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M/D/1 Multiple Vacation Queueing Systems with Deterministic Service Time

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Abstract: *In this paper we assumed that the service times are deterministic and the two vacation types are exponentially distributed but with different means. The steady-state solution is obtained. Multiple vacations queueing system is that in which a vacation following a busy period has a different distribution from a vacation that is taken without serving at least one customer.*

Keywords: *Queueing Model; Vacation; Deterministic; Steady state.*

I. Introduction

In the modern world there is tough competition between service providers. So in order to survive service systems have to be managed efficiently and economically. Demand for service often fluctuates. There may be periods of low customer inflow. During such periods, it may not be economical from the system point of view to retain idle servers in the system. At the same time no system can afford to lose its customers and goodwill. So there is a need to strike a balance between the two extreme situations. A vacation queueing system is one in which a server may become unavailable for a random period of time from a primary service center. The time away from the primary service center is called a vacation, and it can be the result of many factors. In some cases the vacation can be the result of server breakdown, which means that the system must be repaired and brought back to service. It can also be a deliberate action taken to utilize the server in a secondary service center when there are no customers present at the primary service center.

II. Literature Review and Previous work

Queues with vacations have been extensively studied by several authors. Doshi [2] provides an exhaustive survey of such work through 1985. Since then the vacation models have been studied in different contexts. Among these include stochastic decomposition of queue length and that of stationary waiting time and we refer the reader to the recent book by Tian and Zhang [17] for details.

Server vacations are useful for those systems in which the server wishes to utilize his idle time for different purposes, and this makes the queueing model be applicable to a variety of real world stochastic service systems.

Recently vacation models have gained significance in telecommunication networks. However, compared to continuous time models discrete time models are more appropriate for modeling computer and telecommunication systems. Servi and Finn [15] introduced a working vacation model with the idea of offering services but at a lower rate whenever the server is on vacation. They applied the M/M/1 queue with multiple working vacations to model a Wavelength- Division Multiplexing optical access network and derived the probability-generating function (PGF) of the number of customers in the system. Queueing systems with server vacations have attracted the attention of many researchers since the idea was first discussed in the paper of Levy and Yechiali [28].

Their model was generalized to the case of M/G/1 in ([14], [6]), and to GI/M/1 model in [27]. A survey of working vacation models with emphasis on the use of matrix analytic methods is given in Tian and Li [23]. Working vacation models have a number of applications in practice. Two such examples are given in [23]. Li and Tian [11] studied an M/M/1 queue with working vacations in which vacationing server offers services at a lower rate for the first customer arriving during a vacation. Upon completion of the service at a lower rate the server will

(a) continue the current vacation (if not already completed) or take another vacation (if the working vacation expired) if there are no customers waiting; or

(b) resume at a normal rate (irrespective of whether the vacation expired or not) if there are customers waiting.

Resuming services at a normal rate while the vacation is still in progress corresponds to the vacation being interrupted. Several excellent surveys on these vacation models have been done by Doshi [2, 3], and the books by Takagi [7] and Tian and Zhang [17] are devoted to the subject. There are different types of vacation queueing systems. In the *single vacation scheme*, the server takes a vacation of a random duration when the

queue is empty. At the end of the vacation the server returns to the queue. If there is at least one customer waiting when the server returns from vacation, the server performs one of the following actions depending on the service policy.
(a) Under the *exhaustive service* policy, the server will serve all waiting customers as well as those that arrive while he is still serving at the station. He takes another vacation when the queue becomes empty.
(b) Under the *gated service* policy, the server will serve only those customers that he finds at the queue upon his return from vacation. At the end of their service the server will commence another vacation and any customers that arrive while the server was already serving at the station will be served when the server returns from the vacation.
(c) Under the *limited service* policy, the server will serve only a predefined maximum number of customers and then will commence another vacation. The single service scheme in which exactly one customer is served is a special type of this policy.

