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Invariant Submanifolds in a Indefinite Trans-Sasakian Manifold

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Abstract: *In this paper, invariant submanifolds in a indefinite trans-Sasakian manifold are studied. Necessary and sufficient condition are given on submanifold of a indefinite trans-Sasakian manifold to be invariant submanifold.Here we shown that an invariant submanifold of a indefinite trans-Sasakian manifold is totally geodesic.*

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I. Introduction

In 1973 and 1974 B.Y.Chen and K. Ogive introduced geometry of submanifolds and totally real submanifolds in [1] and [2]. In [3] D.E Blair discussed contact manifold in Riemannian geometry in 1976. Light like submanifolds and hypersurfaces of indefinite sasakian manifolds introduced in 2007 and 2003 [4] and [5]. In 2010, F. Massamba introduced light like hypersurfaces in indefinite trans sasakian manifolds in [6]. In recent works many authors for example [7] C.S. Bagewadi and P.Venkatesha study trans sasakian manifolds,[8] Aysel Turgut Vanli and Ramazan sari study invariant submanifolds of trans sasakian manifolds. [9] Arindam Bhattacharya and Bandana Das study some properties of Contact CR-Submanifolds of an indefinite trans sasakian manifold. [10] B.Ravi and C.S. Bagewadi study invariant sub manifolds in a conformal K- Contact Riemannian manifold.

II. Preliminaries

Let $\overline{M}$ be an (2n+1)-dimensional indefinite almost contact metric manifold with indefinite almost contact metric structure (ϕ, ξ, η, g) then they satisfies

(2.1) $$\phi^2 = -I + \eta \otimes \xi,$$

(2.2) $$\eta(\xi) = 1, \quad \phi\xi = 0,$$

(2.3) $$g(\phi X, \phi Y) = g(X,Y) - \epsilon\eta(X)\eta(Y),$$

(2.4) $$g(X,\xi) = \epsilon\eta(X)$$

where X, Y are vector fields on $\overline{M}$ and where $\epsilon = g(\xi,\xi) = \pm 1$

An indefinite almost contact metric structure (ϕ, ξ, η, g) on M is called indefinite trans-Sasakian if

(2.5) $$(\overline{\nabla}_X\phi)(Y) = \alpha\{g(X,Y)\xi - \epsilon\eta(Y)X\} + \beta\{g(\phi X,Y)\xi - \epsilon\eta(Y)\phi X\}$$

where α and β are non zero scalar funtions on $\overline{M}$ of type (α,β). $\overline{\nabla}$ is a Riemannian connection on $\overline{M}$. In particular, an indefinite trans-Sasakian manifold is normal.

From above formula, one easily obtains

(2.6) $$\overline{\nabla}_X\xi = -\alpha\epsilon\phi X + \beta\{\epsilon X - \epsilon\eta(X)\xi\},$$

Let M be an (2m+ 1) dimensional (n > m) manifold imbedded in $\overline{M}$. The induced metric g of M is given by $g(X,Y) = \overline{g}(\overline{X}, \overline{Y})$ for any vector fields X,Y on M.

Let $T_x(M)$ and $T_x(M)^{\perp}$ denote that tangent and normal bundles of M and x ε M. Let ∇_X denote the Riemannian connection on M determined by the induced metric g and R denote the Riemannian curvature tensor of M. Then Gauss-Weingarten formula is given by

(2.7) $$\overline{\nabla}_X Y = \nabla_X Y + B(X,Y),$$

(2.8) $$\overline{\nabla}_X N = -A_N(X) + D_X N$$

for any vector fields X,Y tangent to M and any vector field *N* normal to M, where D is the operator of covariant differentiation with respect to the linear connection induced in the normal bundle $T_x(M)^{\perp}$. Both A and B are called the second fundamental forms of they satisfy

$$g(B(X, Y), N) = g(A_N(X, Y)).$$

A submanifold M of $\overline{M}$ is said to be invariant if ξ tangent to M everywhere on M and $\overline{\phi}$ X is tangent to M for any tangent vector X to M. An invariant submanifold M has the induced structure tensor (ϕ, ξ, η, g).

