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Hall Current Effects On Mhd Free Convecttive Heat And Mass Transfer Flow Over An Infinite Vertical Porous Plate With Radiation And Chemical Reaction

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Abstract: *The present study deals with unsteady two-dimensional hydro magnetic free convective boundary layer flow of an incompressible and electrically-conducting fluid along an infinite vertical plate embedded in the porous medium with heat and mass transfer is analyzed, by taking into account the effect of Radiation, Chemical Reaction, viscous dissipation, heat source and Hall current. The governing equations for this investigation are solved analytically using two-term harmonic and non-harmonic functions. The expressions for velocity, temperature and concentration are obtained and discussed through graphs. The Skin friction, the rate of heat transfer in the form of Nusselt number and the rate of mass transfer in the form of Sherwood number are also derived and discussed through tables.*

Keywords: *Radiation, Chemical Reaction, MHD, heat and mass transfer, free convection flow, porous medium, viscous dissipation*

I. Introduction

The study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering. In particular, the study of chemical reaction, heat and mass transfer with radiation is of considerable importance in chemical and Hydrometallurgical industries. Hall current effects are likely to be important in many astrophysical and geophysical situations as well as in engineering problems such as Hall accelerators, constructions of turbines and centrifugal machines.

There has been a renewed interest in studying magneto hydrodynamic (MHD) flow and heat transfer in porous medium due to the effect of magnetic fields on the boundary layer flow control and on the performance of many systems using electrically conducting fluids. In addition, this type of flow finds applications in many engineering problems such as MHD generators, plasma studies, nuclear reactors and geothermal energy extractions. The phenomenon of heat and mass transfer frequently exist in chemically processed industries such as food processing and polymer production. Free convection flow involving coupled heat and mass transfer occurs frequently in nature. Magneto hydrodynamic has attracted the attention of a large number of scholars due to its diverse applications. Convective heat transfer through porous media has been a subject of great interest for the last three decades.

Kim et al. [1] and Harris et al. [2] solved the problem of natural convection flow through porous medium past a plate. The effects of heat and mass transfer on a free convection flow near an infinite vertical porous plate has been extensively investigated by Takhar et al.[3], Hossain et al. [4], Israel et al.[5], Sahoo et al [6], Ali [7], Chaudhary and Jain [8]. The vertical free convection boundary layer flow in porous media owing to combined heat and mass transfer has been investigated by Bejan and Khair [9]. Soundalgekar [10] analysed the effects of variable suction and the horizontal magnetic field on the free convection flow past infinite vertical porous plate and made a comparative discussion of different parameters and the free convection flow of mercury and ionized air.

Similarity solutions for hydro magnetic simultaneous heat and mass transfer by natural convection from an inclined plate with internal heat generation or absorption studied by Chamkha and Khaled [11]. Soundalgekar [12] presented an exact solution to the flow of a viscous fluid past an impulsively started infinite isothermal vertical plate with mass transfer. Muthucumaraswamy and Ganesan [13] have studied numerical solution of flow past an impulsively started semi-infinite isothermal vertical plate with uniform mass diffusion.

Chaudhary et.al. [14] have studied the MHD flow past an infinite vertical oscillating plate through porous medium, taking account of the presence of free convection and mass transfer. Chaudhary and Jain [15] have discussed the combined effects of oscillating surface temperature and fluctuating surface velocity, on hydro magnetic convection with internal heat absorption.

A study of Hall effects over the heat and mass transfer flow of viscoelastico fluid is made by Chaudhary et al. [16]. Recently Singh and Kumar [17] have investigated the heat and mass transfer MHD flow through porous medium.

Anjalidevi and Kandasamy [18] have studied the effect of a chemical reaction on the flow in the presence of heat transfer and magnetic field. Muthucumaraswamy and Ganesan [19] have analyzed the effect of a chemical reaction on the unsteady flow past an impulsively started semi-infinite vertical plate, which is subject to uniform heat flux. McLeod and Rajagopal [20] have investigated the uniqueness of the flow of a Navier Stokes fluid due to a linear stretching boundary.

Rajput and Sahu [21] studied the effect of a uniform transverse magnetic field on the unsteady transient free convection flow of an incompressible viscous electrically conducting fluid between two infinite vertical parallel porous plates with constant temperature and variable mass diffusion. Nagamanemma et al. [22] have studied unsteady MHD free convective Heat and mass transfer flow near a moving vertical porous plate with radiation & thermo diffusion effects. Analytical study of MHD free convective heat and mass transfer flow bounded by an infinite vertical plate with thermal radiation and chemical reaction effects have been investigated by Nagamanemma et al. [23]. Hemant Poonia and Chaudhary, R.C., [24] have studied the MHD free convection and mass transfer flow over an infinite vertical porous plate with viscous dissipation.

The objective of the present paper is to analyze the heat and mass transfer effects on an unsteady hydromagnetic free convective boundary layer flow of viscous, incompressible, electrically conducting fluid, along a vertical plate with suction, embedded in porous medium, in the presence of transverse magnetic field, by taking into account the effects of radiation, Hall current, chemical reaction, viscous dissipation with heat source. The equation of continuity, motion, energy and mass transfer, which govern the flow field are solved by using a regular perturbation method. The behavior of velocity, temperature, concentration has been discussed for variations in the governing parameters.

II. Mathematical Model

An unsteady two-dimensional hydro magnetic free convective boundary layer flow of a viscous incompressible electrically conducting fluid past an infinite vertical flat plate through porous medium in presence of uniform transverse magnetic field B0 applied on this plate. The co-ordinate system is such that the *x*- axis* is taken along the plate and *y* - axis* is normal to the plate (Fig.1). All the fluid properties are considered to be constant except the influence of the density variation in the buoyancy term. Assume the suction velocity to be time dependent.

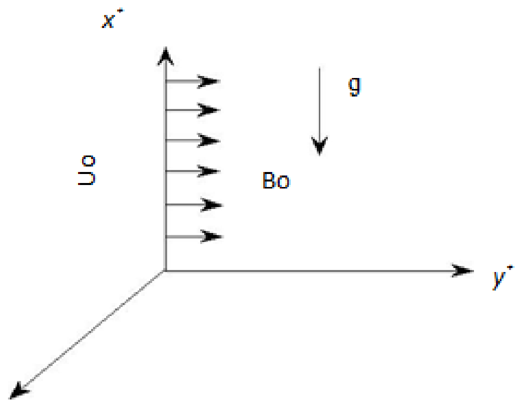

Fig.1 A schematic of the problem and coordinate system

Under the usual Boussinesq's approximation, the governing boundary layer equations are:

Equation of Continuity:

$$\frac{\partial v'}{\partial y'}=0 \tag{1}$$

Equation of Motion:

$$\upsilon\frac{\partial^2 u'}{\partial y'^2}-v'\frac{\partial u'}{\partial y'}=\frac{\partial u'}{\partial t'}-g\,\beta(T'-T'_\infty)-g\beta^*(C'-C'_\infty)+\frac{\sigma B_0{}^2}{\rho(1+m^2)}u'+\frac{\upsilon}{k'}u' \tag{2}$$

Equation of Energy:

$$\frac{\partial T'}{\partial t'}+v'\frac{\partial T'}{\partial y'}=a\frac{\partial^2 T'}{\partial y'^2}+\frac{\mu}{\rho C_p}\left(\frac{\partial u'}{\partial y'}\right)^2-\frac{1}{\rho C_p}\frac{\partial q_r^{*}}{\partial y^{*}}-\frac{Q_0}{\rho Cp}(T'-T'_\infty)+\frac{\sigma B_0{}^2}{\rho C_p(1+m^2)}u'^2 \qquad (3)$$

Equation of Mass Transfer:

$$\frac{\partial C'}{\partial t'}+v'\frac{\partial C'}{\partial y'}=D_M\frac{\partial^2 C'}{\partial y'^2}+D_T\frac{\partial^2 T'}{\partial y'^2}-K_1(C'-C'_\infty) \qquad (4)$$

Where, u', v' - denote the components of velocity in the boundary layer in x' and y' direction respectively; T' - the temperature in the boundary; T'_∞ - the temperature of the free stream; t' - the time; β and β^* the volumetric coefficient of thermal and concentration expansion respectively; ρ - the density of the fluid; μ - the coefficient of viscosity; g - the acceleration due to the gravity; υ - the kinematics viscosity; σ - the electrical conductivity; C_P - the heat capacity of the fluid; $a=\frac{k}{\rho Cp}$ (the thermal diffusivity); K - the coefficient of thermal conductivity; B_0 - the magnetic induction; C' - the concentration in the boundary layer; C'_∞ - the concentration in the fluid far away from the plate; D - the mass diffusivity.

Cogley et al. [25] have shown that, in the optically thin limit for a non-gray gas near equilibrium, the radiative heat flux is represented by the following form:

$$\frac{\partial q_r^*}{\partial y^*}=4\left(T'-T'_\infty\right)I^* \qquad (5)$$

Where $I^*=\int K_{\lambda w}\frac{\partial e b\lambda}{\partial T'}d\lambda \qquad (6)$

The boundary conditions for the velocity, temperature and concentration fields are:

$$\begin{aligned} &u' = 0,\quad T' = T'_\infty+T_0(t)(T'_0-T'_\infty),\quad C'=C'_\infty+C_0(t)(C'_0-C'_\infty)\quad for\ y'=0\\ &u'\to 0,\quad T'\to T'_\infty\ ,\quad C'\to C'_\infty\quad as\ y'\to\infty \end{aligned} \qquad (7)$$

From equation (1), it is clear that the suction velocity normal to the plate is either a constant or a function of the time. Hence, it is assumed in the form

$$v'=-v_0(1+\varepsilon\alpha e^{-nt}) \qquad (8)$$

Where, α is a real positive constant, ε and $\varepsilon\alpha$ are small less than unity, n is positive constant and v_0 is a non-zero positive constant suction velocity, the negative sign indicates that the suction is towards the plate.

