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Block-wise Distribution of Primes less than a Trillion in Arithmetical Progressions 8*n* + *k*

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Abstract: *After classifyingprimes into forms 8n + k for k = 1, 3, 5 and 7, their abundance in these forms is compared.For each block of 1–10^n,for $1 \le n \le 12$, the first & last prime in them are determined.Minimum number of primes in blocks, their minimum occurrence frequency, first and last minimum prime containing blocks are given. Similar analysis for maximum number of primes in blocks is also carried out.*
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I. Introduction

Simple and elementary ideas can hide many mysterious properties in them. Prime numbers are the best example of that. Their definition is so naïve that they get introduced in primary school level, immediately after the concept of divisibility is defined.By definition, primes are those positive integers p greater than 1 that have only trivial divisors ±1 and $\pm p$, and no others.

II. Prime Distributions

That the number of primes is infinite is classically known fact [1]. Their irregular distribution was also guessed from earlier days. Now we have proved that there are as large gaps between successive primes as we desire [2]. In contrast with that at the same time, there are ampleoftwin primes, those that differ by only 2.They too have been strongly conjectured to be infinite and have also been analyzed in detail [3]. So, successive primes are at times closer to each other and at other times they are farther from each other. This poses problem in knowing them perfectly. The first consequence is that we don't have a straightforward formula in which all primes fit. Nor do we know most of their properties accurately. So we have to adopt the approximation techniques [4].

Prime density, i.e., the number of primes amongst positive integers in specific ranges, is a keen point of study. The symbol $\pi(x)$, for $x> 1$, stands for the number of primes p such that $1 \le p \le x$.

III. Prime Distributions In Arithmetical Progressions

First prime is 2 and it is very special. Apart from being the prime list starter, it enjoys the uniqueness of being the only even prime. So it is considered to be an *odd man out* candidate in primes. There will be one more occasion to prove its peculiarity in present work.

For fixed positive integers a and b, an arithmetical progression is an expression of the form $an + b$, which contains infinite integers; to be precise, all those integers, which when divided by a give remainder (or residue in the language of congruences) b. Clearly there a number of distinct arithmetical progressions $an + b$ for $0 \le b < a$. Each integer occurs in one and only one of them.

We continue to investigate primes in arithmetical progressions. Earlier, their occurrence in arithmetical progressions of forms $3n + k$, $4n + k$,$5n + k$,$6n + k$, and $7n + k$has been analyzed in detail [5], [6], [7], [8], [9]. Now it is turn of form $8n + k$.
We recall that the number of primes p of form $an + b$ such that $1 \le p \le x$ is given by $\pi_{a,b}(x)$.

IV. Prime Distributions In Arithmetical Progressions 8n + K

The division algorithm states that one and only one of the numbers $0, 1, 2, \cdots, m - 1$ are remainders after dividing any positive integer by m. Choosing $m = 8$, the possible values of remainders in division by 8 are 0, 1, 2, 3, 4, 5, 6, and 7.Due to aforementioned property, every positive integer must be of either of the forms $8n + 0 = 8n$ or $8n + 1$ or $8n + 2$ or $8n + 3$ or $8n + 4$ or $8n + 5$ or $8n + 6$ or $8n + 7$, which are arithmetical progressions $8n + k$.

First few numbers of the form $8n = 8n + 0$ are

$$8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, \cdots$$

All these are perfectly divisible by 8 and none of them is prime.

First few numbers of the form $8n + 1$ are

1, 9, 17, 25, 33, 41, 49, 57, 65, 73, 81, 89, · · ·

This progression does contain infinitely many primes as gcd(8, 1) = 1 as per requirement of Dirichlet's Theorem [10].

First few numbers of the form 8*n* + 2 are

2, 10, 18, 26, 34, 42, 50, 58, 66, 74, 82, 90, · · ·

Each of these is even and hence divisible by 2. Except the first member, viz., 2, none of these is prime. Thus this sequence contains only one prime, viz., 2 and its all other members are composite numbers.

First few numbers of the form 8*n* + 3 are

3, 11, 19, 27, 35, 43, 51, 59, 67, 75, 83, 91, · · ·

By Dirichlet's Theorem [10], this contains infinitely many primes as gcd(8, 3) = 1.

First few numbers of the form 8*n* + 4 are

4, 12, 20, 28, 36, 44, 52, 60, 68, 76, 84, 92, · · ·

In this case gcd(8, 4) = 4 > 1 and this cannot contain any primes.

