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Contra Ir*-Continuous And Almost Contra ir*-Continuous Functions in Ideal Topological Spaces

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Abstract: *In this paper we apply the notion of IR*-closed sets to present and the study a new class of function called contra IR*-continuous & almost contra IR*-continuous functions in ideal topological space. The relationship between their new sets and other sets of functions are established and some properties are discussed.*

Keywords: *IR*-continuous functions, contra IR*-continuous functions, almost contra IR*- continuous functions, IR*-T_1 space, IR*-T_2space, IR*-$T_{1/2}$ space.*

I. Introduction

The notion of ideal topological spaces was studied by Kuratowski [15] and Vaidynathaswamy [21]. In 1996, Dontchev [4] introduced the notion of contra continuity. Almost contra continuous functions was introduced by Ekici [17].The purpose of this paper is to introduce and study the notion of contra IR*-continuous & almost contra IR*-continuous in ideal topological space. C.Janaki and Renu Thomas [12] introduced the concepts of contra R*-continuous & almost contra R*-continuous functions in topological spaces and IR*-closed sets in ideal topological spaces [11].

II. Preliminaries

An ideal [15] I on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies (i) A ∈ I and B ⊂ A implies B ∈ I and (ii)A ∈ I and B ∈ I implies A ∪ B ∈ I. Given a topological space (X, τ) with an ideal I on X and if P(X) is the set of all subsets of X, a set operator $(.)^*$: $\rho(X) \rightarrow \rho(X)$, called a local function [15] of A with respect to τ and I is defined as follows. A ⊆ X, A^* (I, τ) = { x ∈ X | U ∩ A ∉ I for every U ∈ τ (x) } where τ (x) = { U ∈ τ | x ∈ U }.A Kuratowski closure operator [14] cl *(.) for a topology τ^* (X, τ) called the *- topology finer than τ is defined by cl *(A) =A ∪ A*(I, τ). cl* A and int * A will denote the closure and interior of A in (X, τ*).When there is no chance for confusion, A* is substituted for A* (I, τ) and τ^* or τ^* (I) for τ^* (I, τ). A subset A of an ideal space (X, τ, I) is *-closed (τ^* -closed) [14] if A* ⊂ A.

Definition 2.1: [16] A subset A of a topological space (X, τ) is called a regular open if A = int (cl(A)) and regular closed if A = cl(int(A)).The intersection of all regular closed subset of (X, τ) containing A is called the regular closure of A and is denoted by rcl(A).

Definition 2.2: [5] A subset A of a topological space (X, τ) is called a regular semi open set if there is a regular open set U such that U ⊂ A ⊂ cl (U).The family of all regular semi open sets of X is denoted by RSO(X).

Definition 2.3: A function f : (X, τ) → (Y,σ) is called
1. [4] contra continuous if f^{-1}V) is closed in (X, τ) for every open set V of (Y,σ).
2. [7] R- map if f^{-1}(V) is regular closed in (X, τ) for every regular closed set V of (Y, σ).
3. [1, 6] perfectly continuous if f^{-1}(V) is clopen in (X, τ) for every open set V of (Y, σ).
4. [17] almost continuous if f^{-1}(V) is open in (X, τ) for every regular open set V of (Y,σ).
5. [9] regular set connected if f^{-1}(V) is clopen in (X, τ) for every regular open set V of (Y,σ).
6. [17] RC-continuous if f^{-1}(V) is regular closed in (X, τ) for every open set V of (Y, σ)

Definition 2.4: A subset A of an ideal topological space (X, τ, I) is called
1. [10] IR-closed if A = cl^*(int (A)) and is denoted by IR-C(X). The intersection of all IR-closed sets containing A is called the IR*-closure and is denoted by r_i^{**}cl (A).
2. [10] IR*-closed if r_i^{**}cl (A) ⊂ U whenever A ⊂ U and U is regular semi-open and is denoted by IR*-C(X).
3. [10] IR*-open if A^c is IR*-closed in (X, τ, I).
4. [17] regular I-closed if A = $(int(A))^*$

Definition 2.5: A function f : (X, τ, I) → (Y,σ) is called
1. [13] IR*-continuous if f^{-1}(V) is IR*- closed in (X, τ, I) for every closed set V of (Y,σ).
2. [13] IR*- irresolute if f^{-1}(V) is IR*- closed in (X, τ, I) for every IR*- closed set V of (Y,σ).