In the original formulation of the working vacation scheme the server cannot be interrupted when he is on vacation; he resumes full service only when his vacation ends. The working vacation scheme has attracted a lot of research effort, and several authors have extended the original model. Wu and Takagi [6] generalized the model in [15] to an M/G/1 queue with general working vacations. Baba [27] studied a GI/M/1 queue with working vacations by using the matrix analytic method. Banik et al. [1] analyzed the GI/M/1/N queue with working vacations. Liu et al. [24] established a stochastic decomposition result in the M/M/1 queue with working vacations. If the queue is empty on the server's return, the server waits to complete a busy period using one of the service policies before taking another vacation.

In the *multiple vacation schemes*, if the server returns from a vacation and finds the queue empty, he immediately commences another vacation. If there is at least one waiting customer, then he will commence service according to the prevailing service policy. Observe that in the vacation queueing system we have described the server completely stops service or is switched off when server is on vacation.

For the batch arrival queues, Xu et al. [25] studied a batch arrival M^X/M/1 queue with single working vacation. Using the matrix analytic method, they derived the PGF of the stationary system length distribution. Baba [26] studied a batch arrival M^X/M/1 queue with *multiple* working vacations. He obtained the PGF of the stationary system length distribution and the stochastic decomposition structure of system length that indicates the relationship with that of M^X/M/1 queue without vacation. The GI/M/1 queue with working vacations and vacation interruption was studied by Li et al. [12]. Similarly, Zhang and Hou [16] discussed an M/G/1 queue with multiple working vacations and vacation interruption. Recall that in the multiple vacation queueing system it is assumed that the vacation times are independent and identically distributed. However, there are practical environments where this assumption may not be valid. Specifically, a vacation taken after "long duration" during which many customers have been served may be longer than a vacation taken after the server returns from vacation and finds the queue empty. We define a vacation queueing system that distinguishes between two kinds of vacations that a server can take as a vacation queueing system with differentiated vacations. The analysis of the M/M/1 version of this type of vacation queueing system is the subject of this paper.

Thus, the paper deals with an M/D/1 queueing system in which two types of vacations can be taken by the server: a vacation taken immediately after the server has finished serving at least one customer and a vacation taken immediately after the server has just returned from a previous vacation to find that there are no customers waiting. The model is motivated by certain aspects of human and physical system behavior. For example, a computer system can suffer one of two types of failures: *permanent failure* and *discontinuous (soft) failure* [19]. A permanent failure, which is sometimes called a hard failure, requires the physical repair of the failed system, which usually takes a long time because it requires the presence of the field services personnel. By contrast, after a system has suffered a soft failure, no physical repair is required. The system is restored to operation by means of a system reboot or some other repair function that does not require the presence of the field services personnel. As long as the system is not being used for the intended service, it can be modeled as being on vacation.

Some researchers have also considered discrete-time working vacation systems. Tian et al. [18] considered the discrete time Geo/Geo/1 queue with multiple working vacations. Li and Tian [8] analyzed the discrete-time Geo/Geo/1 queue with single working vacation. Gao and Liu [21] analyzed the performance of a discrete-time Geo^X/G/1 queue with single working vacation. Li et al. [9] discussed a discrete-time batch arrival Geo^X/GI/1 queue with working vacations. Li and Tian [10] analyzed a GI/Geo/1 queue with working vacations and vacation interruption. Under such a policy, the server can come back to the normal working level before the vacation ends. They obtained the steady-state distributions for the number of customers in the system at arrival epochs and waiting time for an arbitrary customer using the matrix geometric solution method. Another example is the following. Consider a gas station attendant who operates under the following policy. When there is no customer waiting to be attended he will take a break that he can use to perform other functions at the station. At the end of the break if there is still no waiting customer, he will take another break, but if there is at least one waiting customer, he will serve exhaustively and will take a break when all customers have been served. This is the traditional multiple vacation model. Suppose now that there are two types of breaks that he can take. Specifically, after serving all customers in a busy period that includes at least one customer, he will take a personal break whose length has a given distribution. If he returns from a break and there is no waiting customer, he goes back on another break whose length has another distribution. This time can be used to attend to other duties at the station and usually has a shorter mean duration than the personal break. Thus, long breaks are associated with the completion of a busy period with at least one service completion while short breaks are associated with busy periods of zero length. In general, differentiated vacations occur in environments where "breaks" of different durations can occur.