The Non-dimensional quantities are defined as:

$$u=\frac{u'}{v_0},\ y=\frac{v_0 y'}{\upsilon},\ t=\frac{t'v_0^2}{4\upsilon},\ Sc=\frac{\upsilon}{D_M},\ \theta=\frac{T'-T'_\infty}{T'_0-T'_\infty},\ K=\frac{K'v_0^2}{\upsilon^2},$$

$$\Pr=\frac{\mu c_p}{k},\ M=\frac{\sigma B_0^2\upsilon}{\rho v_0^2},\ Gr=\frac{g\beta\upsilon(T'_0-T'_\infty)}{v_0^3},\ S=\frac{Q_0\upsilon}{\rho c_p v_0^2},\ R=\frac{4\upsilon I^*}{\rho c_p v_0^2}$$

$$T_0(t)=1+\varepsilon e^{-nt},\ \psi=\frac{C'-C'_\infty}{C'_0-C'_\infty},\ Gc=\frac{g\beta^*\upsilon(C'_0-C'_\infty)}{v_0^3},\ Ec=\frac{v_0^2}{Cp(T'_0-T'_\infty)}$$

$$S_0=\frac{D_T}{\upsilon}\frac{(T'_0-T'_\infty)}{(C'_0-C'_\infty)},\ \gamma=\frac{K_1\upsilon}{v_0^2} \qquad (9)$$

Using (9), the governing equations (2), (3) and (4) reduce to the following non-dimensional form:

$$\frac{\partial^2 u}{\partial y^2} + (1+\varepsilon\alpha e^{-nt})\frac{\partial u}{\partial y} = \frac{1}{4}\frac{\partial u}{\partial t} - G_r\,\theta - Gc\,\psi + N u \quad (10)$$

$$\frac{\partial^2 \theta}{\partial y^2} + \Pr(1+\varepsilon\alpha e^{-nt})\frac{\partial \theta}{\partial y} = \frac{1}{4}\Pr\frac{\partial \theta}{\partial t} - Ec\,P_r\,(\frac{\partial u}{\partial y})^2 + P_r\,S_1\,\theta - Ec\Pr M_1\,u^2 \quad (11)$$

$$\frac{\partial^2 \psi}{\partial y^2} + Sc\;(1+\varepsilon\alpha e^{-nt})\frac{\partial \psi}{\partial y} = \frac{1}{4}\,Sc\,\frac{\partial \psi}{\partial t} - Sc\,S_0\,\frac{\partial^2 \theta}{\partial y^2} + Sc\;\gamma\,\psi \quad (12)$$

Where $M_1 = \dfrac{M}{1+m^2}$, $N = M_1 + \dfrac{1}{K}$, $S_1 = R + S$, T_0 (t) - the temperature at the wall; M - the Magnetic parameter; $\Pr$ - the Prandtl number; K - the Porosity parameter; Gr - the thermal Grashof number; Gc - the solutal Grashof number; Ec - the Eckert number; Sc - the Schmidt number; S_0 - the Soret number; S - the Heat source; R - Radiation parameter, m is the hall current parameter and γ - Chemical reaction.

The boundary condition (7) in the dimensionless form can be written as

$$u = 0,\quad \theta = T_0(t),\quad \psi = C_0(t) \qquad for\;\; y = 0$$
$$u \to 0,\quad \theta \to 0\;,\quad \psi \to 0 \qquad as\;\; y \to \infty \quad (13)$$

III. Solution Of The Problem

For the solution of equations (10), (11) and (12), we assume

$$u(y,t) = u_1(y) + \varepsilon\,e^{-nt}\,u_2(y)$$
$$\theta(y,t) = 1+\varepsilon\,e^{-nt} - \theta_1(y) - \varepsilon\,e^{-nt}\;\theta_2(y)$$
$$\psi(y,t) = 1+\varepsilon\,e^{-nt} - \psi_1(y) - \varepsilon\,e^{-nt}\;\psi_2(y)$$

(14)

Substituting equation (14) in equations (10), (11) and (12), equating harmonic terms and neglecting coefficient of ε^2, we get

$$u_1''(y) + u_1'(y) - N\,u_1(y) = -Gr[1-\theta_1(y)] - Gc\,[1-\psi_1(y)] \quad (15)$$

$$u_2''(y) + u_2'(y) - N_1\,u_2(y) = -Gr[1-\theta_2(y)] - Gc[1-\psi_2(y)] - \alpha\,u_1'(y) \quad (16)$$

$$\theta_1''(y) + \Pr\,\theta_1'(y) - \Pr\,S_1\,\theta_1(y) = Ec\;\Pr\,[u_1'(y)]^2 - \Pr\,S_1 + Ec\Pr M_1 u_1^{\,2} \quad (17)$$

$$\theta_2''(y) + \Pr\,\theta_2'(y) - \Pr S_2\;\theta_2(y) = 2\,Ec\;\Pr u_1'(y)\,u_2'(y) - \alpha\Pr\,\theta_1'(y) - 2Ec\Pr M_1 u_1 u_2 - \Pr S_2 \quad (18)$$

$$\psi_1''(y) + Sc\,\psi_1'(y) - Sc\,\gamma\,\psi_1(y) = -Sc\,S_0\,\theta_1''(y) - Sc\,\gamma \quad (19)$$

$$\psi_2''(y) + Sc\,\psi_2'(y) - Sc\,\gamma_1\,\psi_2(y) = -Sc\,S_0\,\theta_2''(y) - \alpha\,Sc\,\psi_1'(y) - Sc\,\gamma_1 \quad (20)$$

Where $N_1 = N - \dfrac{n}{4}$, $S_2 = S_1 - \dfrac{n}{4}$, $\gamma_1 = \gamma - \dfrac{n}{4}$

The corresponding boundary conditions are

$$u_1 = 0,\quad u_2 = 0,\quad \theta_1 = 0,\quad \theta_2 = 0,\quad \psi_1 = 0,\quad \psi_2 = 0 \qquad for\;\; y = 0$$
$$u_1 \to 0,\quad u_2 \to 0,\quad \theta_1 \to 1,\quad \theta_2 \to 1,\quad \psi_1 \to 1,\quad \psi_2 \to 1 \qquad as\;\; y \to \infty \quad (21)$$

The equations (15) to (20) are still coupled and non-linear, whose exact solution are not possible, so we can expand u_1, u_2, θ_1, θ_2, ψ_1, ψ_2 in terms of Ec (Eckert number) in following form, as the Eckert number is very small for incompressible flows.

$$u_1(y) = u_{11}(y) + Ec\,u_{12}(y)$$
$$u_2(y) = u_{21}(y) + Ec\,u_{22}(y)$$
$$\theta_1(y) = \theta_{11}(y) + Ec\;\theta_{12}(y)$$
$$\theta_2(y) = \theta_{21}(y) + Ec\;\theta_{22}(y)$$
$$\psi_1(y) = \psi_{11}(y) + Ec\;\psi_{12}(y)$$

$$\psi_2(y) = \psi_{21}(y) + Ec\ \psi_{22}(y) \tag{22}$$

Introducing equations (22) into equations (15) to (20), we obtain the following systems of equations.

$$u_{11}''(y) + u_{11}'(y) - N\, u_{11}(y) = -Gr[1-\theta_{11}(y)] - Gc[1-\psi_{11}(y)] \tag{23}$$

$$u_{12}''(y) + u_{12}'(y) - N\, u_{12}(y) = Gr\ \theta_{12}(y) + Gc\ \psi_{12}(y) \tag{24}$$

$$u_{21}''(y) + u_{21}'(y) - N_1 u_{21}(y) = -Gr[1-\theta_{21}(y)] - Gc[1-\psi_{21}(y)] - \alpha u_{11}'(y) \tag{25}$$

$$u_{22}''(y) + u_{22}'(y) - N_1 u_{22}(y) = Gr\ \theta_{22}(y) + Gc\,\psi_{22}(y) - \alpha u_{12}'(y) \tag{26}$$

$$\theta_{11}''(y) + \Pr\theta_{11}'(y) - \Pr S_1\ \theta_{11}(y) = -\Pr S_1 \tag{27}$$

$$\theta_{12}''(y) + \Pr\theta_{12}'(y) - \Pr S_1\ \theta_{12}(y) = \Pr[u_{11}'(y)]^2 + \Pr M_1 u_{11}{}^2 \tag{28}$$

$$\theta_{21}''(y) + \Pr\theta_{21}'(y) - \Pr S_2\,\theta_{21}(y) = -\alpha\ \Pr\theta_{11}'(y) - \Pr S_2 \tag{29}$$

$$\theta_{22}''(y) + \Pr\theta_{22}'(y) - \Pr S_2\,\theta_{22}(y) = 2\Pr u_{11}'(y) u_{21}'(y) - \alpha\ \Pr\theta_{12}'(y) - 2\Pr M_1 u_{11}u_{21} \tag{30}$$

$$\psi_{11}''(y) + Sc\,\psi_{11}'(y) - Sc\,\gamma\,\psi_{11}(y) = -Sc\ S_0\ \theta_{11}''(y) - Sc\,\gamma \tag{31}$$

$$\psi_{12}''(y) + Sc\,\psi_{12}'(y) - Sc\,\gamma\,\psi_{12}(y) = -Sc\ S_0\ \theta_{12}''(y) \tag{32}$$

$$\psi_{21}''(y) + Sc\,\psi_{21}'(y) - Sc\gamma_1\psi_{21}(y) = -Sc\ S_0\,\theta_{21}''(y) - \alpha Sc\,\psi_{11}'(y) - Sc\gamma_1 \tag{33}$$

$$\psi_{22}''(y) + Sc\,\psi_{22}'(y) - Sc\gamma_1\ \psi_{22}(y) = -Sc\ S_0\,\theta_{22}''(y) - \alpha Sc\,\psi_{12}'(y) \tag{34}$$