Definition 2.6: The collection of all IR* open subset of X containing a fixed point x is denoted by IR*-O (X, x).
Definition 2.7: A function f: A → B is said to be injective (or 1-1) if for each pair of distinct points of A their image under f are distinct.

Definition 2.8: [18] A topological space (X, τ) is called a ultra normal space if each pair of disjoint closed sets can be separated by disjoint clopen sets.

Definition 2.9: [6] For a function f: X → Y the subset {(x, f(x)) : $x \in$ X}$\subset X \times Y$ is called the graph of f and is denoted by G(f).

Definition 2.10: [8] A subset A of a topological space (X, τ) is said to be clopen if it is both open and closed in (X, τ).

Definition 2.11: [19] A topological space X is said to be hyperconnected if every open set is dense.

Definition 2.12: A subset A of a topological space X is called dense (in X) if every point x in X either belongs to A (or) is a limit point of A. Also A is dense in X if $\bar{A}$ = X.

Definition 2.13 : A topological space X is termed a Urysohn space if for any two distinct points $x, y \in X$ there exist disjoint open subsets $x \in U$ and $y \in V$ such that the closures $\bar{U}$ and $\bar{V}$ are disjoint closed subsets of X.

Definition 2.14: A topological space X is termed a T_1-space (or Frechet space or accessible space) if it satisfies the following equivalent conditions:
1. Given two distinct points $x, y \in X$ there exists an open subset U and V of X such that $x \in U$ and $y \notin V$.
2. For every $x \in X$ the singleton set $\{x\}$ is a closed subset.
3. For every $x \in X$ the intersection of all open subsets of X containing $\{x\}$ is precisely $\{x\}$.

Definition 2.15: A space (X, τ) is said to be an Ultra Hausdroff space if for pair of distinct points x and y in X there exist two clopen sets U and V containing x and y such that U∩V = ϕ .

Definition 2.16: [9] Let X be a space such that one point set closed in X. Then X is said to be regular if for all $x \in$ X and for all closed set B not containing x there exist disjoint open sets U and V containing x and B respectively.

Definition 2.17: [20] A space (X, τ) is said to be weakly Hausdroff is each element of X is an intersection of regular closed sets.

III. Contra Ir*- Continuous In Ideal Topological Space.

Definition 3.1: A function f : (X, τ, I) → (Y,σ) is called contra IR*-continuous if f^{-1}(V) is IR*-closed in (X, τ, I) for every open set V in (Y,σ).

Example 3.2: Let X = {a, b, c, d}=Y**,** τ = {X, ϕ,{a},{b},{c},{a, b},{a, c},{b, c},{a, b, c}}, and I = { ϕ, {a}}, IR*- C(X) = {X, ϕ, {a}, {d},{a, d},{b, d},{c, d},{a, b, c},{a, b, d},{a, c, d}, {b, c, d}}**,** σ = {Y, ϕ,{a},{c},{a, c},{a, b, c},{a, c, d}}**.** Define a mapping f: (X, τ, I) → (Y,σ) as f(a) = a, f(b) = b, f(c) = d, f(d) = c so the function f is contra IR*-continuous.

Remark 3.3: The composition of two contra IR*-continuous function need not be contraIR*-continuous.

