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Analytic Approach in Solving Steady Laminar Flow of Fluid over a Stretching Sheet in Presence of Magnetic Field

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Abstract: *We have considered the steady laminar flow over a linearly stretching sheet subjected to an order of chemical reaction. A similarity transformation is utilized to convert the governing nonlinear partial differential equations into ordinary differential equations. The exact solution to this study may obtained by exploiting the fact that some features of free-parameter method and the "separation of variables" method are alike. Similarity transformations are used to convert unsteady boundary layer equations to a non-linear ordinary differential equation. Finally we have solved the equation analytically.*

Keywords: *Laminar flow, Electric field, Chemical reaction, Similarity transformation, Heat and mass transfer, Stretching sheet.*

I. Introduction

The heat and mass transfer in laminar boundary layer flow over a linearly stretching sheet have important applications in many fields of engineering. In addition, the diffusing species can be generated or absorbed due to some kind of chemical reaction with the ambient fluid. Mahapatra and Gupta studied the heat transfer in stagnation-point flow towards a stretching sheet [1]. On the other hand, Afify analyzed the MHD free convective flow and mass transfer over a stretching sheet with homogeneous chemical reaction [2]. Further, Alam and Ahammad investigated the effects of variable chemical reaction and variable electric conductivity on free convective heat and mass transfer flow over an inclined stretching sheet with variable heat and mass fluxes under the influence of Dufour and Soret effects [3]. Ferdows and Qasem Al-Mdallal studied effects of order of chemical reaction on a boundary layer flow with heat and mass transfer over a linearly stretching sheet [4]. Hayata et al. studied the unsteady three dimensional flow of couple stress fluid over a stretching surface with chemical reaction based on using homotopy analysis method [5]. Singh investigated the MHD Flow with Viscous Dissipation and Chemical Reaction over a Stretching Porous Plate in Porous Medium [6]. Makinde and Sibanda investigated the effects of chemical re-action on boundary layer flow past a vertical stretching surface in the presence of internal heat generation [7]. We have concerned with two dimensional steady, laminar flow of a fluid over a linearly stretching sheet. In this method a partial differential equation may reduce to an ordinary differential equation.

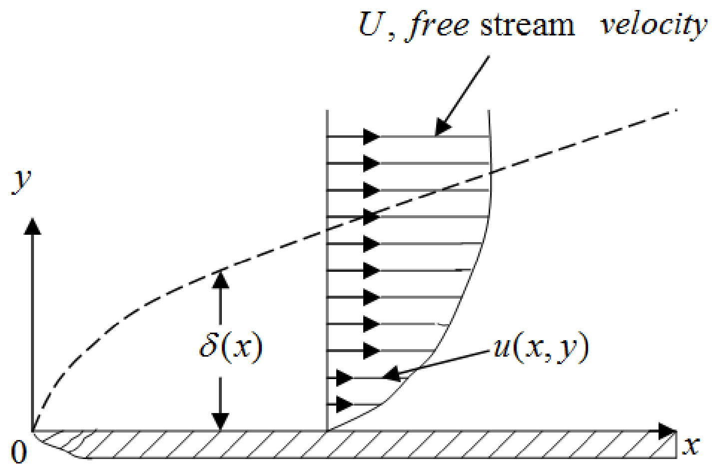
In the present analysis, we consider the problem of steady laminar flow of a viscous incompressible electrically conducting fluid over a stretching sheet subjected to a transverse magnetic field. The exact solution to this problem obtained by exploiting the fact that some features of free-parameter method and the "separation of variables" method are alike.

In this paper we have investigated analytically the effects of chemical reaction on the steady laminar two dimensional boundary layer flow and heat and mass transfer over a stretching sheet. The method of solution is based on the well-known similarity transformations.

