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Common Fixed Point Theorem in Intuitionistic Fuzzy Metric Space Weakely Compactible Set Maps by Using General Contractive Condition of Integral Type

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Abstract: *The aim of this paper is to obtain common fixed points theorems in an intuitionistic fuzzy metric space for point wise R-weakly commuting mappings using contractive condition of integral type and to establish a situation in which a collection of maps has a fixed point which is a point of discontinuity.*
Keywords*: Fuzzy set, Intuitionistic fuzzy set, Intuitionistic fuzzy metric space, pointwiseR-weakly commuting, reciprocally, non-compatible, Integraltype*

I. Introduction

Atanassov [3] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [22] and later there has been much progress in the study of intuitionistic fuzzy sets by many authors [4, 7]. In 2004, Park [17] introduced a notion of intuitionistic fuzzy metric spaces with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [12]. Fixed point theory has important applications in diverse disciplines of mathematics, statistics, engineering and economics in dealing with problems arising in:Approximation theory, potential theory, game theory, mathematical economics, etc. Several authors [9, 10, 12, 13, and 19] proved some fixed point theorems for various generalizations of contraction mappings in probabilistic and fuzzy metric space. Branciari [6] obtained a fixed point theorem for a single mapping satisfying an analogue of Banach's contraction principle for an integral type inequality. Sedhi.at.el [20] established a common fixed point theorem for weakly compatible mappings in intuitionistic fuzzy metric space satisfying a contractive condition of integral type.

In this paper, we prove a common fixed point theorem for six self mapsin an intuitionistic fuzzy metric space for pointwise R-weakly commuting mappings using contractive condition of integral type and to establish a situation in which a collection of maps has a fixed point which is a point of discontinuity.

II. Preliminaries

Definition 2.1. [22] Let X be any set. A fuzzy set A in X is a function with domain X and values in [0, 1].
Definition 2.2. [3] Let a set E be fixed. An intuitionistic fuzzy set (IFS) A of E is an object having the form,

$A = \{< x, \mu_A(x), V_A(x) > / x \in E\}$

where the function $\mu_A : E \to [0, 1]$, $V_A : E \to [0, 1]$ define respectively, the degree of membership and degree of non-membership of the element $x \in E$ to the set A, which is a subset of E, and for every $x \in E$, $0 \leq \mu_A(x)+V_A(x) \leq 1$.
Definition 2.3. [19] A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-norm if it satisfies the following conditions:
(a) $*$ is commutative and associative;
(b) $*$ is continuous;
(c) $a* 1 = a$ for all $a \in [0, 1]$;
(d) $a* b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0, 1]$.
Definition 2.4. [19] A binary operation $\Diamond : [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-conorm if it satisfies the following conditions:
(a) $\Diamond$ is commutative and associative;
(b) $\Diamond$ is continuous;
(c) $a\Diamond 0 = a$ for all $a \in [0, 1]$;
(d) $a\Diamond b \leq c\Diamond d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0, 1]$.
Definition 2.5. [1] A 5-tuple $(X,M, N, *, \Diamond)$ is said to be an intuitionistic fuzzy metric space (shortly IFM-Space) if X is an arbitrary set, $*$ is a continuous t-norm, $\Diamond$ is a continuous t-conorm and M,N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions: for all $x, y, z \in X$ and $s, t > 0$;
(IFM-1) $M(x, y, t) + N(x, y, t) \leq 1$;
(IFM-2) $M(x, y, 0) = 0$;

(IFM-3) M(x, y, t) =1 if and only if x = y;
(IFM-4) M(x, y, t) = M(y, x, t);
(IFM-5) M(x, y, t) ∗ M(y, z, s) ≤ M(x, z, t + s);
(IFM-6) M(x, y, .) : [0, ∞) → [0, 1] is left continuous;
(IFM-7) $\lim_{t\to\infty}$M(x, y, t) =1;
(IFM-8) N(x, y, 0) = 1;
(IFM-9) N(x, y, t) =0 if and only if x = y;
(IFM-10) N(x, y, t) = N(y, x, t);
(IFM-11) N(x, y, t) ◊N(y, z, s) ≥ N(x, z, t + s);
(IFM-12) N(x, y, .): [0, ∞) → [0, 1] is right continuous;
(IFM-13) $\lim_{t\to\infty}$N(x, y, t) =0;

Then (M,N) is called an intuitionistic fuzzy metric on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and degree of non-nearness between x and y with respect to t, respectively.