Example 3.4: Let X = Y = Z = {a, b, c, d}**,** τ = { X, ϕ,{a},{b},{c},{a, b},{ac},{b, c},{a, b, c}}**,**and I = { ϕ, {a}},σ = {Y, ϕ,{a}, {d}, {a, d}}, η = { Z, ϕ,{a},{c},{a, c},{b, c},{a, b, c},{b, c, d}}**.** Define f : (X, τ, I) → (Y,σ) be the identity mapping and g : (Y,σ) → (Z,η) by g(a) = a, g(b) = b, g(c) = d, g(d) = c. Here both f and g are contra IR*-continuous but gof is not contra IR*-continuous.

Remark 3.5: contra IR*-continuity and contra continuity are independent concepts.

Example 3.6: Let X = {a, b, c, d} = Y, τ = {X, ϕ, {a}, {b},{c},{a, b},{a, c},{b, c},{a, b, c}}, and I = { ϕ, {a}}, τ* = {X, ϕ ,{d},{c, d},{a, d},{b, d},{b, c, d},{a, c, d},{a, b, d}}, IR*-C(X) = { X, ϕ,{a},{d},{a, d},{b, d},{c, d},{a, b, c},{a, b, d},{a, c, d},{b, c, d} }, σ = { Y, ϕ, {a},{d},{a, d} }. Define a mapping f: (X, τ, I) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c, f(d) = d then f is contra IR*-continuous function but not contra continuous. Since f^{-1}{a} = a is not closed in (X, τ, I).

Example 3.7: Let X = {a, b, c, d} = Y, τ = {X, ϕ,{a},{c},{a, c},{a, b, c},{a, c, d}}, and I = { ϕ, {a}}, τ* = {X, ϕ ,{a},{b},{d},{a ,d},{b, d},{a, b, d},{b, c, d}}, IR*- C(X) = {X, ϕ,{a}, {a, c},{a, b, c},{a, c, d},{b, c, d}}, σ = {Y, ϕ,{b},{d},{b, d}}. Define f : (X, τ, I) → (Y,σ) by the identity mapping. Hence f is continuous but not contra IR*-continuous. Since f^{-1}{b} = b is not IR*- closed.

Theorem 3.8: If f: (X, τ, I) → (Y,σ) is a contra IR*- continuous function an g : (Y,σ) → (Z, η) is a continuous function then the function gof : (X, τ, I)→ (Z, η) is contra IR*-continuous.

Proof: Let V be open in (Z,η). Since g is continuous, g^{-1}(V) is open in (Y,σ).Since f is contra IR*-continuous. So f^{-1}(g^{-1}(V) is IR*-closed in X. That is $(gof)^{-1}$(V) is IR*-continuous. Hence gof is contra IR*-continuous.

Theorem 3.9: If f: (X, τ, I) → (Y,σ) is IR*- irresolute and g: (Y,σ) → (Z,η) is a contra IR*- continuous function then gof : (X, τ, I)→ (Z, η) is contra IR*-continuous.

Proof: Let V be open in (Z,η). Since g is contra IR*-continuous g^{-1}(V) is IR*- closed in (Y,σ). Since f is IR*-irresolute f^{-1}(g^{-1}(V)) is IR*-closed in (X, τ, I). Hence gof is contra IR*-continuous.

Theorem 3.10: Suppose IR*-O(X) is closed under arbitary union then the following are equivalent for a function f : (X, τ ,I) → (Y,σ).
(i) f is contra IR*- continuous
(ii) for every closed subset V of (Y, σ), f^{-1}(V) ∈ IR*-O(X)
(iii) for each x ∈ X and each V ∈ C(Y,f(x)) there exist a set U ∈ IR*-O(X, x) such that f(U)⊂ V

Proof: (i) ⇒ (ii) Let f be contra IR*- continuous. Then f^{-1}(V) is IR*-closed in (X, τ, I) for every open set V of (Y, σ). That is f^{-1}(V) is IR*-open in (X, τ, I) for every closed set V of (Y, σ).
Hence f^{-1} (V) ∈ IR*-O(X).
(ii) ⇒ (i) obvious
(ii) ⇒ (iii) For every closed subset V of Y, f^{-1}(V) ∈ IR*-O(X) then for each x ∈ X and each
V ∈ C(Y, f(x)), there exists a set U ∈ IR*-open (X) such that f(U) ⊂ V
(iii) ⇒(ii) For each x ∈ X and each V ∈ C(Y, f(x)) there exists a set U_x ∈ IR*-O(X, x) Such that
f (U_x) ⊂ V. That is $x \in f^{-1}$(V) and f(x) ⊂V. So there exists U ∈ IR*-O(X, x),
f^{-1}(V) = ∪{U_x: $x \in f^{-1}$(V)} and Hence f^{-1}(V) is IR*- O(X).