First few numbers of the form 8*n* + 5 are

5, 13, 21, 29, 37, 45, 53, 61, 69, 77, 85, 93, · · ·

This progression also contains infinitely many primes as gcd(8, 5) = 1 as necessary by Dirichlet's Theorem [10].

First few numbers of the form 8*n* + 6 are

6, 14, 22, 30, 38, 46, 54, 62, 70, 78, 86, 94, · · ·

In this case gcd(8, 6) = 2 > 1 and this also doesn't contain any primes.

Finally, first few numbers of the form 8*n* + 7 are

7, 15, 23, 31, 39, 47, 55, 63, 71, 79, 87, 95, · · ·

This progression does contain infinitely many primes because here gcd(8, 7) = 1 as Dirichlet's Theorem's [10] pre-requisite.

There are independent proofs showing that particular arithmetical progressions contain infinite number of primes [4]. Those can be sketched for our candidates also.

V. Prime Number Race

The term prime number race [11] in the context of arithmetical progressions has been used to check which form of the progression contains how many more or less number of primes compared to others.

Here we compare the number of primes of form 8*n* + 1, 8*n* + 3, 8*n* + 5 and 8*n* + 7 for dominance till one trillion, i.e., 1,000,000,000,000 (10^{12}). The huge prime data was available by the best choice of the algorithms compared thoroughly in [12]. [13], [14], [15], [16], [17] and [18].For implementation, Java Programming Language was used [5].

Table 1 : Number of Primes of form 8*n* + *k* in First Blocks of 10 Powers

Sr. No.	Range 1-*x* (1 to *x*)	Number of Primes of Form			
		8*n* + 1 ($\pi_{8,1}(x)$)	8*n* + 3 ($\pi_{8,3}(x)$)	8*n* + 5 ($\pi_{8,5}(x)$)	8*n* + 7 ($\pi_{8,7}(x)$)
	1-10	0	1	1	1
	1-100	5	7	6	6
	1-1,000	37	44	43	43
	1-10,000	295	311	314	308
	1-100,000	2,384	2,409	2,399	2,399
	1-1,000,000	19,552	19,653	19,623	19,669
	1-10,000,000	165,976	166,161	166,204	166,237
	1-100,000,000	1,439,970	1,440,544	1,440,534	1,440,406
	1-1,000,000,000	12,711,220	12,712,340	12,712,271	12,711,702
	1-10,000,000,000	113,758,759	113,763,027	113,764,516	113,766,208
	1-100,000,000,000	1,029,502,984	1,029,511,402	1,029,517,296	1,029,523,130
	1-1,000,000,000,000	9,401,951,850	9,401,994,474	9,401,972,490	9,401,993,203

This has covered all primes till 1 trillion except 2. It is uniquely present in progression 8*n* + 2, which we have dropped from analysis. Barring this exception, for which $\pi_{8,3}(x) = 1$ for $x \geq 2$, the deviation from respective other four from their averages is plotted.

The number of primes of the form 8*n* + 7 and 8*n* + 3 seem most of the times ahead of the average up to 10^{12} in discrete blocks of 10 powers; while those of form 8*n* + 1 lag behind in more instances. This trend is a subject matter of future explorations.