III. Invariant Submanifolds in Indefinite Trans-Sasakian Manifold

Let $\overline{M}$ be a (2n+1) dimensional indefinite trans-Sasakian manifold and M a (2m+1) dimensional (n > m) manifold imbedded in $\overline{M}$. For the second fundamental form B of an invariant submanifold M of a indefinite trans-Sasakian manifold. We define its covariant derivative $(\widetilde{\nabla}_X B)$ by

(3.1) $$(\overline{\nabla}_X B)(Y,Z) = D_X(B(Y,Z)) - B(\nabla_X Y, Z) - B(Y, \nabla_X Z),$$

where X, Y,Z ε χ(M) - the set of all differential vector field on M.

Then by (2.7).We obtain

(3.2) $$\bar{R}(X,Y)Z = R(X,Y)Z - A_{B(Y,Z)}(X) + A_{B(X,Z)}(Y) + (\overline{\nabla}_X B)(Y,Z) - (\overline{\nabla}_Y B)(X,Z)$$

Lemma 3.1. If M is an invariant submanifold of a indefinite trans-Sasakian manifold $\overline{M}$, then its second fundamental form B satisfies B(X, ξ) = 0, for any X ε χ(M).

Proof: Since $\bar{\xi}$ is tangent to M everywhere on M, we have

(3.3) $$\bar{\nabla}_X \bar{\xi} = \bar{\nabla}_X \xi = \nabla_X \xi + B(X,\xi).$$

Since by equation,

$$\bar{\nabla}_X \bar{\xi} = \bar{\nabla}_X \xi = -\alpha\epsilon\phi X + \beta\{\epsilon X - \epsilon\eta(X)\xi\}$$

$\bar{\nabla}_X$ ξ is tangent to M for any X ε χ(M).

$$\bar{\nabla}_X \xi = \nabla_X \xi + B(X,\xi).$$

$$-\alpha\epsilon\phi X + \beta\{\epsilon X - \epsilon\eta(X)\xi\} = \nabla_X \xi + B(X,\xi).$$

then by taking the normal parts of (3.3)we get B(X, ξ) = 0.

Lemma 3.2. Any invariant submanifolds M with induced structure tensors of a indefinite trans-Sasakian manifold $\overline{M}$ is also indefinite trans-Sasakian manifold.

Proof: From (3.2) and lemma (3.1), we have

(3.4) $$\bar{R}(X,\bar{\xi})\bar{\xi} = R(X,\xi)\xi + (\overline{\nabla}_X B)(\xi,\xi) - (\overline{\nabla}_\xi B)(X,\xi).$$

Again from equation From (3.1) and lemma (3.1), we get

(3.5) $$(\overline{\nabla}_X B)(\xi,\xi) = 0\,, \;\; (\overline{\nabla}_X B)(X,\xi) = 0.$$

Finally using From (3.5) in (3.4), we obtain

$$\begin{aligned} \bar{R}(X,\bar{\xi})\bar{\xi} &= R(X,\xi)\xi + 0 + 0, \\ R(X,\xi,\xi) &= -\alpha(\epsilon\eta(X)\xi - X) + \beta(\phi X). \end{aligned}$$

Hence the lemma.

Lemma 3.3. Let M be an invariant submanifold of a indefinite trans-Sasakian manifold $\overline{M}$, then $\overline{R}(X, \xi)Y$ is tangent to M iff $\phi B(X, \phi Y) = B(X, \phi Y)$ for any X, Y ε χ (M).
Proof:

$$\begin{aligned}(\overline{\nabla}_X\phi)Y &= \overline{\nabla}_X\phi Y - \phi(\overline{\nabla}_X Y)\\ &= \nabla_X\phi Y + B(X,\phi Y) - \phi(\nabla_X, Y) - \phi(B(X,Y))\\ &= (\nabla_X\phi)Y + B(X,\phi Y) - \phi(B(X,Y))\end{aligned}$$