II. Governing Equations

We have considered steady, laminar flow of a fluid over a stretching sheet in presence of Magnetic field. Again we have considered the stretched with a velocity proportional to x axis as shown below. We have assumed that the fluid far away from the sheet is at rest and at temperature T and concentration C. Consider the steady laminar flow of a viscous incompressible, electrically conducting Newtonian fluid past an impermeable flat elastic stretching sheet. Two equal and opposite forces are applied along the x-axis so that the sheet is stretched by a velocity proportional to the distance from the origin $x = 0$ keeping the origin fixed. The proportionality constant is called "stretching rate".

The equation governing the motion are:



$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\sigma B^2 u \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right). \tag{3}$$

Boundary conditions:

$$u=cx,\ v=o,\quad p=p_w,\ at\ \ y=0$$

$$(4)$$

and $\qquad u\rightarrow 0,\ as\ \ y\rightarrow\infty$

Nomenclature:

$u\rightarrow$ velocity component in the x direction

$v\rightarrow$ velocity component in the y direction

$B\rightarrow$ appearing in equation is the uniform magnetic field acting perpendicular to the flow direction .

$\mu\rightarrow$ dynamic viscosity

$\omega\rightarrow$ similarity variable

$c\rightarrow$ concentration of the fluid

$l\rightarrow$ dimensionless stream function

$\psi\rightarrow$ stream function

$\varepsilon\rightarrow$ similarity variable

III. Mathematical Formulation

The continuity equation (1) is identically satisfied by stream function $\psi(x, y)$ defined as:

$$\left.\begin{array}{l} u=\dfrac{\partial\psi}{\partial y} \\ and \\ v=-\dfrac{\partial\psi}{\partial x}\end{array}\right\} \tag{5}$$

Now, substituting (5) in (2) and (3), we obtain easily

$$\rho\left(\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^3\psi}{\partial x^2\partial y}+\frac{\partial^3\psi}{\partial y^3}\right)-\sigma B^2\frac{\partial\psi}{\partial y} \qquad (6)$$

$$\rho\left[\frac{\partial\psi}{\partial y}(-\frac{\partial^2\psi}{\partial x^2})+\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial x\partial y}\right]==-\frac{\partial p}{\partial x}+\mu\left(-\frac{\partial^3\psi}{\partial x^3}-\frac{\partial^3\psi}{\partial x\partial y^2}\right) \qquad (7)$$

At this stage we introduce coordinate transformations of the independent variables as:
In order to solve Equations, we introduce the following similarity transformation

$$\mathrm{x}=\varepsilon,\ y=\frac{\omega}{g(\varepsilon)} \qquad (8)$$

and

$$p=p_w-\frac{1}{2}c\mu h(\omega) \qquad (9)$$

We concentrate now on the equation (6).Applying the transformations (9) on all the derivatives appearing in equation (6) the following expressions are obtained:

$$\frac{\partial\psi}{\partial y}=\frac{\partial\psi}{\partial\omega}\cdot g(\varepsilon)=l_1\text{, say}$$

$$\frac{\partial^2\psi}{\partial x\partial y}=g(\varepsilon)\cdot\frac{\partial^2\psi}{\partial\omega^2}\cdot\omega\cdot\frac{dIng(\varepsilon)}{d\varepsilon}+\left[\frac{\partial^2\psi}{\partial\varepsilon\partial\omega}\cdot g(\zeta)+\frac{\partial\psi}{\partial\omega}\cdot g'(\varepsilon)\right]=l_2\text{, say}$$

$$\frac{\partial\psi}{\partial x}=\frac{\partial\psi}{\partial\omega}\cdot\omega\cdot\frac{dIng(\varepsilon)}{d\varepsilon}+\frac{\partial\psi}{\partial\varepsilon}=l_3\text{, say}$$

$$\frac{\partial^2\psi}{\partial y^2}=\frac{\partial^2\psi}{\partial\omega^2}\cdot g^2(\varepsilon)=l_4\text{, say}$$