The corresponding boundary conditions are

$$u_{11}=0,\ u_{12}=0,\ u_{21}=0,\ u_{22}=0,\ \theta_{11}=0,\ \theta_{12}=0,\ \theta_{21}=0,\ \theta_{22}=0,$$
$$\psi_{11}=0,\ \psi_{12}=0,\ \psi_{21}=0,\ \psi_{22}=0 \quad for \quad y=0$$

$$u_{11}\to 0,\ u_{12}\to 0,\ u_{21}\to 0,\ u_{22}\to 0,\ \theta_{11}\to 1,\ \theta_{12}\to 0,\ \theta_{21}\to 1,\ \theta_{22}\to 0,$$
$$\psi_{11}\to 1,\ \psi_{12}\to 0,\ \psi_{21}\to 1,\ \psi_{22}\to 0 \quad as \quad y\to\infty \tag{35}$$

Solving the equations (23) to (34), under the boundary conditions, we get

$$u_{11} = A_3 e^{-m_1 y} + A_4 e^{-m_2 y} + A_5 e^{-m_3 y} \tag{36}$$

$$u_{12} = A_{21} e^{-m_1 y} + A_{22} e^{-m_2 y} + A_{29} e^{-m_3 y} + A_{23} e^{-2m_1 y} + A_{24} e^{-2m_2 y} + A_{25} e^{-2m_3 y} + A_{26} e^{-(m_1+m_2)y} + A_{27} e^{-(m_1+m_3)y} + A_{28} e^{-(m_2+m_3)y} \tag{37}$$

$$u_{21} = A_{36} e^{-m_1 y} + A_{37} e^{-m_2 y} + A_{38} e^{-m_3 y} + A_{39} e^{-m_4 y} + A_{40} e^{-m_5 y} + A_{41} e^{-m_6 y} \tag{38}$$

$$u_{22} = A_{78} e^{-m_1 y} + A_{79} e^{-m_2 y} + A_{80} e^{-m_3 y} + A_{81} e^{-m_4 y} + A_{82} e^{-m_5 y} + A_{98} e^{-m_6 y} + A_{83} e^{-2m_1 y}$$
$$+ A_{84} e^{-2m_2 y} + A_{85} e^{-2m_3 y} + A_{86} e^{-(m_1+m_2)y} + A_{87} e^{-(m_1+m_3)y} + A_{88} e^{-(m_1+m_4)y} + A_{89} e^{-(m_1+m_5)y}$$
$$+ A_{90} e^{-(m_1+m_6)y} + A_{91} e^{-(m_2+m_3)y} + A_{92} e^{-(m_2+m_4)y} + A_{93} e^{-(m_2+m_5)y} + A_{94} e^{-(m_2+m_6)y} + A_{95} e^{-(m_3+m_4)y}$$
$$+ A_{96} e^{-(m_3+m_5)y} + A_{97} e^{-(m_3+m_6)y} \tag{39}$$

$$\theta_{11} = 1 - e^{-m_1 y} \tag{40}$$

$$\theta_{12} = A_{12} e^{-m_1 y} + A_6 e^{-2m_1 y} + A_7 e^{-2m_2 y} + A_8 e^{-2m_3 y} + A_9 e^{-(m_1+m_2)y} + A_{10} e^{-(m_1+m_3)y} + A_{11} e^{-(m_2+m_3)y} \tag{41}$$

$$\theta_{21} = 1 + A_{30} e^{-m_1 y} + A_{31} e^{-m_4 y} \tag{42}$$

$$\theta_{22} = A_{42} e^{-m_1 y} + A_{58} e^{-m_4 y} + A_{43} e^{-2m_1 y} + A_{44} e^{-2m_2 y} + A_{45} e^{-2m_3 y} + A_{46} e^{-(m_1+m_2)y} + A_{47} e^{-(m_1+m_3)y}$$
$$+ A_{48} e^{-(m_1+m_4)y} + A_{49} e^{-(m_1+m_5)y} + A_{50} e^{-(m_1+m_6)y} + A_{51} e^{-(m_2+m_3)y} + A_{52} e^{-(m_2+m_4)y} + A_{53} e^{-(m_2+m_5)y}$$
$$+ A_{54} e^{-(m_2+m_6)y} + A_{55} e^{-(m_3+m_4)y} + A_{56} e^{-(m_3+m_5)y} + A_{57} e^{-(m_3+m_6)y} \tag{43}$$

$$\psi_{11} = 1 + A_1 e^{-m_1 y} + A_2 \; e^{-m_2 y} \tag{44}$$

$$\psi_{12} = A_{13} e^{-m_1 y} + A_{20} e^{-m_2 y} + A_{14} e^{-2m_1 y} + A_{15} e^{-2m_2 y} + A_{16} e^{-2m_3 y} + A_{17} e^{-(m_1+m_2) y} + A_{18} e^{-(m_1+m_3) y} + A_{19} e^{-(m_2+m_3) y} \tag{45}$$

$$\psi_{21} = 1 + A_{32} e^{-m_1 y} + A_{33} e^{-m_2 y} + A_{34} e^{-m_4 y} + A_{35} e^{-m_5 y} \tag{46}$$

$$\psi_{22} = A_{59} e^{-m_1 y} + A_{60} e^{-m_2 y} + A_{61} e^{-m_4 y} + A_{77} e^{-m_5 y} + A_{62} e^{-2m_1 y} + A_{63} e^{-2m_2 y} + A_{64} e^{-2m_3 y} + A_{65} e^{-(m_1+m_2)y} + A_{66} e^{-(m_1+m_3)y} + A_{67} e^{-(m_1+m_4)y} + A_{68} e^{-(m_1+m_5)y} + A_{69} e^{-(m_1+m_6)y} + A_{70} e^{-(m_2+m_3)y} + A_{71} e^{-(m_2+m_4)y} + A_{72} e^{-(m_2+m_5)y} + A_{73} e^{-(m_2+m_6)y} + A_{74} e^{-(m_3+m_4)y} + A_{75} e^{-(m_3+m_5)y} + A_{76} e^{-(m_3+m_6)y}$$

(47)

The Non-dimensional **skin friction** at the surface is given by

$$\tau = (\frac{\partial u}{\partial y})_{y=0} = -(m_1 A_3 + m_2 A_4 + m_3 A_5) - Ec(m_1 A_{21} + m_2 A_{22} + m_3 A_{29} + 2m_1 A_{23} + 2m_2 A_{24} + 2m_3 A_{25} + (m_1+m_2)A_{26} + (m_1+m_3)A_{27} + (m_2+m_3)A_{28}) - \varepsilon e^{-nt}(m_1 A_{36} + m_2 A_{37} + m_3 A_{38} + m_4 A_{39} + m_5 A_{40} + m_6 A_{41}) - Ec\, \varepsilon e^{-nt}(m_1 A_{78} + m_2 A_{79} + m_3 A_{80} + m_4 A_{81} + m_5 A_{82} + m_6 A_{98} + 2m_1 A_{83} + 2m_2 A_{84} + 2m_3 A_{85} + (m_1+m_2)A_{86} + (m_1+m_3)A_{87} + (m_1+m_4)A_{88} + (m_1+m_5)A_{89} + (m_1+m_6)A_{90} + (m_2+m_3)A_{91} + (m_2+m_4)A_{92} + (m_2+m_5)A_{93} + (m_2+m_6)A_{94} + (m_3+m_4)A_{95} + (m_3+m_5)A_{96} + (m_3+m_6)A_{97}) \tag{48}$$

The **rate of heat transfer** in terms of the Nusselt number is given by

$$Nu = -(\frac{\partial \theta}{\partial y})_{y=0} = m_1 - Ec(m_1 A_{12} + 2m_1 A_6 + 2m_2 A_7 + 2m_3 A_8 + (m_1+m_2)A_9 + (m_1+m_3)A_{10} + (m_2+m_3)A_{11}) - \varepsilon e^{-nt}(m_1 A_{30} + m_4 A_{31}) - Ec\, \varepsilon e^{-nt}(m_1 A_{42} + m_4 A_{58} + 2m_1 A_{43} + 2m_2 A_{44} + 2m_3 A_{45} + (m_1+m_2)A_{46} + (m_1+m_3)A_{47} + (m_1+m_4)A_{48} + (m_1+m_5)A_{49} + (m_1+m_6)A_{50} + (m_2+m_3)A_{51} + (m_2+m_4)A_{52} + (m_2+m_5)A_{53} + (m_2+m_6)A_{54} + (m_3+m_4)A_{55} + (m_3+m_5)A_{56} + (m_3+m_6)A_{57}) \tag{49}$$