Definition 3.11: A space (X, τ, I) is said to be IR*-T_1 if for each pair of distinct points x and y in (X, τ, I) there exist IR*- open set U and V containing x and y respectively. Such that y ∉U and x ∉V.

Definition 3.12: A space (X, τ, I) is said to be IR*-T_2 if for each pair of distinct points x and y in (X, τ, I) there exist IR*- open sets U and V containing x and y respectively. Such that U⋂V= ϕ.

Definition 3.13: A space (X, τ, I) is said to be IR*-$T_{1/2}$ if every IR*-closed set is regular I-closed.

Theorem 3.14: If (X, τ, I) is an ideal topological space and for each pair of distinct points x_1 and x_2 in X there exists a function f into a Urysohn space (Y,σ). Such that f(x_1) ≠ f(x_2) and f is contra IR*- continuous at x_1 and x_2 then the space (X, τ, I) is IR*-T_2.

Proof: Let x_1 and x_2 be any distinct points in (X, τ, I). Then by hypothesis there is a Urysohn space (Y,σ) and a function f: (X, τ, I) → (Y,σ) which statisfies the condition of this theorem. Let y_i= f (x_i) for i = 1,2 then $y_1 \neq y_2$. Since (Y,σ) is Urysohn space, there exists open neighborhoods U_{y_1} and U_{y_2} of y_1 and y_2 respectively in Y. Such that cl (U_{y_1}) ⋂ cl (U_{y_2}) = ϕ. Since f is contra IR*- continuous at x , there exists a IR*- open neighbourhoods

w_{xi} of x_i in X. such that f (w_{xi}) ⊂ cl (U_{y_i}), for i =1, 2. Hence (w_{x_1}) ∩ (w_{x_2}) = ϕ. Because cl (U_{y_1}) ∩ cl(U_{y_2}) = ϕ. Then (X, τ, I) is IR* - T_2.

Corollary 3.15: If f is a contra IR*- continuous injection of an ideal topological space (X, τ, I) into a Urysohn space (Y,σ) then (X, τ, I) is IR*-T_2 space.

Proof: Suppose that f : (X, τ, I) → (Y,σ) is contra IR*-continuous injection and Y is a Urysohn space. Then for each pair of distinct points x_1 and x_2 in X f(x_1) ≠ f(x_2). Therefore by the above theorem 3.14, X is IR*-T_2 space.

Corollary 3.16: If f is a contra IR*-continuous injection of an ideal topological space (X, τ, I) into a Ultra Hausdroff space (Y,σ) then (X, τ, I) is IR*-T_2.

Proof: Let x_1 and x_2 be any distinct points in (X, τ, I). Then f is injective and Y is Ultra Hausdroff, f(x_1) ≠ f(x_2) and there exist two clopen sets V_1 and V_2 in (Y,σ). Such that f(x_1) ∈ V_1 and f(x_2) ∈ V_2 and $V_1 \cap V_2 = \phi$ .Then $x_i \in f^{-1}(V_i)$ ∈ IR*-O(X) for i = 1,2 and $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$. Then X is IR*-T_2.

Theorem 3.17: If f: (X, τ, I) → (Y,σ) is a contra IR*-continuous injection and Y is weakly Hausdroff then X is IR*-T_1.