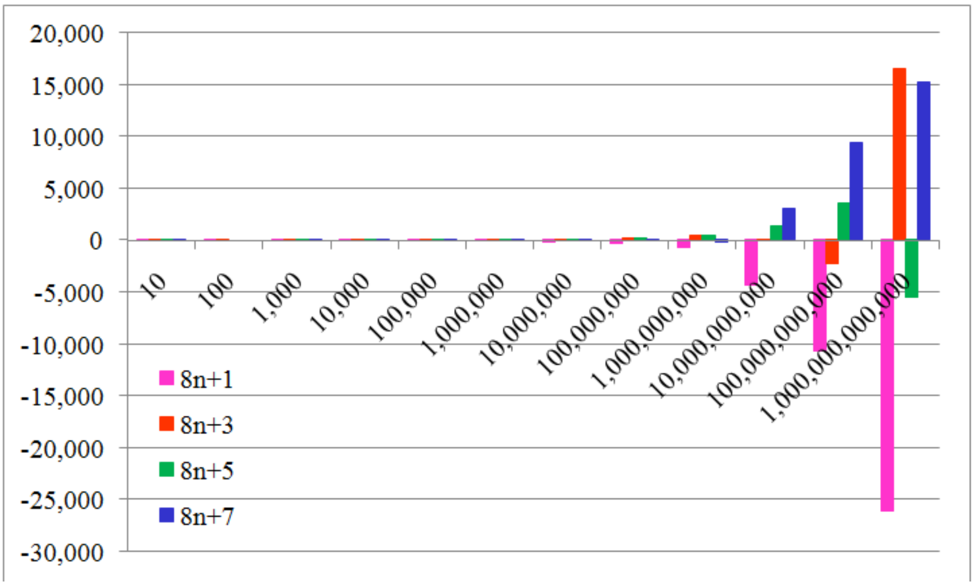

Figure 1 : Deviation of $\pi_{8,k}(x)$ from Average

VI. Block-wise Distribution of Primes

Lack of a simple formula to consider all primes together forces us to adopt a plain approach of considering all primes up to a certain limit;which here we have chosen as high as one trillion (10^{12}).Further we divide number range under consideration in all possible blocks of powers of 10 :

$$1\text{-}10,\ 11\text{-}20,\ 21\text{-}30,\ 31\text{-}40,\ \cdots$$
$$1\text{-}100,\ 101\text{-}200,\ 201\text{-}300,\ 301\text{-}400,\ \cdots$$
$$1\text{-}1000,\ 1001\text{-}2000,\ 2001\text{-}3000,\ 3001\text{-}4000,\ \cdots$$
$$\vdots$$

As our range is 1-10^{12}, there come out 10^{12-n} number of blocks of 10^n size for each $1 \le n \le 12$.

A. The First and the Last Primes in the First Blocks of 10 Powers

We have determined the first and the last prime in each first block of 10 powers till the range of 10^{12}. We give the initial data till availability for first prime as it continues for higher sized blocks.

Table 2: First Primes of form 8*n* + *k*FirstBlocks of 10 Powers

Sr. No.	Blocks of Size (of 10 Power)	First Prime in the First Block				
		Form 8*n* + 1	Form 8*n* + 2	Form 8*n* + 3	Form 8*n* + 5	Form 8*n* + 7
1.	10	Not Found	2	3	5	7
2.	100 and All Higher	17	2	3	5	7

The largest primes of these forms in first blocks of 10 powers are as follows.

Table 3 : Last Primes of form 8*n* + *k* First Blocks of 10 Powers

Sr. No.	Blocks of Size (of 10 Power)	Last Prime in the First Block				
		Form 8*n* + 1	Form 8*n* + 2	Form 8*n* + 3	Form 8*n* + 5	Form 8*n* + 7
	10	NOT FOUND	2	3	5	7
	100	97	2	83	61	79
	1,000	977	2	971	997	991
	10,000	9,929	2	9,931	9,973	9,967
	100,000	99,961	2	99,971	99,989	99,991
	1,000,000	999,961	2	999,979	999,917	999,983
	10,000,000	9,999,937	2	9,999,971	9,999,973	9,999,991
	100,000,000	99,999,721	2	99,999,971	99,999,989	99,999,959
	1,000,000,000	999,999,937	2	999,999,883	999,999,893	999,999,751
	10,000,000,000	9,999,999,929	2	9,999,999,851	9,999,999,781	9,999,999,967
	100,000,000,000	99,999,999,977	2	99,999,999,947	99,999,999,829	99,999,999,943
	1,000,000,000,000	999,999,999,961	2	999,999,999,899	999,999,999,989	999,999,999,959

The first primes in all the first blocks, whenever found, have same values ahead and the deviations of the last primes of these forms in the first blocks have a random trend.

