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Study of Fuzzy Set Theory and Its Applications

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Abstract: *The purpose of this paper is to study fuzzy sets and their real applications. Also, we study some properties of fuzzy sets. As an application of fuzzy sets, we solve some test problems and their solutions are represented graphically using Mathematica.*
Keywords: *Fuzzy set, Membership function, Fuzzy Norm, α-Cuts, Support.*

I. Introduction

The sets are first studied formally by German Mathematician Gorge Cantor [1845-1918]. His theory of sets met much resistance during his life time. Now a day, many Mathematicians believe that it is possible to express most of the Mathematics in the language of set theory. In the classical set theory, it is not allowed that an element is in a set and not in the set at the same time. Therefore, many real-world application problems cannot be described and handled by the classical set theory, including all those involving elements with only partial membership of a set. So in this connection, in the year 1965, Zadeh published his Pioneering paper on fuzzy sets and many examples have been supplied to understand the concept of fuzzy sets. Furthermore, in the years 1973 and 1975, Zadeh, explain the theory of fuzzy sets that result from the extension as well as a fuzzy logic based on the set theory. After that, Zimmermann 1993) introduced recent application of fuzzy set theory which simplifies the concepts of fuzzy sets. Fuzzy set theory accepts partial memberships, and therefore, in a sense generalizes the classical set theory to some extent. Recently, many researchers studied the consequences of "fuzzifying" set theory and found the applications of fuzzy logic in the area of science, engineering and mathematical biology; it allows controlling complex processes based on small number of expert rules. In the connection of this we study fuzzy sets and their real applications.

We organize this paper as follows: In section 2, we study definition and some examples of fuzzy set and also these are represented graphically. The section 3 is devoted for some basic concepts associated with fuzzy set. In the section 4, we study basic operations of fuzzy sets. The last section is devoted for the norms of fuzzy sets. In the next section, we study definitions and some examples of fuzzy set.

II. Classical Set and Fuzzy Set

To understand what fuzzy set is, first consider what is meant by classical set.

- A classical set is a container that wholly includes or excludes any given element.
- Let U be the universe of discourse or universal set, which contains all the possible elements of concern in each particular context or application. Recall that a classical (crisp) set ***A,*** or simply a set A, in the universe of discourse U can be defined by listing all of its members (the list method) or by specifying the properties that must be satisfied by the members of the set (the rule method). The ***list method*** can be used only for finite sets and is therefore of limited use. The ***rule method*** is more general. In the rule method, a set A is represented as

***A* = {*x* $\in$ U /*x* meets some conditions)**

There is yet, a third method to define a set A-***the membership method***, which introduces a zero- one membership function.

Definition 2.1: M***embership Function***

A Membership *Function* (characteristic function, discrimination function, or indicator function) for set ***A***, is denoted by $\mu_A(x)$, and defined by

$\mu_A(x) = 1$, if x $\in$ Aotherwise 0

Definition 2.2: Fuzzy Set

A fuzzy *set* in a universe of discourse U is characterized by a membership function $\mu_A(x)$ that takes values in the interval [0, 1]. Therefore, a fuzzy set is a generalization of a classical set by allowing the membership function to take any values in the interval [0, 1]. In other words, the membership function of a classical set can only take two values-zero and one, whereas the membership function of a fuzzy set is a continuous function with range [0, 1].

We see from the definition that there is nothing **"fuzzy" about a fuzzy set; it is simply a set with a continuous membership function.**

A fuzzy set ***A*** in ***U*** may be represented as a set of ordered pairs of a generic element *x* and its membership value, that is,

$$A = \{(x, \mu_A(x)) \mid x \in U\}$$

The set ***A*** is mathematically equivalent to its membership function $\mu_A(x)$, in the sense that knowing $\mu_A(x)$, is the same **as** knowing *A* itself.

The following are the notations for representing fuzzy sets.