The **rate of mass transfer** in the form of Sherwood number is given by

$$S_h = -(\frac{\partial \psi}{\partial y})_{y=0} = -(m_1 A_1 + m_2 A_2) - Ec(m_1 A_{13} + m_2 A_{20} + 2m_1 A_{14} + 2m_2 A_{15} + 2m_3 A_{16} + (m_1+m_2)A_{17} + (m_1+m_3)A_{18} + (m_2+m_3)A_{19}) - \varepsilon e^{-nt}(m_1 A_{32} + m_2 A_{33} + m_4 A_{34} + m_5 A_{35}) - Ec\, \varepsilon e^{-nt}(m_1 A_{59} + m_2 A_{60} + m_4 A_{61} + m_5 A_{77} + 2m_1 A_{62} + 2m_2 A_{63} + 2m_3 A_{64} + (m_1+m_2)A_{65} + (m_1+m_3)A_{66} + (m_1+m_4)A_{67} + (m_1+m_5)A_{68} + (m_1+m_6)A_{69} + (m_2+m_3)A_{70} + (m_2+m_4)A_{71} + (m_2+m_5)A_{72} + (m_2+m_6)A_{73} + (m_3+m_4)A_{74} + (m_3+m_5)A_{75} + (m_3+m_6)A_{76}) \tag{50}$$

IV. Results And Discussion

The non linear coupled equations (10)-(12) subject to boundary conditions (13), which illustrate the hydro magnetic free convective boundary layer flow of an incompressible and electrically-conducting fluid along an infinite vertical plate embedded in the porous medium with heat and mass transfer flow by taking into account the effect of Radiation, Chemical Reaction, viscous dissipation Hall current with heat source were solved analytically using a perturbation method and the expressions for the velocity, temperature, concentration were obtained. In order to get a physical insight of the problem, the above physical quantities are computed numerically for different values of the governing parameters.

The velocity profile for the different values of Magnetic parameter (M), thermal Grashof number (Gr), solutal Grashof number (Gc), Prandtl number (Pr), Schmidt number (Sc), Porosity parameter (K), Radiation parameter (R), Hall current parameter (m), Soret number (S_0), Eckert number (Ec) and Heat source parameter (S) are shown in the figures 2 -12 respectively. From these figures it is observed that the velocity increases as K, Gr, Gc, m, S_0 , and Ec increases. While velocity decreases as M, R, S, Pr and Sc increases.

Figure 13 - figure 16 shows that the temperature profile for the different values of the Prandtl number (Pr), the Radiation parameter (R), the Eckert number (Ec) and the Heat source parameter (S). It is noticed that temperature decreases as Pr, R and S increases. While it increases as Ec increases.

The concentration profile for different values of the Schmidt number (Sc), the Soret number (S_0) and the Chemical reaction parameter (γ) are shown in figures 17-19 respectively. It is noticed that the concentration decreases as Sc and γ increases. While it increases as S_0 increases.

From table 1 it is noticed that an increasing the thermal Grashof number (Gr), the solutal Grashof number (Gc), the Soret number (S_0), the hall current parameter (m), the Eckert number (Ec), ε and the Porosity parameter (K) results an increasing Skin friction. While it decreases with increase of Prandtl number (Pr), the Magnetic parameter (M), the Radiation parameter (R), the Heat source parameter (S), the Chemical reaction parameter (γ) and the Schmidt number (Sc) respectively.

Table 2 discuss the effects of Prandtl number (Pr), the Radiation parameter (R), the Heat source parameter (S), the Eckert number (Ec), the Magnetic parameter (M), the hall current parameter (m), ε , n and time (t) numerically on rate of heat transfer (Nu). It is noticed that the rate of heat transfer increases with increasing of Pr, R, M, ε and S. While it decreasing with increasing Ec, m, n and time (t) respectively.

Table 3 shows the effects of the Schmidt number (Sc), the Soret number (S_0), the Chemical reaction parameter (γ), the Eckert number (Ec), ε , n and time (t) on rate of mass transfer (Sh) numerically. It is observed that the rate of mass transfer increases with increasing γ , Ec and ε . While it decreases with increasing Sc, S_0 , n and time (t) respectively.