In this paper we have associated these breaks with the durations of busy periods. Note that this model is different from the traditional multiple vacation models because in the traditional multiple vacation model the durations of vacations are identically distributed and are independent of the number of customers served in the busy period preceding the vacation. In the differentiated vacation model that we are proposing, there are two distributions of the durations of vacations: one is associated with a vacation taken after a nonzero busy period and the other is associated with a vacation taken after a zero busy period. The practical application of this model is that durations of vacations taken after a nonzero busy period can be longer than those that are taken when the server did not serve any customer prior to the vacation in order to give the server a sufficient time to rest following some hectic busy period.

III. Mathematical Model

We consider a multiple vacation queueing system where customers arrive according to a Poisson process with rate λ. The time to serve a customer is assumed to be deterministic (Here, we use deterministic service time model. The approach we use is similar to to be found in saaty [22] and which is essentially originally due to Crommelin [4]. In the M/D/1 queueing model the arrival rate is λ and the constant service time (say b=1/μ).with mean $1/\mu$, where $\mu > \lambda$.

Now we rescale our parameter as λ= λ/b and μ=1[5] so that the traffic intensity 'ρ' $\left(=\frac{\lambda}{\mu}\right)$ remains unaffected.

We assume that there are two types of vacations: type 1 vacation that is taken after a busy period of nonzero duration, and type 2 vacation that is taken when no customers are waiting for the server when it returns from a vacation.

For ease of analysis we assume that the durations of type 1 vacations are independent of the busy period and are exponentially distributed with mean $1/\gamma_1$ (As discussed earlier, there are cases where this independence assumption is not valid. However, we make this assumption to simplify the analysis.) Similarly, durations of type 2 vacations are assumed to be exponentially distributed with mean $1/\gamma_2$.

Let the state of the system be denoted by (r, k), where r is the number of customers in the system, k = 0 if the server is active serving customers, k = 1 if the server is on a type 1 vacation, and k = 2 if the server is on a type 2 vacation.

Steady-State Analysis

Let $P_{n,k}(t)$ denotes the probability that the process is in state (n,k) at time t, and let

$$P_{n,k} = \lim_{n\to\infty} P_{n,k}(t) \qquad (1)$$

Consider the result given by Oliver C. Ibe and Olubukola A. Isijola[20]

Using

$$(\lambda + \gamma_1)\, P_{0,1} = bP_{1,0} \qquad (2)$$

Therefore

$$P_{0,1} = \frac{bP_{1,0}}{(\lambda + \gamma_1)} = \alpha_1 P_{1,0} \qquad (3)$$

where $\alpha_1 = \frac{b}{(\lambda+\gamma_1)} < 1$

In similar way, we can write

$$\lambda P_{0,2} = \gamma_1 P_{1,0} \qquad (4)$$

which gives

$$P_{0,2} = \frac{\gamma_1}{\lambda} P_{0,1} = \left(\frac{\gamma_1}{\lambda}\right)\left(\frac{b}{(\lambda+\gamma_1)}\right) P_{1,0} = \alpha_2 P_{1,0} \qquad (5)$$

where $\alpha_2 = \frac{b\gamma_1}{\lambda(\lambda+\gamma_1)} < 1$

For n = 0, 1, 2, ……..,

we have $\lambda P_{n,1} = (\lambda + \gamma_1)P_{n+1,1}$

$$\lambda P_{n,2} = (\lambda + \gamma_2)P_{n+1,2} \quad (6)$$

Implies that

$$P_{n+1,1} = \frac{\lambda}{(\lambda+\gamma_1)}P_{n,1} \quad n = 0,1,2,\ldots,$$

$$P_{n+1,2} = \frac{\lambda}{(\lambda+\gamma_1)}P_{n,2} \quad n = 0,1,2,\ldots, \quad (7)$$