Then we have

$$\alpha(g(X,Y)\xi - \epsilon\eta(Y)X) + \beta(g(\phi X,Y)\xi - \epsilon\eta(Y)\phi X)$$

$$= \alpha(g(X,Y)\xi - \epsilon\eta(Y)X) + \beta(g(\phi X,Y)\xi - \epsilon\eta(Y)\phi X) + B(X,\phi Y) - \phi(B(X,Y))$$

thus we get

$$B(X,\phi Y) = \phi(B(X,Y))$$

Lemma 3.4. Let M be invariant submanifold of the indefinite trans saskian manifold $\overline{M}$ then,

$$\overline{\nabla}_X B(Y,\xi) = -B(Y,\overline{\nabla}_X\xi)$$

for any X, Y ε χ (M)
Proof: By using Lemma 3.1 we get

$$\overline{\nabla}_X B(Y,\xi) = \nabla_X B(Y,\xi) - B(\nabla_X Y,\xi) - B(Y,\overline{\nabla}_X\xi)$$

Then, we have

$$\overline{\nabla}_X B(Y,\xi) = -B(Y,\overline{\nabla}_X\xi)$$

Theorem 3.1. Let M be an invariant submanifold of an indefinite trans sasakian manifold $\overline{M}$. Then B is parallel if and only if M is totally geodesic.
Proof: Suppose that B is parallel. For each X,Y ε χ(M) and using lemma 3.4 we get,

$$\nabla_X B(Y,\xi) = 0$$

$$B(Y,\nabla_X\xi) = 0$$

$$\bar{\nabla}_X\xi = -\alpha\epsilon\phi X + \beta\{\epsilon X - \epsilon\eta(X)\xi\}$$

BY equation (2.1), we have

$$\bar{\nabla}_X\xi = -\alpha\epsilon\phi X - \beta\epsilon\phi^2$$

Hence

$$B(Y, -\alpha\epsilon\phi X - \beta\epsilon\phi^2 X) = 0$$

$$-\alpha B(Y,\epsilon\phi X) - \beta B(Y,\epsilon\phi^2 X) = 0$$

Since M is an invariant submanifold of $\overline{M}$, we have $\phi(B(X, Y)) = 0$.

From Lemma 3.3 it follows that

$$\phi(B(X,Y)) = B(X,\phi Y) = 0$$

Then we get

$$\beta B(Y,\epsilon\phi^2 X) = 0$$

hence it follows that

$$B(Y, -X + \epsilon\eta(X)\xi) = 0$$

so

$$B(Y, X) = 0$$

viceversa let M is totally geodesic, Then B=0, for all X, Y,Z εTM.

$$(\overline{\nabla}_X B)(Y,Z) = D_X(B(Y,Z)) - B(\nabla_X Y,Z) - B(Y,\nabla_X Z) = 0$$

thus we have $\nabla B = 0$

References

[1]. B.Y Chen , Geometry of submanifolds, M.Dekker,Newyork,1973
[2]. B.Y.Chen and K.Ogive , on totally real submanifolds, Transaction of the Amer.Math.Society, 193(1974), 257-266
[3]. D.E. Blair, Contact manifold in Riemannian Geometry, Lecture Notes in Mathematics, 509, Springer-Verlag, Berlin, 1976.
[4]. K.L.Duggal and B.Sahin, Lightlike submanifolds of Indefinite sasakian manifolds, International Journal of mathematics and Matematical Sciences, Hindawi Publishing Corporation (2007),1-22
[5]. T.H.Kang,S.D.Jung,B.H.Kim,H.K.pak and J.S.Pak, Light like hypersurfaces of indefinite sasakian manifolds, Indian Journal of Pure and Applied mathematics, Vol.34,no.9(2003),1369-1380.
[6]. F. Massamba, light like hypersurfaces in indefinite trans sasakian manifolds , IC,044(2010),1-29.
[7]. C.S. Bagewadi and P.Venkatesha, Torseforming vector field in a 3-dimentional Trans sasakian manifold , Differ.Geom.Dyn.Syst.,8(2006),23-28.
[8]. Aysel Turgut Vanli and Ramazan sari, Contact CR-Submanifolds of an indefinite trans sasakian manifold., Differ.Geom.Dyn.Syst.,12(2010),277-288
[9]. Arindam Bhattacharya and Bandana Das, invariant submanifolds of trans sasakian manifolds, Int.J.Contemp.Math.Science,Vol 6,2011,no.26,1271-1282
[10]. C.S. Bagewadi and B. Ravi, Invariant submanifolds of a conformal k-contact manifold, Indian J. Pure and Appl. Math., 20 (1989), 1119-1125.