \in \nabla \}$ be a cover of A by ĝ*s –open subsets of X. Then M* = M ∪ A^c is a ĝ*s –open cover of X. i.e., X = ($\cup\{ G_\alpha : \alpha \in \nabla \}$) $\cup$ Ac. Since X is g*s –compact, M* is reducible to a finite cover of X, say X = *Gα1∪ Gα2 ∪ Gα3∪* ……………………… $\cup G_{\alpha_m} \cup A^c$ where $G_{\alpha_k} \in$ M. Since A and A^c are disjoint, we have A ⊆ $G_{\alpha_1} \cup G_{\alpha_2} \cup G_{\alpha_3} \cup$ ……………………… $\cup G_{\alpha_m}$ where $G_{\alpha_k} \in$ M. Thus any ĝ*s –open cover M of A contains a finite sub cover. Hence, A is ĝ*s –compact relative to X.

Proposition 4.8: (i) If f: X → Y is a ĝ*s- continuous surjection and X is ĝ*s – compact, then Y is compact.

(ii) If f: X → Y is a $\hat{g}^*$s- irresolute and a subset B of X is $\hat{g}^*$s – compact relative to X, then the image f(B) is $\hat{g}^*$s – compact relative to Y.

Proof: (i) Let f: X → Y be a $\hat{g}^*$s- continuous map from a $\hat{g}^*$s – compact space X onto a topological space Y. Let $\{ A_\alpha : \alpha \in \nabla \}$ be an open cover of Y. Then $\{f^{-1}(A_\alpha) : \alpha \in \nabla \}$ is a $\hat{g}$*s –open cover of X. Since X is $\hat{g}^*$s – compact, it has a finite sub cover, say $\{f^{-1}(A_1), f^{-1}(A_2), f^{-1}(A_3), \ldots\ldots\ldots\ldots, f^{-1}(A_n)\}$. Since f is onto, $\{ A_1, (A_2, A_3, \ldots\ldots\ldots\ldots, A_n \}$ is a cover of Y and so Y is compact.

(ii) Let $\{ A_\alpha : \alpha \in \nabla \}$ be any collection of $\hat{g}$*s –open subsets of Y such that f(B) $\subseteq \bigcup\{ A_\alpha : \alpha \in \nabla \}$. Then B $\subseteq \bigcup\{f^{-1}(A_\alpha) : \alpha \in \nabla \}$ holds. By hypothesis, there exists a finite subset ∇_0 of ∇ such that B $\subseteq \bigcup\{f^{-1}(A_\alpha) : \alpha \in \nabla0 \}$. Therefore, we have f(B) ⊆ { □□: □ ∈∇0 } which shows that f(B) is g*s –compact relative to Y.

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