$$\frac{\partial p}{\partial x}=-\frac{1}{2}c\mu h'(\omega)\frac{\partial\omega}{\partial x}=-\frac{1}{2}c\mu h'(\omega)yg'(\varepsilon)=-\frac{1}{2}\mu\eta h'(\omega)\frac{g'(\varepsilon)}{g(\varepsilon)}\omega=l_5\text{, say}$$

$$\begin{aligned}\frac{\partial^3\psi}{\partial y\partial x^2}=&\left[\frac{\partial^3\psi}{\partial\omega^3}\cdot\omega\cdot g'(\varepsilon)+2\frac{\partial^2\psi}{\partial\omega^2}\cdot g'(\varepsilon)+g(\varepsilon)\cdot\frac{\partial^3\psi}{\partial\omega^2\partial\varepsilon}\right]\cdot\omega\cdot\frac{g'(\varepsilon)}{g(\varepsilon)}\\&+\frac{\partial^3\psi}{\partial\omega^2\partial\varepsilon}\cdot\omega\cdot\frac{dIng(\varepsilon)}{d\varepsilon}+\frac{\partial^2\psi}{\partial\omega\partial\varepsilon}\frac{dIng(\varepsilon)}{d\varepsilon}\\&+\frac{\partial^2\psi}{\partial\omega^2}\cdot\omega\cdot\frac{d^2Ing(\varepsilon)}{d\varepsilon^2}+\frac{\partial\psi}{\partial\omega}\cdot\frac{d^2Ing(\varepsilon)}{d\varepsilon^2}+\frac{\partial^3\psi}{\partial\omega\partial\varepsilon^2}=l_6\text{,say}\end{aligned} \qquad (10)$$

$$\frac{\partial^3\psi}{\partial y^3}=\frac{\partial^3\psi}{\partial\omega^3}\cdot g^3(\varepsilon)=l_7\text{, say} \qquad (11)$$

In view of the expressions obtained above in (10), equation (6) is transformed to:

$$\rho(l_1l_2-l_3l_4)=-l_5+\mu(l_6+l_7)-\sigma B^2l_1 \qquad (12)$$

where,

l_1,l_2,l_3,l_4,l_5,l_6 and l_7 are all functions of ω and ε.

To solve the equation (10), we apply the “separation of variables” method and accordingly, put
$\psi=H(\varepsilon)\cdot L(\omega)$

Hansen has recommended that “Substitution of the product form of the dependent variable into the equation generally leads to an equation in which the functions of one variable cannot be isolated on the two sides of the equation unless certain parameters are specified”[8]. Keeping this in view we proceed choosing simply $H(\varepsilon)=\varepsilon$ which reduces (11) to

$$\psi=\varepsilon\cdot L(\omega). \qquad (13)$$

But, after substitution $H(\varepsilon)=\varepsilon$ in (10), we found that the equation is still not separable. On inspection, we found that with the choice of $g(\varepsilon)=1$, the equation is reduced finally to an equation solely in $L(\omega)$ e. g.

$$\upsilon \cdot L'''(\omega) - L'^2(\omega) + L(\omega) \cdot L''(\omega) - \sigma B^2 L'(\omega) = 0. \qquad (14)$$

Equation (14) with the boundary conditions (15) indicates that the free parameter method and the separation of variables technique are alike.

$$\begin{aligned} &L'(\omega) = c, \quad L(\omega) = 0 &&\text{for } \omega = 0 \\ &L''(\omega) = 0 &&\text{for } \omega \to \infty. \end{aligned} \qquad (15)$$

IV. Conclusion

We have considered steady laminar flow over a stretching sheet in the present of chemical reaction. By using some suitable transformations we have reduced the partial differential equation into ordinary differential equation. The general procedure of obtaining similarity solutions of a system of partial differential equations is equivalent to the determination of the invariant solutions of these equations under the appropriate one-parameter group of transformations. Usually, from these equations a single differential equation, in terms of a similarity function related to the stream function is derived. So, in the free-parameter method we introduce a stream function ψ and subsequently a non-linear partial differential equation in stream function is derived.

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