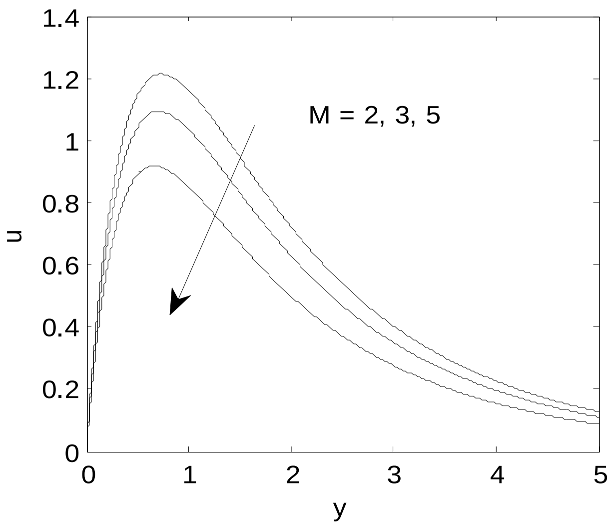

Fig2: Velocity distribution for various values of M

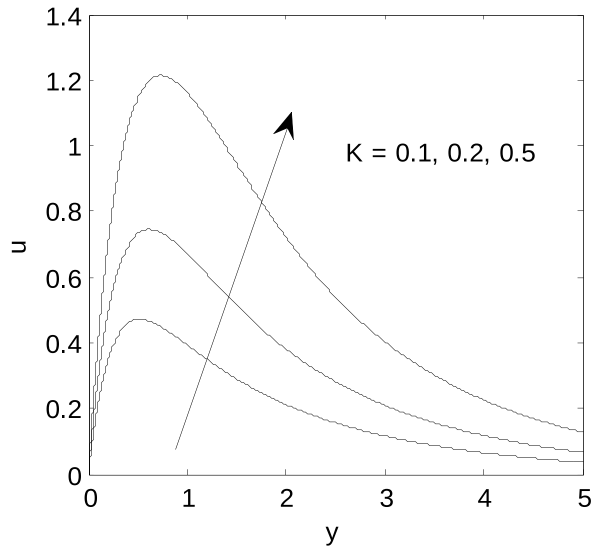

Fig3: Velocity distribution for various values of K

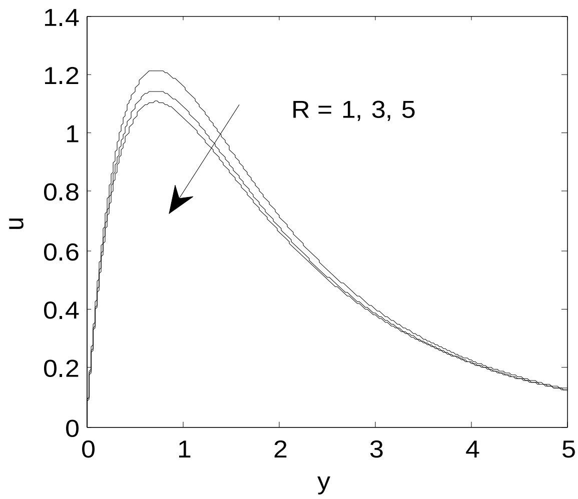

Fig4: Velocity distribution for various values of R

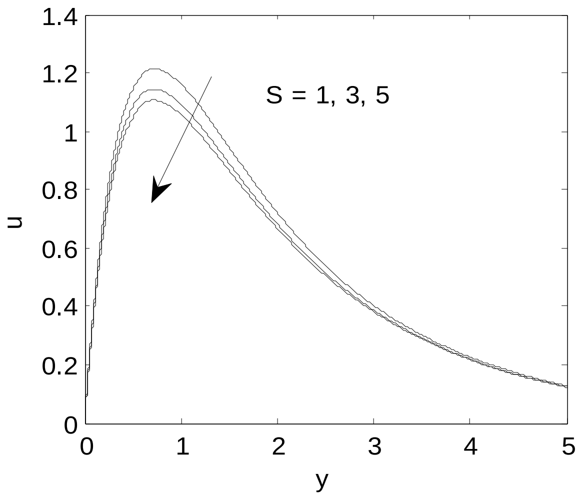

Fig5: Velocity distribution for various values of S

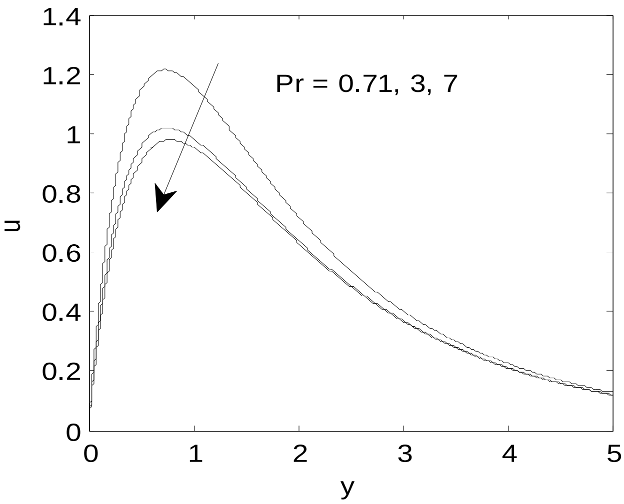

Fig6: Velocity distribution for various values of Pr

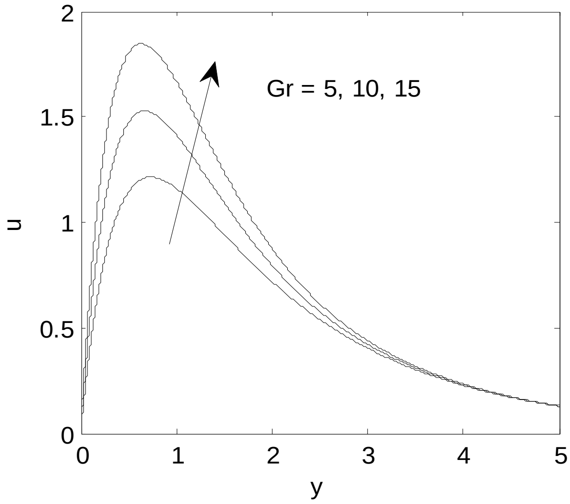

Fig7: Velocity distribution for various values of Gr

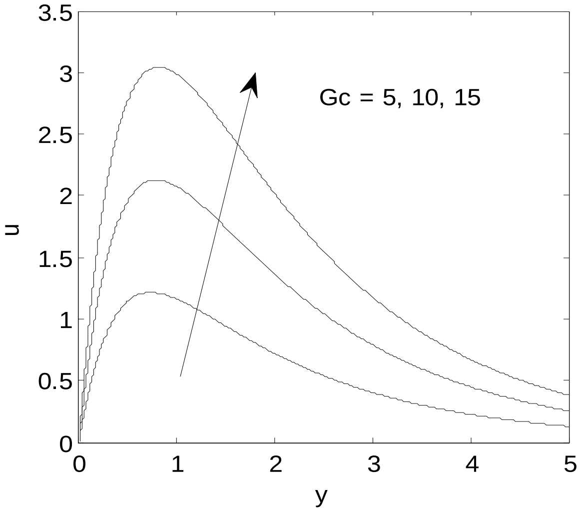

Fig8: Velocity distribution for various values of Gc

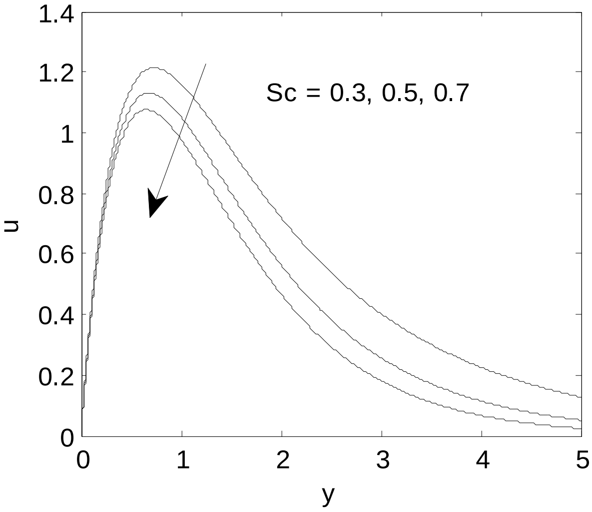

Fig9: Velocity distribution for various values of Sc

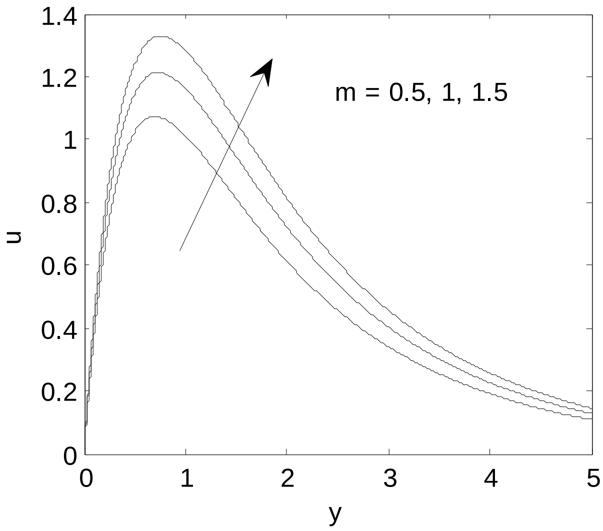

Fig10: Velocity distribution for various values of m

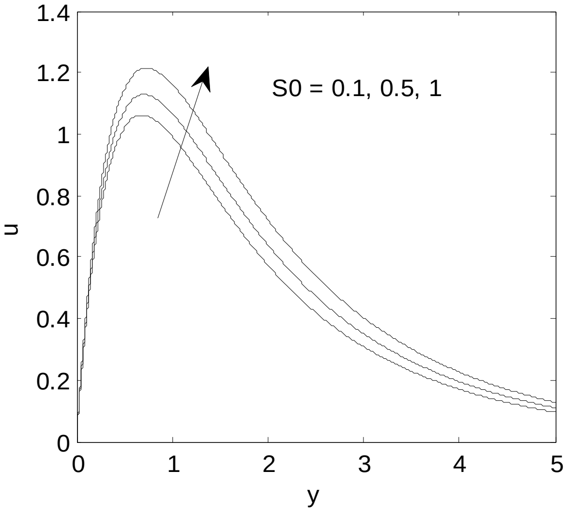

Fig11: Velocity distribution for various values of S0

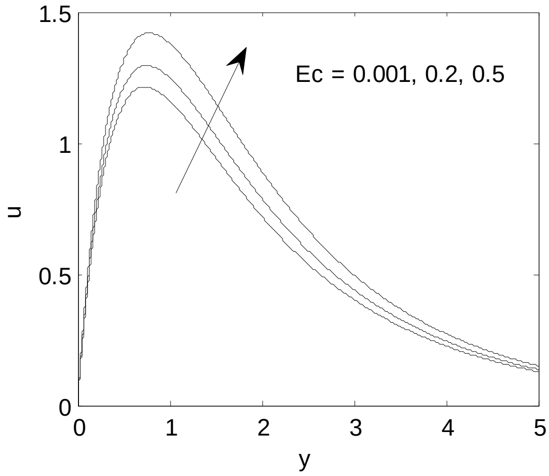

Fig12: Velocity distribution for various values of Ec

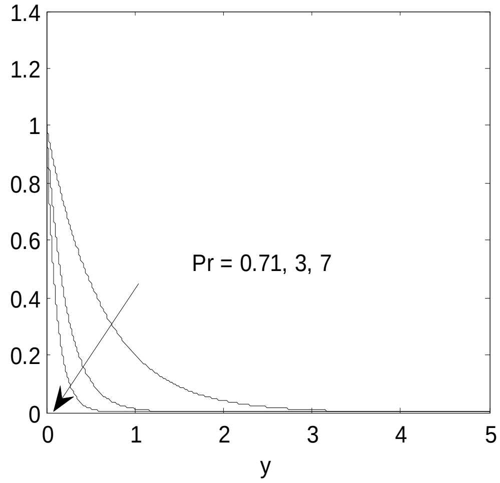

Fig13: Temperature distribution for various values of Pr

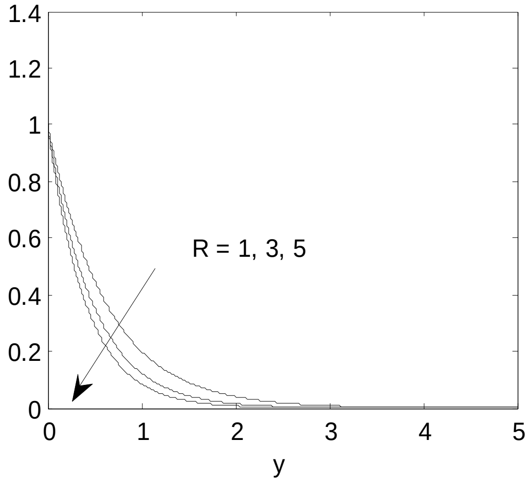

Fig14: Temperature distribution for various values of R

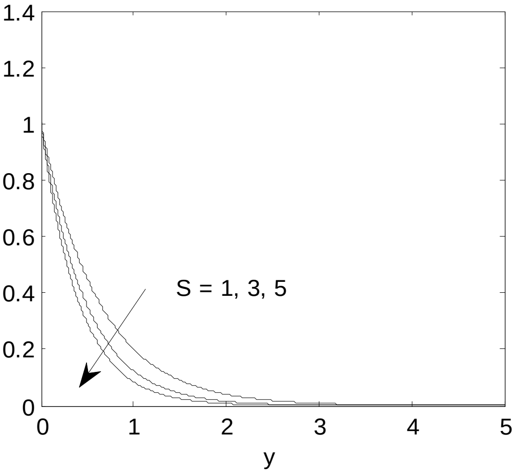

Fig15: Temperature distribution for various values of S

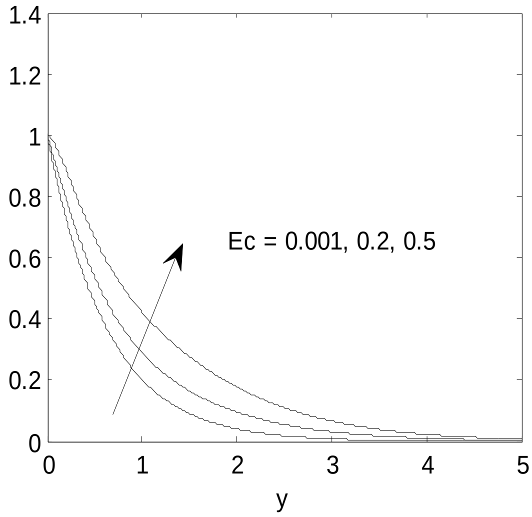

Fig16: Temperature distribution for various values of Ec

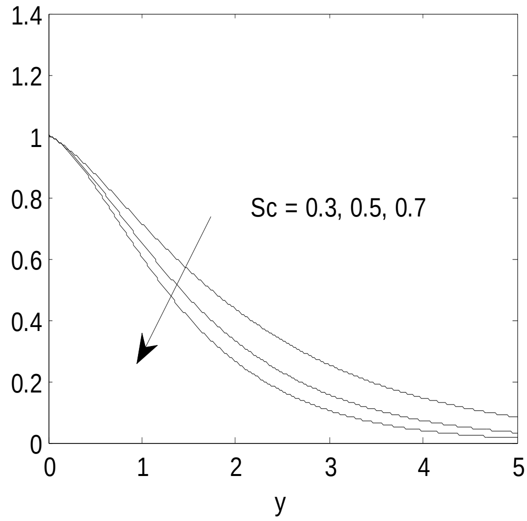

Fig17: Concentration distribution for various values of Sc

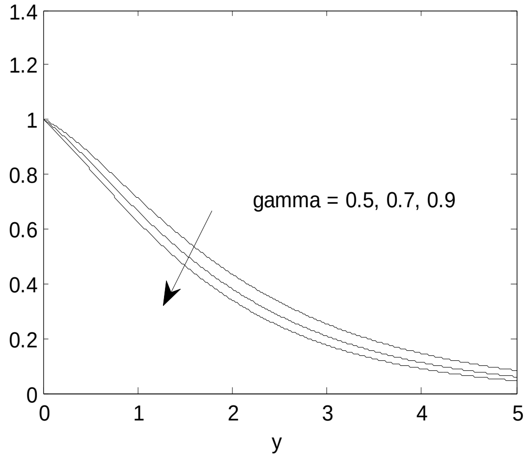

Fig18: Concentration distribution for various values of gamma

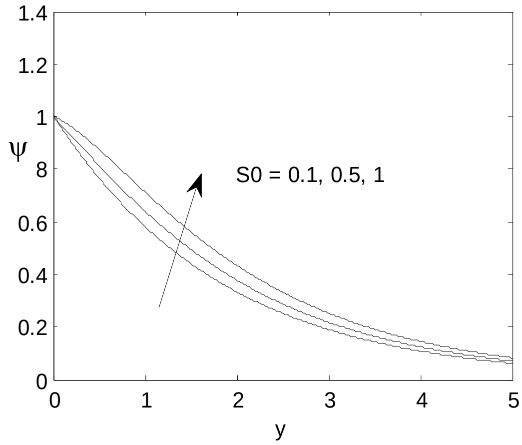

Fig19: Concentration distribution for various values of S0

Table 1: Skin friction (τ) for different parameter

Pr	**M**	**m**	**Gr**	**Gc**	**R**	**S**	**Sc**	**S0**	**K**	γ	**Ec**	ε	τ
0.71	2.00	1.00	5.00	5.00	1.00	1.00	0.30	1.00	0.50	0.50	0.001	0.01	**4.7999**
7.00	2.00	1.00	5.00	5.00	1.00	1.00	0.30	1.00	0.50	0.50	0.001	0.01	3.8738
0.71	**5.00**	1.00	5.00	5.00	1.00	1.00	0.30	1.00	0.50	0.50	0.001	0.01	4.0492
0.71	2.00	**1.50**	5.00	5.00	1.00	1.00	0.30	1.00	0.50	0.50	0.001	0.01	5.0762
0.71	2.00	1.00	**10.00**	5.00	1.00	1.00	0.30	1.00	0.50	0.50	0.001	0.01	6.5324
0.71	2.00	1.00	5.00	**10.00**	1.00	1.00	0.30	1.00	0.50	0.50	0.001	0.01	7.8700
0.71	2.00	1.00	5.00	5.00	**5.00**	1.00	0.30	1.00	0.50	0.50	0.001	0.01	4.5008
0.71	2.00	1.00	5.00	5.00	1.00	**1.50**	0.30	1.00	0.50	0.50	0.001	0.01	4.7409
0.71	2.00	1.00	5.00	5.00	1.00	1.00	**0.70**	1.00	0.50	0.50	0.001	0.01	4.5589
0.71	2.00	1.00	5.00	5.00	1.00	1.00	0.30	**2.00**	0.50	0.50	0.001	0.01	5.1811
0.71	2.00	1.00	5.00	5.00	1.00	1.00	0.30	1.00	**0.70**	0.50	0.001	0.01	5.2307
0.71	2.00	1.00	5.00	2.00	1.00	1.00	0.30	1.00	0.50	**1.00**	0.001	0.01	4.5411
0.71	2.00	1.00	5.00	2.00	1.00	1.00	0.30	1.00	0.50	0.50	**0.50**	0.01	5.2956
0.71	2.00	1.00	5.00	2.00	1.00	1.00	0.30	1.00	0.50	0.50	0.001	**0.50**	5.7567

Table 2: The rate of heat transfer Nu for different parameters