On solving the above equations we get

$$P_{n,1} = \left(\frac{\lambda}{\lambda+\gamma_1}\right)^{n} P_{0,1}$$

$$= \frac{b}{(\lambda+\gamma_1)}\left(\frac{\lambda}{\lambda+\gamma_1}\right)^{n} P_{1,0}$$

$$= \alpha_1\left(\frac{\lambda}{\lambda+\gamma_1}\right)^{n} P_{1,0} = \alpha_1\beta_1^n P_{1,0} \quad (8)$$

Where $\beta_1 = \frac{\lambda}{\lambda+\gamma_1} < 1$

$$P_{n,2} = \left(\frac{\lambda}{\lambda+\gamma_2}\right)^{n} P_{0,2}$$

$$= \frac{b\gamma_1}{\lambda(\lambda+\gamma_1)}\left(\frac{\lambda}{\lambda+\gamma_2}\right)^{n} P_{1,0}$$

$$= \alpha_2\left(\frac{\lambda}{\lambda+\gamma_2}\right)^{n} P_{1,0} = \alpha_2\beta_2^n P_{1,0} \quad (9)$$

Where $\beta_2 = \frac{\lambda}{\lambda+\gamma_2} < 1$

For local balance we have

$$\lambda P_{n,0} + \lambda P_{n,1} + \lambda P_{n,2} = \mu P_{n+1,0}$$

$$\Rightarrow \lambda P_{n,0} + \lambda P_{n,1} + \lambda P_{n,2} = bP_{n+1,0} \quad n= 1, 2, 3\ldots\ldots$$

$$\Rightarrow P_{n+1,0} = (\lambda/b)P_{n,0} + (\lambda/b)P_{n,1} + (\lambda/b)P_{n,2}$$

As we have $\rho = \lambda/\mu$ and for deterministic service time take $\mu = b$

$$\Rightarrow P_{n+1,0} = \rho P_{n,0} + \rho P_{n,1} + \rho P_{n,2} \quad (10)$$

$$\Rightarrow P_{n+1,0} = \rho[P_{n,0} + P_{n,1} + P_{n,2}] \quad (11)$$

$$\Rightarrow P_{n+1,0} = \rho[P_{n,0} + \alpha_1\beta_1^n P_{1,0} + \alpha_2\beta_2^n P_{1,0}] \quad (12)$$

On solving recursively, we obtain

$$P_{n,0} = \rho\left\{\frac{\alpha_1\beta_1(\beta_1^{n-1} - \rho^{n-1}}{\beta_1 - \rho} + \frac{\alpha_1\beta_2(\beta_2^{n-1} - \rho^{n-1}}{\beta_2 - \rho} + \rho^{n-2}\right\} P_{1,0} \quad (13)$$

n = 1, 2,…..

From the law of total probability, we have

$$\sum_{n=1}^{\infty} P_{n,0} + \sum_{n=0}^{\infty} P_{n,1} + \sum_{n=0}^{\infty} P_{n,2} = 1$$

$$P_{1,0}\left\{\frac{\alpha_1}{1-\beta_1} + \frac{\alpha_2}{1-\beta_2} + \frac{\alpha_1\beta_1\rho}{(1-\beta_1)(1-\rho)} + \frac{\alpha_2\beta_2\rho}{(1-\beta_2)(1-\rho)} + \frac{1}{(1-\rho)}\right\} = 1$$

On simplification, we get

$P_{1,0}= \quad ((1-\beta_1)(1-\beta_2)(1-\rho))[\alpha_1(1-\beta_2)(1-\rho) + \alpha_2(1-\beta_1)(1-\rho) + \alpha_1\beta_1\rho(1-\beta_2) + \alpha_2\beta_2\rho(1-$
$\beta 1 + (1-\beta 1)(1-\beta 2)]-1$

(14)