Remark 2.6. Every fuzzy metric space (X, M, ∗) is an intuitionistic fuzzy metric space if X of the form (X, M, 1 − M, ∗, ◊) such that t- norm ∗ and tconorm◊ are associated, that is,x◊ y = 1− ((1−x) ∗ (1−y)) for any x, y ∈ X. But the converse is not true.

Example 2.7. [17] Let (X, d) be a metric space. Denote a ∗ b = ab and a◊ b = min {1, a + b} for all a, b ∈ [0, 1] and let M_d and $N_d$be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows;

$M_d(x, y, t) = \frac{t}{t + d(x,y)}$, $N_d(x, y, t) = \frac{d(x,y)}{t + d(x,y)}$. Then ($M_d$, N_d) is an intuitionistic fuzzy metric on X. We call this intuitionistic fuzzy metric induced by a metric d the standard intuitionistic fuzzy metric.

Remark 2.8. Note the above example holds even with the t-norm a ∗ b = min {a, b} and the t-conorm a◊ b = max {a, b} and hence (M_d, N_d) is an intuitionistic fuzzy metric with respect to any continuous t-norm and continuous t-conorm.

Example 2.9. Let X = N. Define a ∗ b = max{0, a + b − 1} and a◊ b = a + b − ab for all a, b ∈ [0, 1] and let M and N be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows;

$$M(x, y, t) = \begin{cases} \frac{x}{y}, \text{if } x \le y \\ \frac{y}{x}, \text{if } y \le x \end{cases}$$

$$N(x, y, t) = \begin{cases} \frac{y-x}{y}, if x \le y, \\ \frac{x-y}{x}, if\ y \le x \end{cases}$$

for all x, y, z ∈ X and t > 0. Then (X, M, N, ∗, ◊) is an intuitionistic fuzzy metric space.

Remark 2.10. Note that, in the above example, t-norm ∗ and t-conorm◊ are not associated. And there exists no metricd on X satisfying

$M(x, y, t) = \frac{t}{t + d(x,y)}$, $N(x, y, t) = \frac{d(x,y)}{t + d(x,y)}$.

where M(x, y, t) and N(x, y, t) are as defined in above example. Also note the above function (M, N) is not an intuitionistic fuzzy metric with the t-norm and t-conorm defined as

a∗ b = min{a, b} and a◊ b = max{a, b}.

Definition 2.11. [1] Let (X, M, N, ∗, ◊) be an intuitionistic fuzzy metric space.
(a) A sequence {x_n} in X is called Cauchy sequence if for each t > 0 and P > 0,
$\lim_{n\to\infty}$M (x_{n+p}, x_n, t) =1 and $\lim_{n\to\infty}$N (x_{n+p}, x_n, t) =0.
(b) A sequence {x_n} in X is convergent to x ∈ X if $\lim_{n\to\infty}$M (x_n, x, t) =1 and
$\lim_{n\to\infty}$ N(x_n, x, t)=0 for each t > 0.
(c) An intuitionistic fuzzy metric space is said to be complete if every Cauchy sequence is convergent.

Lemma 2.12. [17] In an intuitionistic fuzzy metric spaceX, M(x, y, .) is non-decreasing and N(x, y, .) is non-increasing for all x, y ∈ X.