Pr	**R**	**S**	**Ec**	**ε**	**n**	**t**	**M**	**m**	**Nu**
0.71	1.00	1.00	0.001	0.01	1.00	1.00	2.00	1.00	**1.6021**
7.00	1.00	1.00	0.001	0.01	1.00	1.00	2.00	1.00	8.6512
0.71	**3.00**	1.00	0.001	0.01	1.00	1.00	2.00	1.00	2.0832
0.71	1.00	**3.00**	0.001	0.01	1.00	1.00	2.00	1.00	2.0832
0.71	1.00	1.00	**0.05**	0.01	1.00	1.00	2.00	1.00	1.4640
0.71	1.00	1.00	0.001	**0.05**	1.00	1.00	2.00	1.00	1.6279
0.71	1.00	1.00	0.001	0.01	**3.00**	1.00	2.00	1.00	1.5964
0.71	1.00	1.00	0.001	0.01	1.00	**5.00**	2.00	1.00	1.5957
0.71	1.00	1.00	0.001	0.01	1.00	1.00	**5.00**	1.00	1.6026
0.71	1.00	1.00	0.001	0.01	1.00	1.00	2.00	**1.50**	1.6019

Table 3: The rate of mass transfer Sh for different parameters

Sc	**S0**	**γ**	**Ec**	**ε**	**n**	**t**	**Sh**
0.30	1.00	0.50	0.001	0.01	1.00	1.00	**0.1552**
0.70	1.00	0.50	0.001	0.01	1.00	1.00	0.1153
0.30	**2.00**	0.50	0.001	0.01	1.00	1.00	-0.2566
0.30	1.00	**0.70**	0.001	0.01	1.00	1.00	0.2366
0.30	1.00	0.50	**0.05**	0.01	1.00	1.00	0.1959
0.30	1.00	0.50	0.001	**0.05**	1.00	1.00	0.1565
0.30	1.00	0.50	0.001	0.01	**3.00**	1.00	0.1548
0.30	1.00	0.50	0.001	0.01	1.00	**5.00**	0.1549

V. Conclusion

The Mathematical analysis has been presented on unsteady two dimensional hydromagnetic free convective viscous flow along infinite vertical plate through porous medium in the presence of transverse magnetic field taking into account the effect of radiation, chemical reaction, hall current, viscous dissipation and heat source. The dimensionless momentum, energy and mass conservation equations have been solved using Peturbation method. The numerical results are provided for velocity, temperature and concentration profiles to observe the effects of Prandtl number (Pr), Magnetic parameter (M), thermal Grashof number (Gr), solutal

Grashof number (Gc), Schmidt number (Sc), the hall current parameter (m), Porosity parameter (K), Radiation parameter (R), Heat source parameter (S), Soret number (S_0),Chemical reaction parameter (γ), Eckert number (Ec), ε , n and time (t). Also the numerical results are presented for skin friction, the rate of heat transfer in the form of Nusselt number and the rate of mass transfer in the form of Sherwood number through tables. From the steady the following conclusions are drawn:

- The velocity profile increases with increasing Gr, Gc, m, Ec, S_0 and K. While it decreases with increasing M, R, S, Pr and Sc.
- The temperature decreases with increasing the values of Pr, R and S.
- The concentration increases with increasing So, while it decreases with increasing Sc and γ .
- Velocity on skin friction increases with increasing Gr, Gc, m, Ec, ε , S_0 and K, while it decreases with increasing Pr, M, R, S, Sc and γ .
- The rate of heat transfer increases with increasing Pr, R, M, ε and S. While it decreases with increasing Ec, m, n and time (t).
- The rate of mass transfer in terms of Sherwood number increases with increasing γ , Ec and ε . While it decreases with increasing Sc, S_0 , n and time (t).