(i) When ***U*** **is continuous** (i.e. ***U*** = R), then fuzzy set ***A*** is commonly written as

$$A = \int_U \mu_A(x)/x$$

where the integral sign does not denote integration; it denotes the collection of all points *x* belongs to *U* with the associated membership function μ_A *(x).*

Example 2.1

The set of days of the week includes Monday, Thursday and Saturday. Now consider the set of days comprising a weekend. Most of us believe that Saturday and Sunday belong to weekend, but what about Friday? It "feels" like a part of a weekend, but somehow it seems like it should be techniqually excluded. Friday tries its best to sit on the fence. Classical sets wouldn't tolerate this kind of thing. Either you are in or you are out. Human experience suggest something different, though: fence sitting is a part of life.

Even the dictionary, defining the weekend as "the period from Friday night or Saturday to Monday morning." We are entering the realm where sharp edged yes-no logic stops being helpful. Fuzzy reasoning becomes valuable exactly when we are talking about people really precise the concept Weekend.

Example 2.2

Consider the set of all cars in Berkeley; this is the universe of discourse U. We can define different sets in U according to the properties of cars.

Types of properties that can be used to define sets in U:

(a) US cars or non-US cars,

(b) number of cylinders.

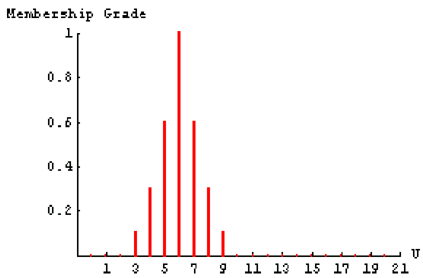
The **c**lassical set theory requires that a set must have a well-defined property, therefore it is unable to define the set like "all US cars in Berkeley."

Example 2.3: Natural Numbers

Problem. Suppose you are asked to define the set of natural numbers close to 6. There are a number of different ways in which you could accomplish this using fuzzy sets.

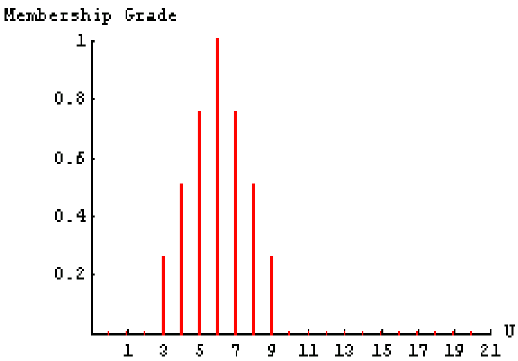
Solution 1. One solution would be to manually create a fuzzy set describing the numbers near 6. This can be done as follows:

```
SetOptions[FuzzySet, UniversalSpace → {0, 20}];
Six1 =
  FuzzySet[{{3, .1}, {4, .3}, {5, .6}, {6, 1.0},
    {7, .6}, {8, .3}, {9, .1}}];
FuzzyPlot[Six1];
```



Solution 2. A second solution would be to use the FuzzyTrapezoid function to create the fuzzy set. For a case such as this, a triangular fuzzy set would probably be better than a trapezoid, so we set the middle two parameters of the FuzzyTrapezoid function to 6.

```
Six2 = FuzzyTrapezoid[2, 6, 6, 10];
FuzzyPlot[Six2];
```

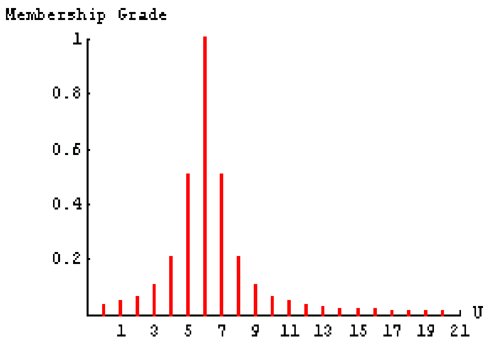


Solution 3. Another solution would be to use a function to create a fuzzy set representing numbers near 6.