Lemma 2.13. [21] Let (X, M, N, ∗, ◊) be an intuitionistic fuzzy metric space. If there exists a constant k ∈ (0, 1) such that

M(y_{n+2}, y_{n+1}, kt) ≥ M(y_{n+1}, y_n, t),
N(y_{n+2}, y_{n+1}, kt) ≤ N(y_{n+1}, y_n, t)

∀t > 0 and n = 1, 2, ..then {y_n} is a Cauchy sequence in X.

Lemma 2.14. [21] Let (X, M, N, ∗, ◊) be an intuitionistic fuzzy metric space. If there exists a constant k ∈ (0, 1) such that

M(x, y, kt) ≥ M(x, y, t), N(x, y, kt) ≤ N(x, y, t),

for x, y ∈ X. Then x = y.

Definition 2.15. [15] Let (X, d) be a metric space. Two self mappings f and g of X are said to be R-weakly commuting if there exists a positive real number R > 0 such that

d(fg(x), gf(x)) ≤ Rd(f(x), g(x))

for all x ∈ X.

Definition 2.16. Let (X, M, N, ∗, ◊) be an intuitionistic fuzzy metric space. Two self mappings f and g of X are said to be pointwise R-weakly commuting on X if given x ∈ X there exists a positive real number R > 0 such that

$M(fg(x), gf(x), t) \geq M(f(x), g(x), t/R)$

$N(fg(x), gf(x), t) \leq N(f(x), g(x), t/R)$

and t > 0.

Definition 2.17. Let A and S be mappings from an intuitionistic fuzzy metric space (X, M, N, ∗, ◊) into itself. Then the mappings are said to be compatible if

$\lim_{n\to\infty} M(ASx_n, SAx_n, t)=1,$

$\lim_{n\to\infty} N(ASxn, SAxn, t)=0,$

for every t > 0, whenever $\{x_n\}$ is a sequence in X such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z,$

for some z ∈ X.

Definition 2.18. Let A and S be mappings from an intuitionistic fuzzy metric space (X, M, N, ∗, ◊) into itself. Then the mappings are said to be non-compatible if whenever $\{x_n\}$ is a sequence in X such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z,$

for some z ∈ X. But

$\lim_{n\to\infty} M(ASx_n, SAx_n, t) \neq 1$

or non-existent,

$\lim_{n\to\infty} N(ASx_n, SAx_n, t) \neq 0$ or non-existent.

Definition 2.19. Let A and S be mappings from an intuitionistic fuzzy metric space (X, M, N, ∗, ◊) into itself. Then the mappings are said to be reciprocally continuous if

$\lim_{n\to\infty} ASx_n = Az$, and $\lim_{n\to\infty} SAx_n = Sz$,

whenever $\{x_n\}$ is a sequence in X such that

$\lim_{n\to\infty} Ax_n = \lim n\to\infty\ Sx_n = z,$

for some z ∈ X.

Remark 2.20. If A and S are both continuous then they are obviously reciprocally continuous. But the converse need not be true.

III. Main Results

Theorem 3.1. Let P,Q,A,B,S & T be mappings from an intuitionisticmetric space(X, M, N, ∗, ◊) in to itself. Let (P, ST) and (Q,AB) be a pointwise R-weakly commuting pairs of self mappings of a complete intuitionistic fuzzy metric space (X,M, N, ∗, ◊) with continuous t-norm ∗ and continuous t-corm ◊ defined by $t * t \geq t$ and $(1 - t)◊ (1 - t) \leq (1 - t)$ for all t ∈ [0, 1] such that,

(i) P(X)⊂AB(X),Q(X)⊂ ST(X)

(ii) there exists a constant k ∈ (0, 1) such that

$$\int_0^{M(Px,Qy,kt)} \phi(t)dt \geq \left(\int_0^{m(x,y,t)} \phi(t)dt\right), \qquad (3.1)$$

$$\int_0^{N(Px,Qy,kt)} \phi(t)dt \leq \left(\int_0^{n(x,y,t)} \phi(t)dt\right), \qquad (3.2)$$

where $\phi: R^+ \to R^+$ is a Lebesgue-integrable mapping which is summable, nonnegative, and such that

$$\int_0^{\epsilon} \phi(t)dt > 0 \text{ for each } \epsilon > 0,$$

where

$m(x, y, t) = \min \{M(ABy, Qy, t), M(STx, Px, t), M(STx, Qy, \alpha t), M(AB y, Px,(2 - \alpha)t), M(AB y, STx, t)\}$