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Appendix:

$m_1=\frac{\Pr+\sqrt{\Pr^2+4\Pr\,S_1}}{2}$; $m_2=\frac{Sc+\sqrt{Sc^2+4Sc\,\gamma}}{2}$; $m_3=\frac{1+\sqrt{1+4N}}{2}$;

$m_4=\frac{\Pr+\sqrt{\Pr^2+4\Pr S_2}}{2}$; $m_5=\frac{Sc+\sqrt{Sc^2+4Sc\gamma_1}}{2}$; $m_6=\frac{1+\sqrt{1+4\,N_1}}{2}$;

$A_1=\frac{Sc\;S_0\;m_1^{\,2}}{m_1^{\,2}-Sc\,m_1-Sc\gamma}$; $A_2=-(A_1+1)$; $A_3=\frac{-Gr+Gc\;A_1}{m_1^{\,2}-m_1-N}$; $A_4=\frac{Gc\;A_2}{m_2^{\,2}-m_2-N}$

$A_5=-(A_3+A_4)$; $A_6=\frac{\Pr\;A_3^{\,2}\left(m_1^{\,2}+M_1\right)}{4m_1^{\,2}-2\Pr m_1-\Pr S_1}$; $A_7=\frac{\Pr\;A_4^{\,2}\left(m_2^{\,2}+M_1\right)}{4m_2^{\,2}-2\Pr m_2-\Pr S_1}$

$A_8=\frac{\Pr\;A_5^{\,2}\left(m_3^{\,2}+M_1\right)}{4m_3^{\,2}-2\Pr m_3-\Pr S_1}$; $A_9=\frac{2\Pr A_3A_4\left(m_1m_2+M_1\right)}{(m_1+m_2)^2-\Pr(m_1+m_2)-\Pr S_1}$;

$A_{10}=\frac{2\Pr A_3A_5\left(m_1m_3+M_1\right)}{(m_1+m_3)^2-\Pr(m_1+m_3)-\Pr S_1}$; $A_{11}=\frac{2\Pr A_4A_5\left(m_2m_3+M_1\right)}{(m_2+m_3)^2-\Pr(m_2+m_3)-\Pr S_1}$;

$A_{12}=-(A_6+A_7+A_8+A_9+A_{10}+A_{11})$; $A_{13}=\frac{-Sc\;S_0\;m_1^{\,2}\;A_{12}}{m_1^{\,2}-Sc\,m_1-Sc\gamma}$;

$A_{14}=\frac{-4Sc\;S_0\;m_1^{\,2}\;A_6}{4m_1^{\,2}-2Sc\,m_1-Sc\gamma}$

$A_{15}=\frac{-4Sc\;S_0\;m_2^{\,2}\;A_7}{4m_2^{\,2}-2Sc\,m_2-Sc\gamma}$; $A_{16}=\frac{-4Sc\;S_0\;m_3^{\,2}\;A_8}{4m_3^{\,2}-2Sc\,m_3-Sc\gamma}$;

$A_{17}=\frac{-Sc\,S_0\,(m_1+m_2)^2\,A_9}{(m_1+m_2)^2-Sc(m_1+m_2)-Sc\,\gamma}$;

$A_{18}=\frac{-Sc\,S_0\,(m_1+m_3)^2\,A_{10}}{(m_1+m_3)^2-Sc(m_1+m_3)-Sc\,\gamma}$; $A_{19}=\frac{-Sc\,S_0\,(m_2+m_3)^2\,A_{11}}{(m_2+m_3)^2-Sc(m_2+m_3)-Sc\,\gamma}$;

$A_{20}=-(A_{13}+A_{14}+A_{15}+A_{16}+A_{17}+A_{18}+A_{19})$;

$A_{21}=\frac{Gr\;A_{12}+Gc\;A_{13}}{m_1^{\,2}-m_1-N}$; $A_{22}=\frac{Gc\;A_{20}}{m_2^{\,2}-m_2-N}$; $A_{23}=\frac{Gr\;A_6+Gc\;A_{14}}{4m_1^{\,2}-2m_1-N}$;

$A_{24}=\frac{Gr\;A_7+Gc\;A_{15}}{4m_2^{\,2}-2m_2-N}$; $A_{25}=\frac{Gr\;A_8+Gc\;A_{16}}{4m_3^{\,2}-2m_3-N}$;

$A_{26}=\frac{Gr\;A_9+Gc\;A_{17}}{(m_1+m_2)^2-(m_1+m_2)-N}$; $A_{27}=\frac{Gr\;A_{10}+Gc\;A_{18}}{(m_1+m_3)^2-(m_1+m_3)-N}$;

$A_{28}=\frac{Gr\;A_{11}+Gc\;A_{19}}{(m_2+m_3)^2-(m_2+m_3)-N}$;

$A_{29}=-(A_{21}+A_{22}+A_{23}+A_{24}+A_{25}+A_{26}+A_{27}+A_{28})$; $A_{30}=\frac{-\Pr\alpha\,m_1}{m_1^{\,2}-\Pr m_1-\Pr S_2}$;

$A_{31}=-(A_{30}+1)$; $A_{32}=\frac{Sc\,m_1(\alpha\,A_1-S_0\,m_1A_{30})}{m_1^{\,2}-Sc\,m_1-Sc\,\gamma_1}$; $A_{33}=\frac{m_2\;A_2\;Sc\,\alpha}{m_2^{\,2}-Sc\,m_2-Sc\,\gamma_1}$;

$$A_{34} = \frac{-Sc\,S_0\,m_4^{\;2} A_{31}}{m_4^{\;2} - Sc\,m_4 - Sc\,\gamma_1} \quad ; \qquad A_{35} = -(1 + A_{32} + A_{33} + A_{34}) \quad ;$$

$$A_{36} = \frac{Gr\,A_{30} + Gc\,A_{32} + \alpha\,m_1 A_3}{m_1^{\;2} - m_1 - N_1} \;;$$

$$A_{37} = \frac{Gc\,A_{33} + \alpha\,m_2 A_4}{m_2^{\;2} - m_2 - N_1} \quad ; \qquad A_{38} = \frac{\alpha\,m_3 A_5}{m_3^{\;2} - m_3 - N_1} \quad ; \qquad A_{39} = \frac{Gr\,A_{31} + Gc\,A_{34}}{m_4^{\;2} - m_4 - N_1} \quad ;$$

$$A_{40} = \frac{Gc\,A_{35}}{m_5^{\;2} - m_5 - N_1} \;;$$

$$A_{41} = -(A_{36} + A_{37} + A_{38} + A_{39} + A_{40}) \;; \qquad A_{42} = \frac{\Pr\alpha\,m_1 A_{12}}{m_1^{\;2} - \Pr m_1 - \Pr S_2} \;;$$

$$A_{43} = \frac{2\Pr[A_3 A_{36}\left(m_1^{\;2} - M_1\right) + \alpha\,m_1\,A_6]}{4m_1^{\;2} - 2\Pr m_1 - \Pr S_2} \;; \qquad A_{44} = \frac{2\Pr[A_4 A_{37}\left(m_2^{\;2} - M_1\right) + \alpha\,m_2\,A_7]}{4m_2^{\;2} - 2\Pr m_2 - \Pr S_2}$$

$$; \qquad A_{45} = \frac{2\Pr[A_5 A_{38}\left(m_3^{\;2} - M_1\right) + \alpha\,m_3\,A_8]}{4m_3^{\;2} - 2\Pr m_3 - \Pr S_2} \quad ;$$

$$A_{46} = \frac{\Pr[2(m_1 m_2 - M_1)(A_3 A_{37} + A_4 A_{36}) + \alpha(m_1 + m_2)A_9]}{(m_1 + m_2)^2 - \Pr(m_1 + m_2) - \Pr S_2} \quad ;$$

$$A_{47} = \frac{\Pr[2(m_1 m_3 - M_1)(A_3 A_{38} + A_5 A_{36}) + \alpha(m_1 + m_3)A_{10}]}{(m_1 + m_3)^2 - \Pr(m_1 + m_3) - \Pr S_2} \quad ;$$

$$A_{48} = \frac{2\Pr A_3 A_{39}(m_1 m_4 - M_1)}{(m_1 + m_4)^2 - \Pr(m_1 + m_4) - \Pr S_2} \quad ;$$

$$A_{49} = \frac{2\Pr A_3 A_{40}(m_1 m_5 - M_1)}{(m_1 + m_5)^2 - \Pr(m_1 + m_5) - \Pr S_2} ; \; A_{50} = \frac{2\Pr A_3 A_{41}(m_1 m_6 - M_1)}{(m_1 + m_6)^2 - \Pr(m_1 + m_6) - \Pr S_2} \;;$$

$$A_{51} = \frac{\Pr[2(m_2 m_3 - M_1)(A_4 A_{38} + A_5 A_{37}) + \alpha(m_2 + m_3)A_{11}]}{(m_2 + m_3)^2 - \Pr(m_2 + m_3) - \Pr S_2} \;;$$

$$A_{52} = \frac{2\Pr A_4 A_{39}(m_2 m_4 - M_1)}{(m_2 + m_4)^2 - \Pr(m_2 + m_4) - \Pr S_2} \quad ;$$

$$A_{53} = \frac{2\Pr A_4 A_{40}(m_2 m_5 - M_1)}{(m_2 + m_5)^2 - \Pr(m_2 + m_5) - \Pr S_2} \;;$$

$$A_{54} = \frac{2\Pr A_4 A_{41}(m_2 m_6 - M_1)}{(m_2 + m_6)^2 - \Pr(m_2 + m_6) - \Pr S_2} \quad ;$$

$$A_{55} = \frac{2\Pr A_5 A_{39}(m_3 m_4 - M_1)}{(m_3 + m_4)^2 - \Pr(m_3 + m_4) - \Pr S_2} \;;$$

$$A_{56} = \frac{2\Pr A_5 A_{40}(m_3 m_5 - M_1)}{(m_3 + m_5)^2 - \Pr(m_3 + m_5) - \Pr S_2} \;; \qquad A_{57} = \frac{2\Pr A_5 A_{41}(m_3 m_6 - M_1)}{(m_3 + m_6)^2 - \Pr(m_3 + m_6) - \Pr S_2}$$