$$\text{CloseTo}[x\_] := \frac{1}{1 + (\#1 - x)^2} \&$$

We can use this function to create a fuzzy set for numbers near 6.

```
Six3 = CreateFuzzySet[CloseTo[6]];
FuzzyPlot[Six3];
```

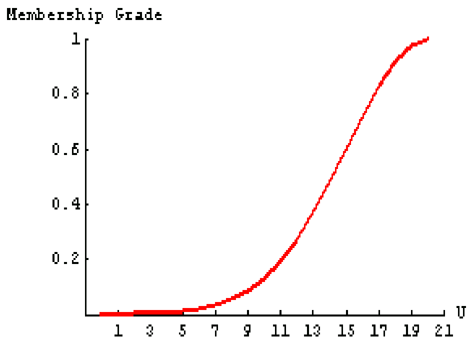


Note that this is a convenient method because the function CloseTo can be called with any integer argument to produce a fuzzy set close to that number.

Example 2.4: Fuzzy Hedges

Problem. Suppose you had already defined a fuzzy set to describe a hot temperature.

```
Hot = FuzzyGaussian[20, 7,
  UniversalSpace → {0, 20}];
FuzzyPlot[Hot, PlotJoined → True];
```



Now, suppose we want to talk about the degree to which something is hot. We need some sort of fuzzy modifier or a hedge to change our fuzzy set. Look at how we can accomplish this.

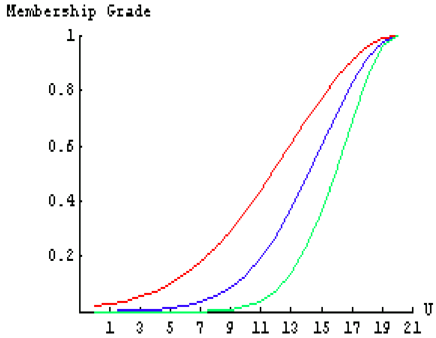
Solution. We can start by defining how a fuzzy set should be modified to represent the hedges "Very" and "Fairly." Two functions in *Fuzzy Logic*, Concentrate and Dilate, can be used to define our two hedges.

```
Very := Concentrate

Fairly := Dilate
```

Now we can look at a graph of the fuzzy sets FairlyHot, Hot, and VeryHot.

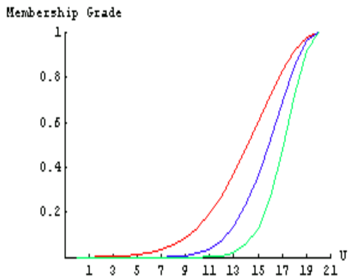
```
FuzzyPlot[Fairly[Hot], Hot, Very[Hot],
  PlotJoined → True];
```



Note that the FairlyHot membership function is a more general, spread-out fuzzy set. The VeryHot fuzzy set is a more focused, concentrated fuzzy set.

We can also apply more than one modifier to a fuzzy set. For instance, let us compare Hot, VeryHot, and VeryVeryHot.

```
FuzzyPlot[Hot, Very[Hot], Very[Very[Hot]],
  PlotJoined → True];
```



As we might expect, the VeryVeryHot fuzzy set is even more concentrated than the VeryHot fuzzy set.

Example 2.5: Distance Relation

Problem. Let *R* be a fuzzy relation between the sets, *X* = {NYC, Paris} and *Y* = {Beijing, NYC, London}, that represents the idea of "very far." In list notation, the relation could be represented as follows [Klir& Folger, 1988].

R(X,Y) = 1.0/NYC, Beijing + 0/NYC, NYC + 0.6/NYC, London + 0.9/Paris, Beijing + 0.7/Paris, NYC + 0.3/Paris, London

Solution. We can represent this fuzzy relation in *Mathematica* in the following way. We can start by creating the membership matrix to represent the relation.