$n(x, y, t) = \max\{N(ABy, Qy, t), N(STx, Px, t), N(STx, Qy, \alpha t), N(AB y, Px,(2 - \alpha)t), N(AB y, STx, t)\}$

for all x, y ∈ X, α ∈ (0, 2) and t > 0. Suppose that (P, ST) or (Q,AB) is a compatible pair of reciprocally continuous mappings. Then P, Q, ST and AB have a unique common fixed point. If the pairs (A,B),(S,T),(Q,B) and (T,P) are commuting mappings then A,B,S,T,P and Q have a unique common fixed point.

Proof. Let x_0 be any point in X. we construct a sequence $\{y_n\}$ in X such that for n = 0, 1, 2...

$$y_{2n} = Px_{2n} = ABx_{2n+1}$$
$$y_{2n+1} = Qx_{2n+1} = STx_{2n+2}.$$

(3.3) we show that $\{y_n\}$ is a Cauchy sequence. By (3.1) and (3.2), for all t > 0 and α = 1 − β withβ ∈ (0, 1), we have

$$\int_0^{M(y_{2n+1},y_{2n+2},kt)} \phi(t)dt = \int_0^{M(Qx_{2n+1},Px_{2n+2},kt)} \phi(t)dt,$$
$$= \int_0^{M(Px_{2n+2},Qx_{2n+1},kt)} \phi(t)dt,$$
$$\geq \int_0^{m(x_{2n+2},x_{2n+1},t)} \phi(t)dt$$

$$\int_0^{N(y_{2n+1},y_{2n+2},kt)} \phi(t)dt = \int_0^{N(Qx_{2n+1},Px_{2n+2},kt)} \phi(t)dt$$

$$= \int_0^{N(Px_{2n+2}, Qx_{2n+1}, kt)} \phi(t)dt$$
$$\leq \int_0^{n(x_{2n+2}, x_{2n+1}, t)} \phi(t)dt$$