Proof: Suppose that Y is weakly Hausdroff for any distinct points x_1 and x_2 in X. There exist regular closed sets U and V in Y. Such that f(x_1) ∈ U but f(x_2)∉U, f(x_1)∉V and f(x_2) ∈ V. Since f is contra IR*- continuous f^{-1}(U) and f^{-1}(V) are IR*- open subset of X. Such that $x_1 \in f^{-1}$(U), $x_1 \notin f^{-1}$(V), $x_2 \in f^{-1}$(V), $x_2 \notin f^{-1}$(U)**.** This shows that X is IR*-T_1.

Theorem 3.18: If f: (X, τ, I) → (Y,σ) is a contra IR*- continuous and (X, τ, I) is IR*-$T_{1/2}$ space then f is RC-continuous.

Proof: Let V be open in (Y,σ). Since f is contra IR*-continuous, f^{-1}(V) is IR*-closed in (X, τ, I) and X is IR*-$T_{1/2}$ space. Hence f^{-1}(V) is regular I-closed in (X, τ, I). "Every regular I-closed set is regular closed". Then for every open set V of (Y,σ), f^{-1}(V) is regular closed in (X, τ, I). Hence f is RC-continuous.

IV. Almost Contra Ir*- Continuous Function In Ideal Topological Space

Definition 4.1: A function f : (X, τ, I) → (Y, σ) is said to be almost contra IR*-continuous if f^{-1}(V) is IR*-closed set in (X, τ, I) for each regular open set V in (Y, σ).

Example 4.2: Let X = Y = {a, b, c, d}**,** τ = {X, ϕ,{a},{c},{a, c},{b, c},{a, b, c},{b, c, d}} and I = { ϕ, {a}},IR*- C(X) = {X,ϕ,{a},{b}, {c},{d},{a, b},{b, c},{c, d},{a, d},{a, c},{b, d}, {a, b, c},{a, c, d},{b, c, d}, {a, b, d}}, σ = {Y, ϕ,{a},{b},{c}{a, b},{a, c},{b, c},{a, b, c}}, Regular open = {Y, ϕ,{a},{b},{c}, {a, b},{a, c},{b, c}}.Define a mapping f : (X, τ, I) → (Y, σ) as f (a) = a, f(b) = b, f(c) = c, f(d) = d, so the function f is almost contra IR*-continuous.

Theorem 4.3: The following are equivalent for a function f : (X, τ, I) → (Y,σ)
1. f is almost contra IR*- continuous
2. for every regular closed set F of (Y, σ), f^{-1}(F) is IR*- open set of (X, τ)

Proof: (1) ⇒ (2) Let F be a regular closed set in (Y, σ), then Y-F is a regular open set in (Y, σ)
By $f^{-1}(Y$ - F) = X - f^{-1}(F) is IR*- closed in (X, τ, I) therefore (2) holds.
(2) ⇒ (1) let G be a regular open set in (Y, σ). Then (Y- G) is regular closed in (Y, σ) by (2) f^{-1}(Y-G) is an IR*- open set in (X, τ, I). This implies X -f^{-1}(G) is IR*- open. This implies f^{-1}(G) is IR*- closed set in (X, τ, I).Therefore (1) holds.

Theorem 4.4: For two functions f: (X, τ, I) → (Y,σ) and k: (Y,σ) → (Z,η). Let the function kof : (X, τ, I) → (Z,η) is a composition function. Then the following holds. If f is almost IR*-continuous and k is perfectly continuous then kof is contra IR*-continuous.

Proof: Let V be an open set in (Z,η). Since k is perfectly continuous, k^{-1}(V) is clopen in (Y,σ). Since f is an almost contra IR*-continuous $f^{-1}(k^{-1}$(V)) = $(kof)^{-1}$(V) is IR*- open and IR*-closed set in (X, τ, I).Therefore kof is contra IR*- continuous.

Theorem 4.5: If f: (X, τ, I) → (Y, σ) is an almost contra IR*- continuous injection and (Y, σ) is weakly Hausdroff then X is IR*- T_1.