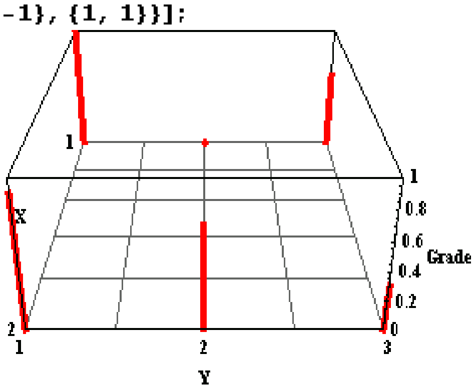
```
DistMat = {{1, 0, 0.6}, {0.9, 0.7, 0.3}}
{{1, 0, 0.6}, {0.9, 0.7, 0.3}}
```

We need to represent the cities in each set with numbers. For set *X*, let NYC be 1, and Paris be 2; for set *Y*, let Beijing be 1, NYC be 2, and London be 3. Now we can create the relation using the FromMembershipMatrix function.

```
DistRel = FromMembershipMatrix[DistMat,
   {{1, 2}, {1, 3}}]
FuzzyRelation[
  {{{1, 1}, 1}, {{1, 2}, 0}, {{1, 3}, 0.6},
   {{2, 1}, 0.9}, {{2, 2}, 0.7}, {{2, 3}, 0.3}},
  UniversalSpace → {{1, 2, 1}, {1, 3, 1}}]
```