$m(x_{2n+2}, x_{2n+1}, t) = \min\{M(AB\, x_{2n+1}, Qx_{2n+1}, t), M(Px_{2n+2}, STx_{2n+2}, t), M(STx_{2n+2}, Qx_{2n+1}, \alpha t),$
$M(AB\, x_{2n+1}, Px_{2n+2}, (2-\alpha)t), M(AB\, x_{2n+1}, STx_{2n+2}, t)\}$
$= \min\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+2}, t), M(y_{2n+1}, y_{2n+1}, \alpha t), M(y_{2n}, y_{2n+2}, (1+\beta)t)),$
$M(y_{2n}, y_{2n+1}, t)\}$
$\geq \min\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+2}, t), 1, M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+2}, \beta t),$
$M(y_{2n}, y_{2n+1}, t)\}$
$\geq \min\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+2}, t), M(y_{2n+1}, y_{2n+2}, \beta t)\}$
$n(x_{2n+2}, x_{2n+1}, t) = \max\{N(AB\, x_{2n+1}, Qx_{2n+1}, t), N(Px_{2n+2}, STx_{2n+2}, t), N(STx_{2n+2}, Qx_{2n+1}, \alpha t),$
$N(AB\, x_{2n+1}, Px_{2n+2}, (2-\alpha)t), N(AB\, x_{2n+1}, STx_{2n+2}, t)\}$
$= \max\{N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n+2}, t), N(y_{2n+1}, y_{2n+1}, \alpha t), N(y_{2n}, y_{2n+2}, (1+\beta)t)),$
$N(y_{2n}, y_{2n+1}, t)\}$
$\leq \max\{N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n+2}, t), 1, N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n+2}, \beta t), N(y_{2n}, y_{2n+1}, t)\}$
$\leq \max\{N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n+2}, t), N(y_{2n+1}, y_{2n+2}, \beta t)\}$
Since t-norm *, t-conorm ◊, M(x, y, .) and N(x ,y,.) is continuous .Letting $\beta \to 1$,we have
$m(x_{2n+2}, x_{2n+1}, t) \geq \min\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+2}, t)\}$
$n(x_{2n+2}, x_{2n+1}, t) \leq \max\{N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n+2}, t)\}$
Therefore,
$\int_0^{M(y_{2n+1}, y_{2n+2}, kt)} \phi(t)dt \geq \int_0^{\min\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+2}, t)\}} \phi(t)dt$,
$$\int_0^{N(y_{2n+1}, y_{2n+2}, kt)} \phi(t)dt \leq \int_0^{\max\{N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n+2}, t)\}} \phi(t)dt$$
Similarly, we can obtain
$\int_0^{M(y_{2n+2}, y_{2n+3}, kt)} \phi(t)dt \geq \int_0^{\min\{M(y_{2n+1}, y_{2n+2}, t), M(y_{2n+2}, y_{2n+3}, t)\}} \phi(t)dt$,
$\int_0^{N(y_{2n+2}, y_{2n+3}, kt)} \phi(t)dt \leq \int_0^{\max\{N(y_{2n+1}, y_{2n+2}, t), N(y_{2n+2}, y_{2n+3}, t)\}} \phi(t)dt$.
In general,
$\int_0^{M(y_{n+1}, y_{n+2}, kt)} \phi(t)dt \geq \int_0^{\min\{M(y_n, y_{n+1}, t), M(y_{n+1}, y_{n+2}, t)\}} \phi(t)dt$,
$$\int_0^{N(y_{n+1}, y_{n+3}, kt)} \phi(t)dt \leq \int_0^{\max\{N(y_n, y_{n+1}, t), N(y_{n+1}, y_{n+2}, t)\}} \phi(t)dt.$$

and, for every positive integer p,
$\int_0^{M(y_{n+1}, y_{n+2}, kt)} \phi(t)dt \geq \int_0^{\min\{M(y_n, y_{n+1}, t), M(y_{n+1}, y_{n+2}, t/k^p)\}} \phi(t)dt$,
$$\int_0^{N(y_{n+1}, y_{n+2}, kt)} \phi(t)dt \leq \int_0^{\max\{N(y_n, y_{n+1}, t), N(y_{n+1}, y_{n+2}, t/k^p)\}} \phi(t)dt$$
Since $M(y_{n+1}, y_{n+2}, t/k^p) \to 1$ as $p\to\infty$, $N(y_{n+1}, y_{n+2}, t/k^p) \to 0$ as $p\to\infty$,
$\int_0^{M(y_{n+1}, y_{n+2}, kt)} \phi(t)dt \geq \int_0^{M(y_n, y_{n+1}, t)} \phi(t)dt$,
$$\int_0^{N(y_{n+1}, y_{n+2}, kt)} \phi(t)dt \leq \int_0^{N(y_n, y_{n+1}, t)} \phi(t)dt$$
By Lemma 2.13, $\{y_n\}$is Cauchy sequence in *X.* Since *X* is a complete, there is a point *z* in *X* such that $y_n \to z \in X$. Hence from (3.3), we have
$y_{2n} = Px_{2n} = ABx_{2+1} \to z$,
$y_{2n+1} = Qx_{2n+1} = STx_{2n+2} \to z$.
Since P and ST are compatible and reciprocally continuous mappings, then$PSTx_{2n} \to Pz$and $STPx_{2n} \to STz$as $n \to \infty$. The compatibility of the pair (P, ST) yields
$\lim_{n\to\infty} M(PSTx_{2n}, STPx_{2n}, t) = 1$
That is,
M (Pz, STz, t) = 1. Hence Pz= STz.
The compatibility of the pair (P, ST) yields
$\lim_{n\to\infty} N(PSTx_{2n}, STPx_{2n}, t) = 0$
That is,