Proof: Suppose Y is weakly Hausdroff for any distinct points x and y in (X, τ, I).There exist V and W regular closed sets in (Y, σ). Such that f(x) ∈ V, f(y) ∉V and f(y) ∈W and f(x) ∉ W. Since f is almost contra IR*-continuous f^{-1} (V) and f^{-1} (W) are IR*- open subset of X. Such that $x \in f^{-1}$(V), $y \notin f^{-1}$(V), $y \in f^{-1}$(W) and $x \notin f^{-1}$(W). Therefore X is IR*- T_1.

Theorem 4.6: If a function f: (X, τ, I) → (Y,σ) is contra IR*-continuous then it is almost contra IR*-continuous.

Proof: obvious because "Every regular open set is open set".

Remark 4.7: The converse of the theorem need not be true in general as seen from the following
X = Y = {a, b, c, d},τ = {X, ϕ,{a},{b},{a, b},{b, c},{a, b, c},{a, b, d}}, and I = { ϕ, {a}},IR* - C(X) = {X, ϕ,{a},{a, b}, {c, d},{a, c},{b, d},{a, b, c},{b, c, d},{a, b, d},{a, c, d}}, σ = {Y, ϕ,{a},{c},{a, c},{c, d},{a, c, d}}, Regular open = {Y, ϕ,{a},{c, d}}. Define f(a) = a, f(b) = b, f(c) = c, f(d) = d, f : (X, τ, I) → (Y, σ) is almost contra IR*-continuous but f^{-1}(c) = c which is not IR*- continuous in (X, τ, I).

Remark 4.8: The composition of two almost contra IR*- continuous function need not be almost contra IR*-continuous as seen in the following example.

Example 4.9: Let X = {a, b, c, d} = Y = Z, τ = { X, ϕ {a},{b},{a, b},{b, c},{a, b, c},{a, b, d}}, and I = { ϕ, {a}},IR*- C(X) = {X, ϕ,{a},{a, b},{c, d},{a, c},{b, d},{a, b, c},{b, c, d},{a, b, d},{a, c, d}}, σ = {Y, ϕ,{a},{c},{a, c},{c, d},{a, c, d}}, IR*-C(Y) = {Y, ϕ,{a},{a, c},{a, d},{b, c},{b, d},{a, b, c},{a, b, d},{a, b, d},{a, c, d},{b, c, d}}, Regular open = {Y, ϕ,{a},{c, d}}**,** η = {Z, ϕ,{a},{b},{a, b},{b, c},{a, b, c},{a, b, d}}. IR*- C(Z) = {Z, ϕ,{a},{a, b},{c, d},{a, c}, {b, d},{a, b, c},{b, c, d},{a, b, d},{a, c, d} }, Regular open = { Z, ϕ ,{a},{b, c} }. Define f : (X, τ, I) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c, f(d) = d and g : (Y, σ) → (Z, η) by g(a) = a, g(b) = b, g(c) = c, g(d) = d. Then f and g are almost contra IR*-continuous. Since $(gof)^{-1}$(b) = $f^{-1}(g^{-1}$(b)) = f^{-1}(b) = b, $(gof)^{-1}$(c) = $f^{-1}(g^{-1}$(c)) = f^{-1}(c) = c, {b, c} is not IR*- closed in (X, τ, I).

Theorem 4.10: For two function f: (X, τ, I) → (Y, σ) and k : (Y, σ) → (Z, η). Let kof : (X, τ, I) → (Z,η) is a composition function. If f is almost contra IR*-continuous and k is an R-map then kof is almost contra IR*-continuous.

Proof: Let V be any regular open set in (Z,η). Since k is an R- map k^{-1}(V) is regular open in (Y, σ). Since f is almost contra IR*-continuous $f^{-1}(k^{-1}$(V)) = $(kof)^{-1}$(V) is IR*- closed in (X, τ, I). Therefore kof is almost contra IR*-continuous.