We can plot this relation using the FuzzyPlot3D function. We will use some of *Mathematica*'s Plot3D options to put the graph in a form that lines up with the membership matrix so that you can see the correlation.

```
FuzzyPlot3D[DistRel,
 AxesLabel → {" X", "Y", "Grade  "},
 ViewPoint → {2, 0, 1},
 AxesEdge → {{-1, -1}, {1, -1}, {1, 1}}];
```




```
ToMembershipMatrix[DistRel] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0.6 \\ 0.9 & 0.7 & 0.3 \end{pmatrix}$$

By customizing the graph, you can get it to match the membership matrix, which makes understanding the fuzzy relation easier.

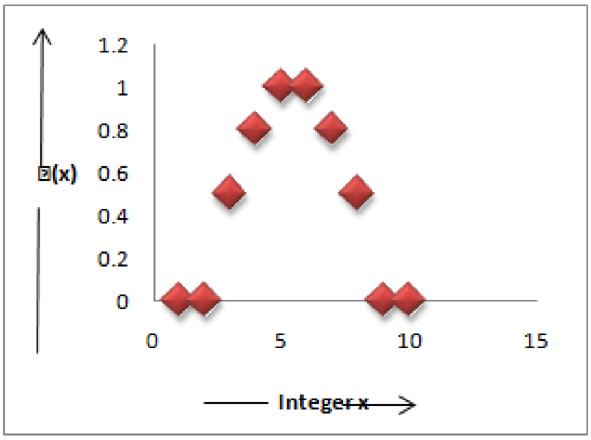
III. Basic Concepts Associated with Fuzzy Set

- **Support of a fuzzy set**
- ***Empty fuzzy set.***
- ***Fuzzy singleton***
- **Center of Fuzzy set:**
- ***Crossover point*** **of a fuzzy set**
- ***Height*** **of a fuzzy set**
- ***Normal fuzzy set***
- ***α-cut*** **of a fuzzy set A**

We will explain these concepts with following example.

EXAMPLE:3 Let U={1,2,3,…,10}

Several = 0.5/3+0.8/4+1/5+1/6+0.8/7+0.5/8 (3.3)



Support of fuzzy set "Several" is the set of integers {3, 4, 5 6, 7, 8}.

Centre of fuzzy set "Several" is the mean of 5 & 6 (i.e.5.5) where its maximum value is finite.

Crossover point of fuzzy set "Several" is 3and 8.

Height of fuzzy set "Several" is 1. Therefore "Several" is the normal fuzzy set.

If = 0.8 then the **-cut of fuzzy** set "Several" is the crisp set {4, 7}.

IV. Basic Operations on Fuzzy Sets

- **Equality and Containment**

  $$\mu_A(x) = \mu_B(x) \text{ for all } x \in U$$
  $$\mu_A(x) \le \mu_A(x) \text{ for all } x \in U.$$

- ***Complement***

  $\mu_{,}(x) = 1 - \mu_A(x)$

- ***Union***

  $\mu_{A\cup B}(x) = \max[\mu_A(x), \mu_B(x)]$

- ***Intersection***

  $\mu_{A\cap B}(x) = \min[\mu_A(x), \mu_B(x)]$

V. Some another Approach to Operations on Fuzzy Sets

- **5.1 Fuzzy Complement:**

Let c: [0, 1] →[0, 1] be a mapping that transforms the membership function of fuzzy set A into the membership function of the complement of A, that is,

$$c[\mu_A(x)] = \mu_{,}(x) \quad (5.1)$$
$$c[\mu_A(x)] = 1 - \mu_A(x)).$$

The function c: [O, 1] x [O, **1]** → [O, 1] that satisfies the following axioms

- **Axiom cl.** c(0) = 1 and c(1) = 0 (boundary condition).
- **Axiom c2.** For all ***a,*** b ∈ [0, 11, if ***a*** < b, then c(a) *2* c(b) (non-increasing condition), where a and b denote membership functions of some fuzzy sets, say, a = $\mu_A(x)$ and b = $\mu_{,}(x)$ is called a fuzzy complement.

5.2 Fuzzy Union: s-norm: Let s : [0, 1]x [0,1] → [0,1] be **a** mapping that transforms the membership functions of fuzzy sets A and ***B*** into the membership function of the union of A and B, that is,

$$s[\mu_A(x), \mu_B(x)] = \mu_{A\cup B}(x)$$

$$s[\mu_A(x), \mu_B(x)] = \max[\mu_A(x), \mu_B(x)]$$

The function s: [0, 1] x [0, **1]** → [0, 1]that satisfies the following axioms

- **Axiom sl.** s(1 ,l) = 1, s(0, a) = s(a, 0) = a (boundary condition).
- **Axiom s2.** s(a, ***b)*** = s(b, a) (commutative condition).
- **Axiom s3.** If a ≤ a' and b ≤ b', then s (a, b) ≤ s(al, b') (nondecreasing condition).
- **Axiom s4.** s (s(a, ***b),*** c) = s(a, s(b, c)) (associative condition)

  is called an s-norm.

5.3 Fuzzy intersection: t-Norm

Let t : [0,1] x [0,1] →[0, 1] be a function that transforms the membership functions of fuzzy sets ***A*** and B into the membership function of the intersection of A and B, that is,

$t[\mu_A(x), \mu_B(x)] = \mu_{A\cap B}(x)$

$t[\mu_A(x), \mu_B(x)] = \min[\mu_A(x), \mu_B(x)]$.

The function ***t*** : [0, 1] x [0, 1] →[0, 1] that satisfies the following axioms

- **Axiom t1:** t (0,0) = 0; t(a, 1) = t(1, a) = a (boundary condition).
- **Axiom t2:** t (a, b) = t (b, a) (commutativity).
- **Axiom t3:** If a ≤ a' and b ≤ b', then t(a, b) ≤ t(a', b') (non-decreasing).
- **Axiom t4**: t [t (a, b), c] = t [a, t(b, c)] (associativity).

  is called a t-norm.

VI. Summary

- **T**he definitions of fuzzy set, basic concepts associated with a fuzzy set and basic operations of fuzzy set.
- The intuitive meaning of membership functions and how to determine intuitively appealing membership functions for specific fuzzy description.
- Solved some test problems on fuzzy sets and basic operations of fuzzy sets and these are represented graphically using Mathematica software.
- The **union of** fuzzy sets is the **smallest fuzzy set** containing both the sets and the **intersection of** fuzzy sets is the **largest fuzzy set** contained by both sets.
- The axiomatic definitions of fuzzy complements, s-norm and t-norm are studie

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