;

$$A_{58} = -(A_{42} + A_{43} + A_{44} + A_{45} + A_{46} + A_{47} + A_{48} + A_{49} + A_{50} + A_{51} + A_{52} + A_{53} + A_{54} + A_{55} + A_{56} + A_{57})$$

$$A_{59} = \frac{Sc\,m_1[\alpha\,A_{13} - S_0 m_1 A_{42}]}{m_1^{\;2} - Sc\,m_1 - Sc\,\gamma_1} \;; \qquad A_{60} = \frac{Sc\;m_2\,\alpha\,A_{20}}{m_2^{\;2} - Sc\,m_2 - Sc\,\gamma_1} \;; \qquad A_{61} = \frac{-Sc\;S_0 m_4^{\;2}\,A_{58}}{m_4^{\;2} - Sc\,m_4 - Sc\,\gamma_1}$$

;

$$A_{62}=\frac{2Sc\,m_1[\alpha A_{14}-2S_0m_1A_{43}]}{4m_1^{\ 2}-2Sc\,m_1-Sc\ \gamma_1}\ ; \qquad A_{63}=\frac{2Sc\,m_2[\alpha A_{15}-2S_0m_2A_{44}]}{4m_2^{\ 2}-2Sc\,m_2-Sc\ \gamma_1}\ ;$$

$$A_{64}=\frac{2Sc\,m_3[\alpha A_{16}-2S_0m_3A_{45}]}{4m_3^{\ 2}-2Sc\,m_3-Sc\ \gamma_1}\ ; \qquad A_{65}=\frac{Sc(m_1+m_2)[\alpha A_{17}-S_0(m_1+m_2)A_{46}]}{(m_1+m_2)^2-Sc(m_1+m_2)-Sc\ \gamma_1}$$

;

$$A_{66}=\frac{Sc(m_1+m_3)[\alpha A_{18}-S_0(m_1+m_3)A_{47}]}{(m_1+m_3)^2-Sc(m_1+m_3)-Sc\gamma_1}\ ; \qquad A_{67}=\frac{-Sc\,S_0(m_1+m_4)^2A_{48}}{(m_1+m_4)^2-Sc(m_1+m_4)-Sc\gamma_1}$$

;

$$A_{68}=\frac{-Sc\,S_0(m_1+m_5)^2A_{49}}{(m_1+m_5)^2-Sc(m_1+m_5)-Sc\gamma_1}\ ; \qquad A_{69}=\frac{-Sc\,S_0(m_1+m_6)^2A_{50}}{(m_1+m_6)^2-Sc(m_1+m_6)-Sc\gamma_1}\ ;$$

$$A_{70}=\frac{Sc(m_2+m_3)[\alpha A_{19}-S_0(m_2+m_3)A_{51}]}{(m_2+m_3)^2-Sc(m_2+m_3)-Sc\gamma_1}\ ; \qquad A_{71}=\frac{-Sc\,S_0(m_2+m_4)^2A_{52}}{(m_2+m_4)^2-Sc(m_2+m_4)-Sc\gamma_1}\ ;$$

$$A_{72}=\frac{-Sc\,S_0(m_2+m_5)^2A_{53}}{(m_2+m_5)^2-Sc(m_2+m_5)-Sc\gamma_1}\ ; \qquad A_{73}=\frac{-Sc\,S_0(m_2+m_6)^2A_{54}}{(m_2+m_6)^2-Sc(m_2+m_6)-Sc\gamma_1}\ ;$$

$$A_{74}=\frac{-Sc\,S_0(m_3+m_4)^2A_{55}}{(m_3+m_4)^2-Sc(m_3+m_4)-Sc\gamma_1}\ ; \qquad A_{75}=\frac{-Sc\,S_0(m_3+m_5)^2A_{56}}{(m_3+m_5)^2-Sc(m_3+m_5)-Sc\gamma_1}\ ;$$

$$A_{76}=\frac{-Sc\,S_0(m_3+m_6)^2A_{57}}{(m_3+m_6)^2-Sc(m_3+m_6)-Sc\gamma_1}\ ;$$

$$A_{77}=-(A_{59}+A_{60}+A_{61}+A_{62}+A_{63}+A_{64}+A_{65}+A_{66}+A_{67}+A_{68}+A_{69}+A_{70}+A_{71}+A_{72}+A_{73}+A_{74}+A_{75}+A_{76})$$

$$A_{78}=\frac{Gr\,A_{42}+Gc\,A_{59}+\alpha\,m_1\,A_{21}}{m_1^{\ 2}-m_1-N_1}\ ; \qquad A_{79}=\frac{Gc\,A_{60}+\alpha\,m_2\,A_{22}}{m_2^{\ 2}-m_2-N_1}\ ; \qquad A_{80}=\frac{\alpha\,m_3\,A_{29}}{m_3^{\ 2}-m_3-N_1}\ ;$$

$$A_{81}=\frac{Gr\,A_{58}+Gc\,A_{61}}{m_4^{\ 2}-m_4-N_1}\ ; \qquad A_{82}=\frac{Gc\,A_{77}}{m_5^{\ 2}-m_5-N_1}\ ; \qquad A_{83}=\frac{Gr\,A_{43}+Gc\,A_{62}+\alpha\,m_1\,A_{23}}{4m_1^{\ 2}-2m_1-N_1}$$

; $$A_{84}=\frac{Gr\,A_{44}+Gc\,A_{63}+2\alpha\,m_2\,A_{24}}{4m_2^{\ 2}-2m_2-N_1}\ ; \qquad A_{85}=\frac{Gr\,A_{45}+Gc\,A_{64}+2\alpha\,m_3\,A_{25}}{4m_3^{\ 2}-2m_3-N_1}$$

; $$A_{86}=\frac{Gr\,A_{46}+Gc\,A_{65}+\alpha(m_1+m_2)A_{26}}{(m_1+m_2)^2-(m_1+m_2)-N_1}\ ;$$

$$A_{87}=\frac{Gr\,A_{47}+Gc\,A_{66}+\alpha(m_1+m_3)A_{27}}{(m_1+m_3)^2-(m_1+m_3)-N_1}\ ;$$

$$A_{88}=\frac{Gr\,A_{48}+Gc\,A_{67}}{(m_1+m_4)^2-(m_1+m_4)-N_1}\ ; \qquad A_{89}=\frac{Gr\,A_{49}+Gc\,A_{68}}{(m_1+m_5)^2-(m_1+m_5)-N_1}\ ;$$

$$A_{90}=\frac{Gr\,A_{50}+Gc\,A_{69}}{(m_1+m_6)^2-(m_1+m_6)-N_1}\ ; \qquad A_{91}=\frac{Gr\,A_{51}+Gc\,A_{70}+\alpha(m_2+m_3)A_{28}}{(m_2+m_3)^2-(m_2+m_3)-N_1}$$

;

$$A_{92}=\frac{Gr\,A_{52}+Gc\,A_{71}}{(m_2+m_4)^2-(m_2+m_4)-N_1}\ ; \qquad A_{93}=\frac{Gr\,A_{53}+Gc\,A_{72}}{(m_2+m_5)^2-(m_2+m_5)-N_1}\ ;$$

$$A_{94}=\frac{Gr\,A_{54}+Gc\,A_{73}}{(m_2+m_6)^2-(m_2+m_6)-N_1}\ ; \qquad A_{95}=\frac{Gr\,A_{55}+Gc\,A_{74}}{(m_3+m_4)^2-(m_3+m_4)-N_1}\ ;$$

$$A_{96} = \frac{Gr\,A_{56} + Gc\,A_{75}}{(m_3 + m_5)^2 - (m_3 + m_5) - N_1} \; ; \qquad A_{97} = \frac{Gr\,A_{57} + Gc\,A_{76}}{(m_3 + m_6)^2 - (m_3 + m_6) - N_1} \; ;$$

$$\begin{aligned} A_{98} = & -(A_{78} + A_{79} + A_{80} + A_{81} + A_{82} + A_{83} + A_{84} + A_{85} + A_{86} + A_{87} + A_{88} + A_{89} + A_{90} + A_{91} + A_{92} + A_{93} \\ & + A_{94} + A_{95} + A_{96} + A_{97}) \end{aligned}$$