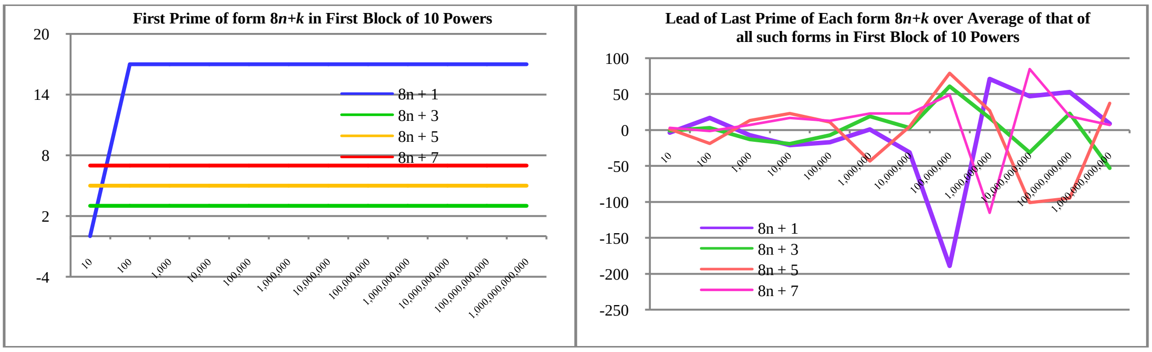

Figure 2 : First & Last Primes of form 8n + k in First Blocks of 10 Powers.

It must be noted that form 8n + 2 is purposefully skipped here as it contains only one prime.

B. Minimum Number of Primes in Blocks of 10 Powers

Examining all blocks of every 10 power from 10^1 to 10^{12}, the minimum number of primes found in each 10 power block has been determined for primes of all forms under consideration.

Table 4 : Minimum Number of Primes of form 8n + k in Blocks of 10 Powers

Sr. No.	Blocks of Size (of 10 Power)	Minimum Number of Primes in Blocks				
		Form 8n + 1	Form 8n + 2	Form 8n + 3	Form 8n + 5	Form 8n + 7
	10	0	0	0	0	0
	100	0	0	0	0	0
	1,000	0	0	0	0	0
	10,000	50	0	50	50	48
	100,000	790	0	793	797	796
	1,000,000	8,757	0	8,792	8,741	8,760
	10,000,000	89,890	0	89,876	89,773	89,850
	100,000,000	903,217	0	903,515	903,679	903,787
	1,000,000,000	9,047,362	0	9,047,329	9,046,213	9,047,276
	10,000,000,000	90,491,447	0	90,495,748	90,494,975	90,495,251
	100,000,000,000	906,480,162	0	906,473,424	906,469,058	906,501,788
	1,000,000,000,000	9,401,951,850	0	9,401,994,474	9,401,972,490	9,401,993,203

The block-wise deviation of minimum number of primes from respective averages, barring 8n + 2, is shown in ahead.

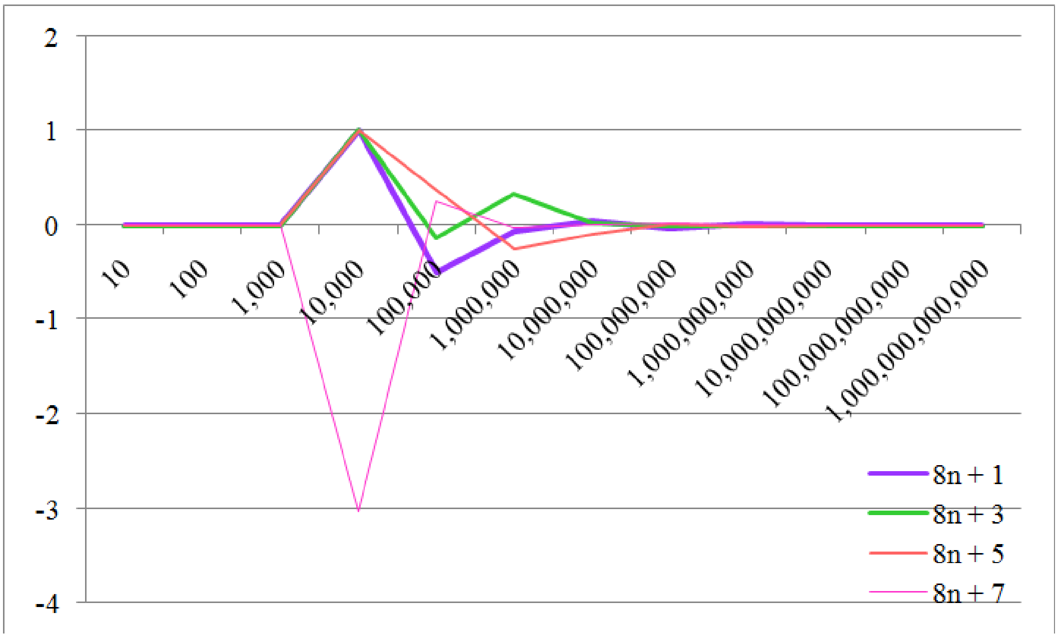

Figure 3 :% Deviation in Minimum Number of Primes of form 8n+k in Blocks of 10 Powers from Average.