Theorem 4.11: For two function f : (X, τ, I) → (Y, σ) and k : (Y, σ) → (Z,η) is a composition function. If f is almost contra IR*- continuous and k is almost continuous then kof is almost contra IR*- continuous.

Proof: Let V be any open set in (Z,η). Since k is almost continuous k^{-1} (V) is open in (Y, σ). Since f is almost contra IR*- continuous $f^{-1}(k^{-1}$(V)) = $(kof)^{-1}$(V) is IR*-closed in (X, τ, I).Therefore kof is almost contra IR*-continuous.

Definition 4.12: A function f: (X, τ, I) → (Y, σ) is said to be almost IR*-continuous if f^{-1}(V) is IR*- open set in (X, τ, I) for each regular open set V in (Y, σ).

Theorem 4.13 : If f : (X, τ, I) → (Y, σ) is a contra IR*-continuous map and g : (Y, σ) → (Z,η) is a regular set connected function then gof : (X, τ, I) → (Z,η) is IR*-continuous and almost IR*-continuous .

Proof: Let V be regular open in (Z,η). Since g is regular set connected g^{-1}(V) is clopen in (Y, σ). Since f is a contra IR*-continuous $f^{-1}(g^{-1}(V))$ is IR*-closed in (X, τ, I). Hence gof is almost IR*-continuous.

Definition 4.14: A function f : (X, τ, I) → (Y, σ) is strongly IR*-open if the image of every IR*-open set of (X, τ, I) is IR*-open (Y, σ).

Theorem 4.15: If f : (X, τ, I) → (Y, σ) is a surjective, strongly IR*-open (or strongly IR*-closed) and g : (Y,σ) → (Z,η) is a function such that gof : (X, τ, I) → (Z,η) is almost contra IR*-continuous then g is almost contra IR*-continuous.

Proof: Let V be any regular closed set (respectively regular open) set in (Z, η). Since gof is almost contra IR*-continuous $(gof)^{-1}$(V) = $f^{-1}(g^{-1}$(V) is IR*-open (respectively IR*-closed) in (X, τ, I). since f is surjective and strongly IR*-open (or) strong IR*-closed f($f^{-1}(g^{-1}$(V)) = g^{-1}(V) is IR*-open (respectively IR*- closed). Therefore g is almost contra IR*- continuous.

Theorem 4.16: Let f : (X, τ, I) → (Y, σ) is a contra IR*-continuous function and g : (Y, σ) → (Z,η) is IR*-continuous. If Y is IR*- $T_{1/2}$ then gof : (X, τ, I) → (Z,η) is an almost contra IR*-continuous function.

Proof: Let V be regular open and hence open set in (Z,η). Since g is IR*- continuous g^{-1}(V) is IR*- open in (Y,σ) and Y is IR*-$T_{1/2}$ space implies g^{-1}(V) is regular open in (Y,σ). Since f is almost contra IR*-continuous $f^{-1}(g^{-1}$(V)) = $(gof)^{-1}$(V) is IR*-closed set in (X, τ, I).Therefore gof is almost contra IR*- continuous.

Definition 4.17: A ideal topological space X is called a IR*-normal space [19] if each pair of disjoint closed sets can be separated by disjoint IR*-open sets.

Theorem 4.18: If f : (X, τ, I) → (Y,σ) is an almost contra IR*-continuous closed injective function and (Y,σ) is ultra normal then (X, τ, I) is IR*- normal.

Proof: Let E and F be disjoint closed subsets of (X, τ, I). Since f is closed and injective f(E) and f(F) are disjoint closed sets in (Y,σ). Since Y is ultra normal there exist disjoint clopen sets in U and V in Y such that f(E) ⊂ U and f(F) ⊂V.This implies E ⊂ f^{-1}(U) and F ⊂ f^{-1}(V). Since f is an almost contra IR*- continuous injection f^{-1}(U) and f^{-1}(V) are disjoint IR*- open sets in (X, τ, I). Therefore X is IR*-normal.

