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On the Gram-Schamidt method and the orthogonal Polynomials

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Abstract: *In this paper we use the gram-Schmid method to define an orthogonal Polynomials such as Legendre – Hermit – Laguir with their corresponding weight functions. Also we get a new generating function for Legendre polynomials.*
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I. Introduction

The basic concepts of Gram-Schamidt method

Gram-Schamidt method constructs an orthogonal vector's from any set of linearly independent vector's $x_1, x_2, x_3, ..., x_n$ the construction as follows:

Put $y_1 = x_1$ (1.1)

and then

$$y_2 = x_2 - \frac{< y_1, x_2 >}{< y_1, y_1 >} y_1 \qquad (1.2)$$

Where $<\cdot,\cdot>$ represent the scalar product on the vector space it is clear from equation $(1,2)$ that y_2 is equal to x_2 minus to projection on y_1.

Following $(1,2)$ we lead generally to

$$y_j = x_j - \sum_{k=1}^{j-1} \frac{< y_k, x_j >}{< y_k, y_k >} y_k \qquad (1,3)$$

II. Finding orthogonal Polynomials using Gram- Schmidt method

In this subsection we construct some important polynomials by using Gram- Schmidt method.

(a) Lagendre Polynomials:

We know that the set $1, x, x^2, ..., x^n$ is linearly independent set. Defined the set of Polynomials $f_1(x), f_2(x), f_3(x), ..., f_n(x)$ let $x_i = x^{i-1}$, $y_j = f_j(x)$, $i, j = 1, 2, 3, ...$

The using equation $(1,3)$ we have

$$y_1 = f_1(x) = x_1 = 1 \qquad (2.1.a)$$

$$y_2 = f_2(x) = x_2 - \frac{< x_2, y_1 >}{< y_1, y_1 >} y_1 \qquad (2.2.a)$$

$$y_3 = f_3(x) = x_3 - \frac{< y_1, x_3 >}{< y_1, y_1 >} y_1 - \frac{< y_2, x_3 >}{< y_2, y_2 >} y_2 \qquad (2.3.a)$$

$$y_4 = f_4(x) = x_4 - \frac{< y_1, x_4 >}{< y_1, y_1 >} y_1 - \frac{< y_2, x_4 >}{< y_2, y_2 >} y_2 - \frac{< y_3, x_4 >}{< y_3, y_3 >} y_3 \qquad (2.4.a)$$

Define the scalar product as $< f, g > = \int_{-1}^{1} f(x) g(x)\, dx$ with weight function equal to one

$$\left.\begin{aligned} &f_1(x)=1\\ &f_2(x)=x\\ &f_3(x)=\frac{1}{3}\left(3x^2-1\right)\\ &f_4(x)=\frac{1}{5}\left(5x^3-3x\right)\\ &\vdots \end{aligned}\right\} \qquad (2.5.a)$$

If we put $f_1(x)=\mathrm{A}_i\,\mathrm{P}_{i-1}(x)$ where $\mathrm{P}_i(x)$ is Legendre Polynomial, A is constant and use the condition $\mathrm{P}_i(1)=1$ then we get

$$\left.\begin{aligned} &P_0(x)=1\\ &P_1(x)=x\\ &P_2(x)=\frac{1}{2}\left(3x^2-1\right)\\ &P_3(x)=\frac{1}{2}\left(5x^3-3x\right)\\ &\vdots \end{aligned}\right\} \qquad (2.6.a)$$

(b) Hermite Polynomials:

Following section (a) and define the scalar product in the form

$$<f\ ,\, g>=\int_{-\infty}^{\infty} e^{-x^2}\ f(x)\ g(x)\ dx \qquad (2.7.a)$$

and using the identity

$$\int_{-\infty}^{\infty} x^{2n}\ e^{-x^2}\ dx=\frac{(2n-1)!}{2^{2n}(n-1)!}\sqrt{i}$$

We can derive Hermit Polynomials as

$$\left.\begin{aligned} &H_0(x)=1\\ &H_1(x)=2x\\ &H_2(x)=4x^2-2\\ &H_3(x)=8x^3-12x\\ &\vdots \end{aligned}\right\} \qquad (2.8.a)$$

III. New generating function for lagendre Polynomails

We know that $L\ J_0(t)=\dfrac{1}{\sqrt{S^2+1}}$ and $L\ f(at)=\dfrac{1}{a}F\left(S/a\right)$ where $F(S)$is Laplace transform for the function for the function $f(t)$ and therefore

$$L\left[e^{xt}\ J_0\left(t\sqrt{1-x^2}\right)\right]=\frac{1}{\sqrt{S^2+1-x^2}}=\frac{1}{S}\sqrt{1-\frac{2x}{S}+\frac{1}{S^2}}=\sum_{n=0}^{\infty}P_n(x)\ S^{-(n+1)}=L\sum_{n=0}^{\infty}\frac{P_n}{n!}t^n$$

$$\therefore\ \left[e^{xt}\ J_0\left(t\sqrt{1-x^2}\right)\right]=L\sum_{n=0}^{\infty}\frac{P_n}{n!}t^n$$

That means the function $e^{xt}\ J_0\left(t\sqrt{1-x^2}\right)$ is generating function for the Legendre Polynomial $P_n(x)$.

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