Next due are the first blocks with those many minimum number of primes of various forms in them.

Table 5: First Blocks of 10 Powers with Minimum Number of Primes of form 8*n* + *k*

Sr. No.	Blocks of Size (of 10 Power)	First Block with Minimum Number of Primes				
		Form 8*n* + 1	Form 8*n* + 2	Form 8*n* + 3	Form 8*n* + 5	Form 8*n* + 7
	10	0	10	20	40	10
	100	8,400	100	4,800	20,800	6,400
	1,000	3,101,819,000	1,000	5,671,637,000	2,257,775,000	1,203,517,000
	10,000	997,642,400,000	10,000	401,414,770,000	326,919,360,000	637,405,700,000
	100,000	905,210,200,000	100,000	917,381,200,000	955,041,700,000	974,435,200,000
	1,000,000	942,153,000,000	1,000,000	938,479,000,000	984,111,000,000	991,533,000,000
	10,000,000	985,230,000,000	10,000,000	992,870,000,000	964,840,000,000	970,280,000,000
	100,000,000	997,500,000,000	100,000,000	998,600,000,000	996,600,000,000	977,200,000,000
	1,000,000,000	998,000,000,000	1,000,000,000	999,000,000,000	997,000,000,000	998,000,000,000
	10,000,000,000	990,000,000,000	10,000,000,000	990,000,000,000	990,000,000,000	990,000,000,000
	100,000,000,000	900,000,000,000	100,000,000,000	900,000,000,000	900,000,000,000	900,000,000,000

The last such blocks in range of 10^{12} with minimum number of primes of these forms in them are as follows.

Table 6 : Last Blocks of 10 Powers with Minimum Number of Primes of form 8*n* + *k*

Sr. No.	Blocks of Size (of 10 Power)	Last Block with Minimum Number of Primes				
		Form 8*n* + 1	Form 8*n* + 2	Form 8*n* + 3	Form 8*n* + 5	Form 8*n* + 7
	10	999,999,999,990	90	999,999,999,990	999,999,999,990	999,999,999,990
	100	999,999,999,700	900	999,999,999,900	999,999,999,700	999,999,999,600
	1,000	999,923,916,000	9,000	999,978,018,000	999,946,050,000	999,948,977,000
	10,000	997,642,400,000	90,000	822,434,990,000	326,919,360,000	637,405,700,000
	100,000	905,210,200,000	900,000	917,381,200,000	955,041,700,000	974,435,200,000
	1,000,000	942,153,000,000	9,000,000	938,479,000,000	984,111,000,000	991,533,000,000
	10,000,000	985,230,000,000	90,000,000	992,870,000,000	964,840,000,000	970,280,000,000
	100,000,000	997,500,000,000	900,000,000	998,600,000,000	996,600,000,000	977,200,000,000
	1,000,000,000	998,000,000,000	9,000,000,000	999,000,000,000	997,000,000,000	998,000,000,000
	10,000,000,000	990,000,000,000	90,000,000,000	990,000,000,000	990,000,000,000	990,000,000,000
	100,000,000,000	900,000,000,000	900,000,000,000	900,000,000,000	900,000,000,000	900,000,000,000

Now comes their comparative graphical representation.

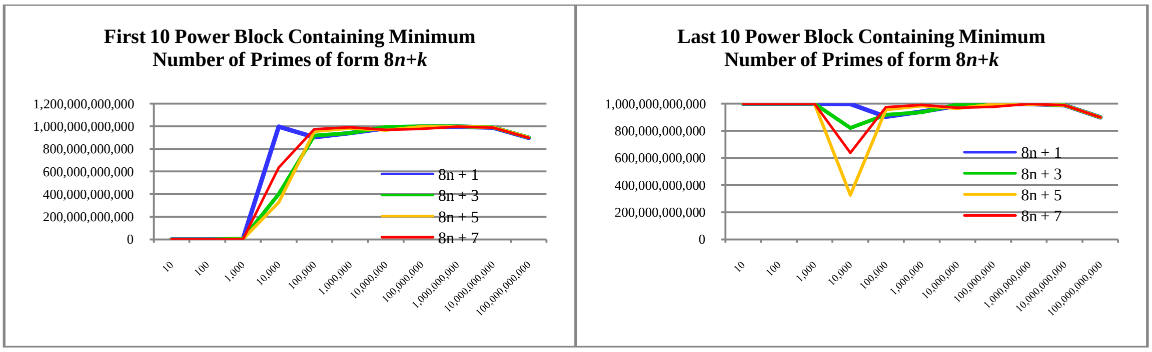

Figure 4 : First & Last Blocks of 10 Powers with Minimum Number of Primes of form 8*n* + *k*.