N (Pz, STz, t) = 0. Hence Pz= STz.
Since P(X)⊂ AB(X), there exist w ∈ X such that Pz= ABw. Using (ii), we get

$\int_0^{M(Pz,Qw,kt)} \phi(t)dt \geq \int_0^{M(z,w,t)} \phi(t)dt$,

$$\int_0^{N(Pz,Qw,,kt)} \phi(t)dt \leq \int_0^{N(z,w,t)} \phi(t)dt$$

Take α = 1,
m(z,w, t) = min{M(ABw,Qw, t),M(STz,Pz, t),M(STz,Qw, t),M(ABw,Pz, t),M(ABw,STz, t)}
= min {M (Pz,Qw, t), 1,M(Pz,Qw, t), 1, 1}
= min {M (Pz,Qw, t), 1},
n(z,w, t) =max{N(ABw,Qw, t),N(STz,Pz, t),N(STz,Qw, t),N(ABw,Pz, t),N(ABw,STz, t)}
= max{N(Pz,Qw, t),1,N(Pz,Qw, t),1,1}
= max{N(Pz ,Qw,t),1}

$\int_0^{M(Pz,Qw,kt)} \phi(t)dt \geq \int_0^{m(Pz,Qw,t)} \phi(t)dt$,

$$\int_0^{N(Pz,Qw,,kt)} \phi(t)dt \leq \int_0^{n(Pz,Qw,t)} \phi(t)dt$$

By using Lemma 2.14, we get Pz= Qw. Thus,
STz= Pz= Qw= ABw.
PointwiseR-weakly commuting of Pand STimplies that there exists R >0 such that
M(PSTz, STPz, t) ≥ M(Pz, STz, t/R) = 1,
N(PSTz, STPz, t) ≤ N(Pz, STz, t/R) = 0.
That is,
PSTz= STPzand PPz= PSTz= STPz= STSTz.
Similarly, PointwiseR-weakly commuting of Q and ABimplies that there exists R >0 such that
M (QABw, ABQw, t) ≥ M(Qw, ABw, t/R) = 1,
N(QABw, ABQw, t) ≤ N(Qw, ABw, t/R) = 0.
That is,
QABw= ABQwand QQw= QABTw= ABQw= ABABw.
Using (ii), we get

$\int_0^{M(Pz,PPz,kt)} \phi(t)dt \geq \int_0^{M(PPz,Qw,kt)} \phi(t)dt$,

$$\geq \int_0^{m(Pz,w,t)} \phi(t)dt$$

$$= \int_0^{M(Pz,PPz,t)} \phi(t)dt$$

$$\int_0^{N(Pz,PPz,,kt)} \phi(t)dt = \int_0^{N(PPz,Qw,kt)} \phi(t)dt$$

$$\leq \int_0^{n(Pz,w,t)} \phi(t)dt$$

$$= \int_0^{N(Pz,PPz,t)} \phi(t)dt$$

By using Lemma 2.14, we get Pz= PPzand Pz= PPz= STPz. Thus, Pzis a common fixed point of P and ST. Similarly, by using (ii), we get Qw (=Pz) is a common fixed point of Q and AB. Uniqueness of the common fixed point follows easily and the proof is similar when Q and AB are assumed compatible and reciprocally continuous.

By using the commntativity of the pairs (A,B),(S,T),(Q,B) and (P,T) we can show that the selfmaps A,B,P,Q,B,T have a Unique common fixed point in X.