The result is given by
The steady state probability is given by

$$P_{n,k} = \begin{cases} \frac{\lambda}{b}\left[\frac{\alpha_1\beta_1(\beta_1^{n-1} - \left(\frac{\lambda}{b}\right)^{n-1})}{\beta_1 - \frac{\lambda}{b}} + \frac{\alpha_1\beta_2(\beta_2^{n-1} - \left(\frac{\lambda}{b}\right)^{n-1})}{\beta_2 - \frac{\lambda}{b}} + \left(\frac{\lambda}{b}\right)^{n-2}\right] P_{1,0} & k = 0 \\ \alpha_1\beta_1^n P_{1,0} & k = 1 \\ \alpha_2\beta_2^n P_{1,0} & k = 2 \end{cases}$$

The mean number of customers in the system is given by

$$\mathrm{E[N]} = \sum_{n=1}^{\infty} nP_{n,0} + \sum_{n=0}^{\infty} nP_{n,1} + \sum_{n=0}^{\infty} nP_{n,2}$$

$$= \left\{ \frac{\alpha_1\beta_1\left(\frac{\lambda}{b}\right)\left(2 - \left(\frac{\lambda}{b}\right) - \beta_1\right)}{(1-\beta_1)^2\left(1 - \left(\frac{\lambda}{b}\right)\right)^2} + \frac{\alpha_2\beta_2\left(\frac{\lambda}{b}\right)\left(2 - \left(\frac{\lambda}{b}\right) - \beta_2\right)}{(1-\beta_2)^2\left(1 - \left(\frac{\lambda}{b}\right)\right)^2} + \frac{\alpha_1\beta_1}{(1-\beta_1)^2} + \frac{\alpha_2\beta_2}{(1-\beta_2)^2} + \frac{1}{(1-\left(\frac{\lambda}{b}\right))^2} \right\} P_{1,0}$$

By Little's formula [13] the mean time a customer spends in the system (or mean delay) is given by

$$E[T] = \frac{E[N]}{\lambda} \qquad (15)$$

IV. Conclusion

We have considered the simple case of M/D/1 model. The results shows that the mean time a customer spends in the system, is more sensitive to the mean duration of the first type of vacation than of the second type, which is to be expected since the duration of the first type of vacation is assumed to be longer than that of the second type. We have considered multiple vacation queueing systems in which two types of vacations are encountered. The first type is a vacation that is taken at the end of a busy period of nonzero duration, and the second is a vacation taken at the end of a busy period of zero duration, which means that no customer was served.

Here we choose the deterministic service time, this queueing system reflects many real life experiences where some vacations can be used for post processing activities while others are actual "breaks" that the server takes.

References

[1] A. D. Banik, U. C. Gupta, and S. S. Pathak, "On the GI/M/1/N queue with multiple working vacations-analytic analysis and computation, "*Applied Mathematical Modeling*, vol. 31,no. 9, pp. 1701–1710, 2007.

[2] B. Doshi, "Queueing systems with vacations—a survey," *Queueing Systems: Theory and Applications*, vol. 1, no.1, pp. 29– 66, 1986.

[3] B. Doshi, "Single server queues with vacations," in *Stochastic Analysis of Computer and Communication Systems*, H. Takag, Ed., pp. 217–265, Elsevier, 1990.

[4] *Crommelin, C. D. 1932. Delay Probability formulae when the holding times are constant. P. O. Electrical Engineering Journal 25, 41-50.*

[5] *D. Gross, J. Shortle, J. Thompson, and C. Harris , Fundamentals of Queueing Theory, Forth edition, Wiley,Poisson Input, Constant Service. pp- 294. 2013.*

[6] D. Wu and H. Takagi, "M/G/1 queue with multiple working vacations," *Performance Evaluation*, vol. 63, no. 7, pp. 654–681, 2006.

[7] H. Takagi, *Queueing Analysis: A Foundation of Performance Analysis*, vol. 1 of *Vacation and Priority Systems*, part 1, Elsevier Science Publishers B.V., Amsterdam, The Netherlands, 1991.

[8] J. Li and N. Tian, "Analysis of the discrete time Geo/Geo/1 queue with single working vacation," *Quality Technology & Quantitative Management*, vol. 5, no. 1, pp. 77–89, 2008.