Definition 4.19: A ideal topological space (X, τ, I) is said to be IR*- ultra connected if every two non empty IR*-closed subsets of X intersect.

Theorem 4.20: If (X, τ, I) is IR*- ultra connected and f : (X, τ, I) → (Y,σ) is an almost contra IR*-continuous surjection then (Y,σ) is hyperconnected.

Proof: Let X be IR*- ultraconnected and f : (X, τ, I) → (Y,σ) is an almost contra IR*-continuous surjection . Suppose Y is not hyperconnected. Then there is an open set V such that V is not dense in Y. Therefore there exist an nonempty regular open subsets B_1= int (cl(V)) and B_2=Y - cl(V) in (Y, σ). Since f is an almost contra IR*-continuous surjection. $f^{-1}(B_1)$ & $f^{-1}(B_2)$ are disjoint IR*-closed in (X, τ). Which is a contradiction to the fact that X is IR*-ultra connected. Therefore Y is hyperconnected.

Reference

[1]. S.P Arya and R.Gupta "on strongly continuous mappings" Kyunpook Math., J.14 (1974), 131-143.
[2]. J. Bhuvaneswari, A. Keskin, N.Rajesh, contra–continity via Topological Ideals.,J.Adv.Res.Pure Math.,3(1)(20),4051
[3]. Cameron D.E, Properties of S-closed spaces, Proc.amer.Math.Soc.72 (1978), 581-586
[4]. J.Dontchev contra-continuous functions and strongly S-closed spaces, Internal J.Math.Math.Sci.,19(1996),303-310
[5]. J.Dontchev On generalizing semi pre opensets, Mem. Fac. Sci. Kochi. Univ. Ser. A. Math 16(1995), 35-48.
[6]. E.Ekici, almost contra pre-continuous functions, Bull Malaysian.Math.Sci.Soc.27:53:65, 2004.
[7]. E.Ekici, "On contra πg-continuous functions", cha08, solitons and fractals 35,(2000) 71-81.
[8]. G.L.Garg and D.Sivraj, On sc-compact and S-closed spaces, Boll.Un.Math.Ital.6 (3B)(1984),321-332.
[9]. Gangster.M and Reilly.I, More on almost S-continuity Indian J.Math.1999, 41:139-146
[10]. C.Janaki and Renu Thomas, On R*- closed sets in topological spaces, International Journal of Mathematical Archive – 3[8], 2012, 3067-3074.
[11]. C.Janaki and Renu Thomas, IR*- closed sets in Ideal Topological spaces [under publishing]
[12]. C.Janaki and Renu Thomas, contra R*-continuous and almost contra R*-contra function
ISSN: 2278-3008, p-ISSN: 2319-7676.volume, Issue 1(Nov –Dec 2013), pp 63-69.
[13]. Renu Thomas and Sheepa. K. S, On IR*-continuty in Ideal Topological spaces [under publishing]
[14]. D. Jankovic and T. R. Hamlett, "New topologies from old via ideals," The American Mathematical Monthly, vol. 97, pp. 295– 310, 1990.
[15]. Kuratowski .K. Topology, Vol.I. New York: Academic Press, 1966.
[16]. Palaniappan and KC Rao, "Regular generalized closed sets" Kyungpook, Math. J.33 (1993), 211-219
[17]. Singal M.K and Singal A.R, Almost continuous mapping, Yokohama. Math.J.16 (1968) 63-73.

[18]. R.Staum, The algebra of bounded continuous functions into a non-archiemedian field. Pacific. J.Math 50:169-185(1974).
[19]. Steen L.A and steeback Jr.J.A 1970 , counter examples in topology, Holt, Rinenhart and wiston, newyork 1970.
[20]. Soundarajan.T weakly Hausdroff spaces and the cardinality of topological spaces in general topology and its relation to modern analysis and algebra. III proc.Taum, conf.Kanpur (1968) Academic, Prague (1971) 301-306
[21]. R. Vaidyanathaswamy, Set Theory, Chelsea Publishing Company, New York, 1960