Example 3.2. Let X = [2, 20] and (X, M, N, *,◊) be a intuitionistic fuzzy metric. Define mappings P, B, S, T:X → X by

P(x) $=\begin{cases} 2\ , if x = 2, \\ 3,\ if x > 3. \end{cases}$

S(x)$=\begin{cases} 2, if x = 2, \\ 6, if x = 6. \end{cases}$

Q(x) $=\begin{cases} 2\ , if x = 2\ or x > 5, \\ 6, if\ 2 < x \leq 5 \end{cases}$

$$A(x) = \begin{cases} 2, & if x = 2, \\ 12, if\ 2 < x < 5, \\ x-3, & if x > 5 \end{cases}$$

T(x) = B(x) = x ,$\forall x \epsilon$ [2,20]

Also, we define,

$$M(Px, Qy, t) = \frac{t}{(t+|x-y|)} \quad N(Px, Qy, t) = \frac{|x-y|}{(t+|x-y|)},$$

For all x, y ∈ X, t >0. Then P, Q, ST and AB satisfy all the conditions of the above Theorem with k = (0, 1) and ϕ(t) = 1 and have a unique common fixed point x = 2. Here, P and ST are reciprocally continuous compatible maps. But neither P nor ST is continuous, even at the common fixed point x = 2. The mapping Qand ABare non-compatible but pointwiseR-weakly commuting. Q and ABare pointwiseR-weakly commuting since they commute at their coincidencepoints. To see that Qand AB are non-compatible, let us consider thesequence $\{x_n\}$ defined by

x_n= 5+1/n, n ≥ 1. Then $ABx_n \to 2$, Qx_n= 2, $ABQx_n$= 2, $QABx_n$= 6.Hence Q and AB are noncompatible.

Remark 3.3. All the mappings involved in this example are discontinuous at the common fixed point.

Remark 3.4. Compatible maps are necessarily pointwiseR weakly commuting since compatible maps commute at their coincidence points. However, as shown in the above example for the mappings Q and AB, pointwiseR-weakly commuting maps need not be compatible.

Remark 3.5. In this remark we demonstrate that pointwiseR-weak commutatively is a necessary condition for the existence of common fixed points of contractive mapping pairs. So, let us assume that the self mappingsA and S of an intuitionistic fuzzy metric space (X, M, N, *, ◊) satisfy the contractive condition

$$\int_0^{M(Px,Py,kt)} \phi(t)dt \geq \int_0^{m(x,y,t)} \phi(t)dt,$$

where

m(x, y, t) = min{M(STx,Sty, t),M(Px, STx, t),M(Py, STy, t),M(Px, STy, t),M(Py, STx, t)},

$$\int_0^{N(Px,Py,,kt)} \phi(t)dt < \int_0^{m(x,y,t)} \phi(t)dt,$$

where

n(x, y, t) = max{N(STx,STy, t),N(Px, STx, t),N(Py, STy, t),N(Px, STy, t),N(Py, STx, t)}.

which is one of the general contractive definitions for a pair of mappings. If possible, suppose that P and STfail to be pointwiseR-weakly commuting and yet have a common fixed point z. Then z = Pz= STzand there exists x in X such that Px= STxbut PSTx≠STPx. clearly, z ≠x since PSTz= STPz= z.Moreover, Px≠Pz. But then we have

$$\int_0^{M(Px,Pz,kt)} \phi(t)dt > \int_0^{m(x,z,t)} \phi(t)dt,$$

where

m (x, z, t) = min{M(STx ,STz, t),M(Px, STx, t),M(Pz, STz, t), M(Px, STz, t),M(Pz, STx, t)}

= M (Px, Pz, t)

$$\int_0^{N(Px,Pz,,kt)} \phi(t)dt < \int_0^{m(x,z,t)} \phi(t)dt,$$

where

n(x, z, t) = max{N(STx,STz,t),N(Px, STz, t),N(Pz, STz, t),N(Px, STz, t),N(Pz, STz, t)}

= N (Px, Pz, t)

$$\int_0^{M(Px,Pz,kt)} \phi(t)dt > \int_0^{m(Px,Pz,t)} \phi(t)dt$$

$$\int_0^{N(Px,Pz,kt)} \phi(t)dt < \int_0^{m(Px,Pz,t)} \phi(t)dt$$

a contradiction. Hence the assertion.

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