[9] J. Li, W. Liu, and N. Tian, "Steady-state analysis of a discrete time batch arrival queue with working vacations," *Performance Evaluation*, vol. 67, no. 10, pp. 897–912, 2010.

[10] J. Li and N. Tian, "The discrete-time GI/Geo/1 queue with working vacations and vacation interruption," *Applied Mathematics and Computation*, vol. 185, no. 1, pp. 1–10, 2007.

[11] J. Li and N.Tian, "TheM/M/1 queue with working vacations and vacation interruptions," *Journal of Systems Science and Systems Engineering*, vol. 16, no. 1, pp. 121–127, 2007.

[12] J. Li, N. Tian, and Z. Ma, "Performance analysis of GI/M/1 queue with working vacations and vacation interruption," *Applied Mathematical Modeling*, vol. 32, no. 12, pp. 2715–2730, 2008.

[13] J. D. C. Little, "A proof for the queuing formula:," *Operations Research*, vol. 9, no. 3, pp. 383–387, 1961.

[14] Kim, J., Choi, D. and Chae, K. : Analysis of queue-length distribution of the M/G/1 queue with working vacations, International Conference on Statistics and Related Fields, Hawaii, 2003.

[15] L. D. Servi and S. G. Finn, "M/M/1 queues with working vacations (M/M/1/WV)," *Performance Evaluation*, vol. 50, no. 1, pp. 41–52, 2002.

[16] M. Zhang and Z. Hou, "Performance analysis of M/G/1 queue with working vacations and vacation interruption," *Journal of Computational and Applied Mathematics*, vol. 234, no. 10, pp. 2977–2985, 2010.

[17] N. Tian and Z. G. Zhang, *Vacation Queueing Models: Theory and Applications*, Springer, New York, NY, USA, 2006.

[18] N. Tian, Z.Ma, andM. Liu, "The discrete time Geom/Geom/1 queue with multiple working vacations," *Applied Mathematical Modeling*, vol. 32, no. 12, pp. 2941–2953, 2008.

[19] Oliver. C. Ibe, R. C. Howe, and K. S. Trivedi, "Approximate availability analysis of VAXcluster systems," *IEEE Transactions on Reliability*, vol. 38, no. 1, pp. 146–152, 1989.

[20] Oliver C. Ibe and Olubukola A. Isijola, "M/M/1 Multiple Vacation Queueing System with Differentiated Vacations," Hindawi Publication Corporation Modeling and Simulation in Engineering vol. 2014

[21] S. Gao and Z. Liu, "Performance analysis of a discrete-time Geo/G/1 queue with single working vacation," *World Academy of Science, Engineering and Technology*, vol. 56, pp. 1162–1170, 2011.

[22] *Saaty, T. L. 1961, Elements of Queueing Theory with Applications McGraw Hill, New York.*

[23] Tian, N. and Li, J. : Matrix Analytic Method and Working Vacations Queues-A Survey. International Journal of Information and Management Sciences, 20, 603-633, 2009.

[24] W. Liu, X. Xu, and N. Tian, "Stochastic decompositions in the M/M/1 queue with working vacations," *Operations Research Letters*, vol. 35, no. 5, pp. 595–600, 2007.

[25] X. Xu, Z. Zhang, and N. Tian, "Analysis for the M^X/M/1 working vacation queue," *International Journal of Information and Management Sciences*, vol. 20, no. 3, pp. 379–394, 2009.

[26] Y. Baba, "The M^X/M/1 queue with multiple working vacation," *American Journal of Operations Research*, vol. 2, no. 2, pp. 217–224, 2012.

[27] Y. Baba, "Analysis of a GI/M/1 queue with multiple working vacations," *Operations Research Letters*, vol. 33, no. 2, pp. 201– 209, 2005.

[28] Y. Levy and U. Yechiali, "Utilization of idle time in an *M/G/*1 queueing system," *Management Science*, vol. 22, no. 2, pp. 202–211, 1975.