These are withoutthe arithmetical progression of form 8*n* + 2. Its second block 100···0 in respective ranges is the first block containing minimum (viz., 0) number of primes and the last block 900···0 in respective ranges is the last block with minimum number of primes.

The blocks here are considered in the following way. Block 0 for 1,000 means the range 0 to 999. Block 1,000 means 1,000 to 1,999. Block 2,000 means 2,000 to 2,999. In general, block *k*00···0 means the number range *k*00···0 to *k*99···9.

Next the frequencies of blocks of minimum number of primes of forms 8*n* + *k*aredue to be dealt with and they are as follows.

Table 7 : Frequency of 10 Power Blocks with Minimum Number of Primes of form 8*n* + *k* in them

Sr. No.	Blocks of Size (of 10 Power)	No. of Times Minimum No. of Primes Occurring in Blocks				
		Form 8*n* + 1	Form 8*n* + 2	Form 8*n* + 3	Form 8*n* + 5	Form 8*n* + 7
	10	90,753,931,476	99,999,999,999	90,753,895,716	90,753,906,166	90,753,879,025
	100	3,568,719,129	9,999,999,999	3,568,698,677	3,568,662,781	3,568,689,528
	1,000	19,515	999,999,999	19,630	19,424	19,531
	10,000	1	99,999,999	2	1	1
	100,000	1	9,999,999	1	1	1
	1,000,000	1	999,999	1	1	1

Sr. No.	Blocks of Size (of 10 Power)	No. of Times Minimum No. of Primes Occurring in Blocks				
		Form 8*n* + 1	Form 8*n* + 2	Form 8*n* + 3	Form 8*n* + 5	Form 8*n* + 7
	10,000,000	1	99,999	1	1	1
	100,000,000	1	9,999	1	1	1
	1,000,000,000	1	999	1	1	1
	10,000,000,000	1	99	1	1	1
	100,000,000,000	1	9	1	1	1

Keeping out 8*n* + 2 with the exceptional behavior, the block-wise deviation of frequency of occurrence of minimum number of primes from respective averages is given.

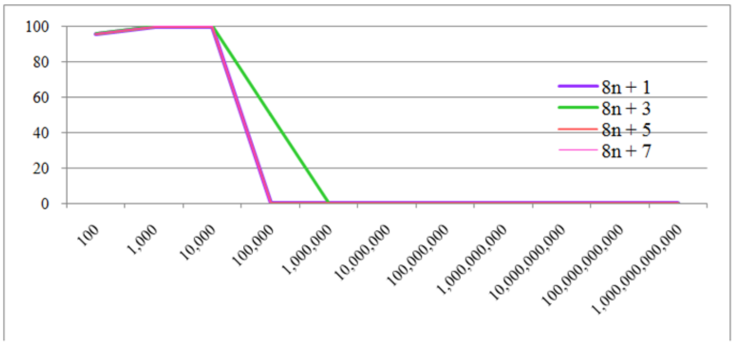

Figure 5 :% Decrease in Occurrences of Minimum Number of Primes of form 8*n*+*k* in Blocks of 10 Powers.

C. Maximum Number of Primes in Blocks of 10 Powers

Now all blocks of 10 powerstill 10^{12} are investigated for the maximum number of primes in each of them.

Table 8 : Maximum Number of Primes of form 8*n* + *k* in Blocks of 10 Powers

Sr. No.	Blocks of Size (of 10 Power)	Maximum Number of Primes in Blocks				
		Form 8*n* + 1	Form 8*n* + 2	Form 8*n* + 3	Form 8*n* + 5	Form 8*n* + 7
	10	2	1	2	2	2
	100	7	1	7	7	7
	1,000	37	1	44	43	43
	10,000	295	1	311	314	308
	100,000	2,384	1	2,409	2,399	2,399
	1,000,000	19,552	1	19,653	19,623	19,669
	10,000,000	165,976	1	166,161	166,204	166,237
	100,000,000	1,439,970	1	1,440,544	1,440,534	1,440,406
	1,000,000,000	12,711,220	1	12,712,340	12,712,271	12,711,702
	10,000,000,000	113,758,759	1	113,763,027	113,764,516	113,766,208
	100,000,000,000	1,029,502,984	1	1,029,511,402	1,029,517,296	1,029,523,130
	1,000,000,000,000	9,401,951,850	1	9,401,994,474	9,401,972,490	9,401,993,203

Analysis of deviation from average shows that the maximum primes of form 8*n* + 1 lie on the lower side and 8*n* + 7 lie on upper side in major discrete cases of 10 power blocks. Here also 8*n* + 2 is not considered owing to its uniqueness of containing maximum unique prime!

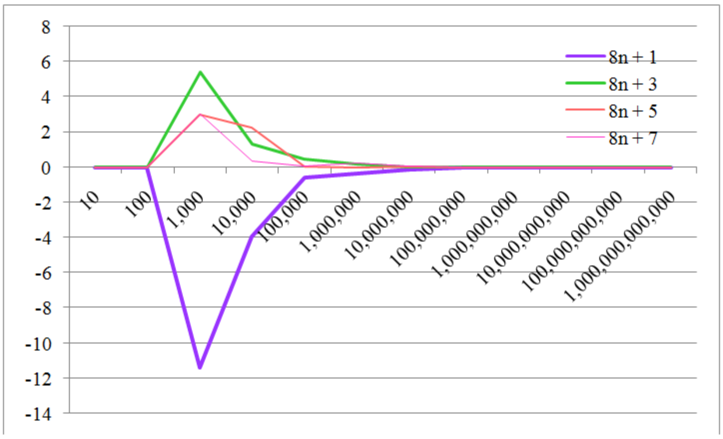

Figure 6 :% Deviation in Maximum Number of Primes of form 8*n*+*k* in Blocks of 10 Powers from Average.

The first blocks in range of one trillion containing maximum number of primes of these forms in them are found to be :

Table 9 : First Blocks of 10 Powers with Maximum Number of Primes of form 8*n* + *k*

Sr. No.	Blocks of Size (of 10 Power)	First Block with Maximum Number of Primes				
		Form 8*n* + 1	Form 8*n* + 2	Form 8*n* + 3	Form 8*n* + 5	Form 8*n* + 7
	10	400	0	10	100	70
	100	2,508,000	0	0	100	2,077,300
	1,000	0	0	0	0	0
	10,000	0	0	0	0	0
	100,000	0	0	0	0	0
	1,000,000	0	0	0	0	0
	10,000,000	0	0	0	0	0
	100,000,000	0	0	0	0	0
	1,000,000,000	0	0	0	0	0
	10,000,000,000	0	0	0	0	0
	100,000,000,000	0	0	0	0	0

Now the last blocks in till one trillion with maximum number of primes of these forms in them are :

Table 10 :Last Blocks of 10 Powers with Maximum Number of Primes of form 8*n* + *k*

Sr. No.	Blocks of Size (of 10 Power)	Last Block with Maximum Number of Primes				
		Form 8*n* + 1	Form 8*n* + 2	Form 8*n* + 3	Form 8*n* + 5	Form 8*n* + 7
	10	999,999,998,440	0	999,999,992,530	999,999,992,140	999,999,993,070
	100	999,838,688,200	0	999,988,689,800	999,732,542,700	998,981,765,500
	1,000	0	0	0	0	0
	10,000	0	0	0	0	0
	100,000	0	0	0	0	0
	1,000,000	0	0	0	0	0
	10,000,000	0	0	0	0	0
	100,000,000	0	0	0	0	0
	1,000,000,000	0	0	0	0	0
	10,000,000,000	0	0	0	0	0
	100,000,000,000	0	0	0	0	0

As, in general, the prime density decreases at higher ranges, for larger sizes, the first as well as the last blocks of maximum number of primes in them tend to be 0, i.e., the first block.

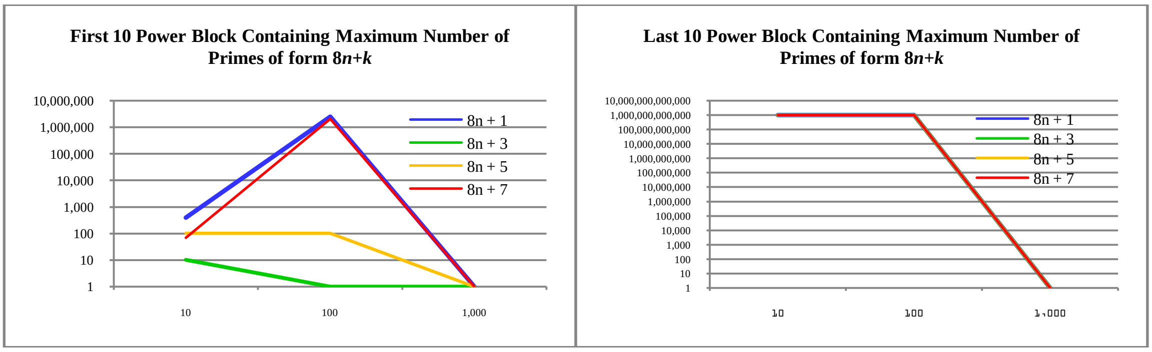

Figure 7 : First & Last Blocks of 10 Powers with Maximum Number of Primes of form 8*n* + *k*.

Decrease in the prime density assures that the maximum number of primes of any form cannot occur more frequently for higher ranges.

Table 11 : Frequency of 10 Power Blocks with Maximum Number of Primes of form 8*n* + *k* in them

Sr. No.	Blocks of Size (of 10 Power)	No. of Times Maximum No. of Primes Occurring in Blocks				
		Form 8*n* + 1	Form 8*n* + 2	Form 8*n* + 3	Form 8*n* + 5	Form 8*n* + 7
	10	155,883,326	1	155,890,190	155,878,656	155,872,228
	100	3,054	1	3,120	3,194	3,099
	1,000	1	1	1	1	1
	10,000	1	1	1	1	1
	100,000	1	1	1	1	1
	1,000,000	1	1	1	1	1
	10,000,000	1	1	1	1	1
	100,000,000	1	1	1	1	1
	1,000,000,000	1	1	1	1	1

Sr. No.	Blocks of Size (of 10 Power)	No. of Times Maximum No. of Primes Occurring in Blocks				
		Form $8n + 1$	Form $8n + 2$	Form $8n + 3$	Form $8n + 5$	Form $8n + 7$
	10,000,000,000	1	1	1	1	1
	100,000,000,000	1	1	1	1	1

Again setting asidethe form of $8n + 2$ due torarity behavior, the block-wise deviation of frequency of occurrence of maximum number of primes from corresponding averages is as follows.

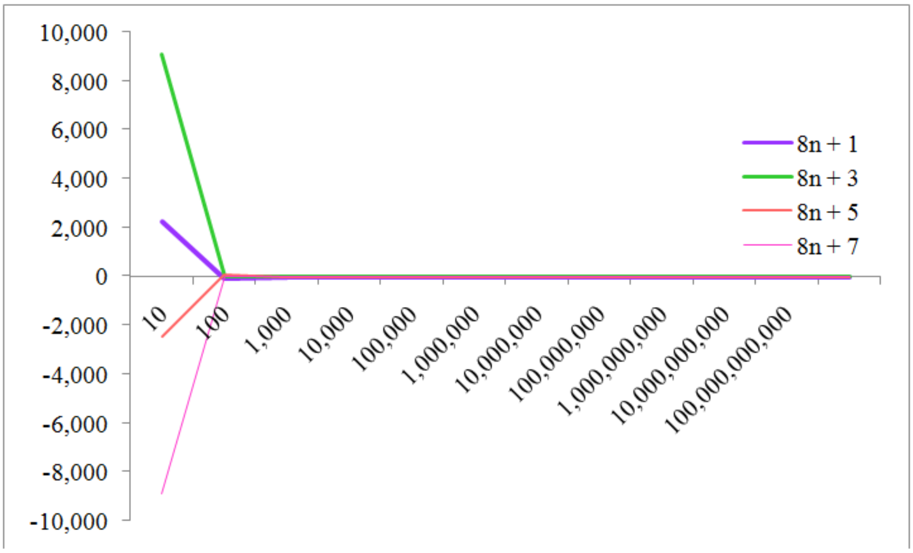

Figure 8 : Deviation in Frequency of Maximum Number of Primes of various forms in Blocks from Average.

All the analysis in this work has just mentioned the values for form $8n + 2$ but ignored it in comparison due to the reason stated in the beginning itself.

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