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Some Stronger Forms of Pre-δgb-Continuous Functions

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Abstract : *In this paper, we introduce perfectly δgb-continuous, strongly pre-δgb-continuous, perfectly pre-δgb-continuous, strongly δgb-closed and regular δgb-closed functions in topological spaces using δgb-closed sets. Study some of their properties and establish characterizations. Also,b^*-normal spaces are introduced and some of its properties and characterizations are obtained.*
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I.Introduction

In 1960,Levine.N [7] introduced and studied the concept of strong continuity. Later T.Noiri [10] and S.S.Benchalli et.al [3]introduced the concepts of perfectly continuous functions and δgb-continuous functions in topological spaces respectively. In 2007, E.Ekici[6] defined the concept of γ-normal spaces and established results related to it and J.H.Park[11] introduced the concept of b-regular set in topological spaces.

Throughout this paper,(X, τ),(Y,σ) and (Z,η) (or simply X,Y and Z) represents topological spaces on which no separation axioms are assumed unless explicitly stated.

II.Preliminaries

Let us recall the following definitions which are useful in the sequel,

Definition 2.1 A subset A of a topological space X is called,
(i) b-closed [1] if cl(int(A))∩ int(cl(A))⊆A.
(ii) regular-closed [12] if A=cl(int(A)).
(iii)δ-closed[13] if $A=cl_\delta(A)$where $cl_\delta(A)=\{x\in X:int(cl(U))\cap A=\varphi, U\in\tau \text{ and } x\in U\}$.
(iv)delta generalized b-closed (briefly,δgb-closed)[2] if bcl(A)⊆G whenever A⊆G and G is δ-open in X.

The complements of the above mentioned closed sets are their respective open sets.The b-closure of a subset A of X is the intersection of all b-closed sets containing A and is denoted by bcl(A).

Definition 2.2 A function f:X→Y from a topological space X into a topo- logical space Y is called,
(i) b-continuous [5] if f^{-1}(G) is b-closed in X for every closed set G of Y.
(ii)b-irresolute [5] if f^{-1}(G) is b-closed in X for every b-closed set G of Y.
(iii)δ-continuous [9] if f^{-1}(G) is δ-closed in X for every δ-closed set G of Y.
(iv)perfectly continuous [10] if f^{-1}(G) is both open and closed in X for every open set G of Y.
(v)strongly continuous [7] if f^{-1}(G) is both open and closed in X for every subset G of Y.
(vi) pre-δgb-continuous [3] if f^{-1}(G) is δgb-closed in X for every b-closed set G of Y.
(vii) δgb-irresolute [3] if f^{-1}(G) is δgb-closed in X for every δgb-closed set G of Y.
(viii)b-closed(resp,b-open) [6] if for every b-closed (resp,b-open) subset A of X, f(A) is b-closed (resp,b-open) in Y.
(ix) δ-closed [8] if for every δ-closed subset A of X, f(A) is δ-closed in Y.
(x) δgb^*-closed [4] if f(A) is δgb-closed in Y for each δ-closed set A of X.
(xi) b-δgb-closed [4] if f(A) is δgb-closed in Y for each b-closed set A of X.
(xii)almost δgb-closed [4] if f(A) is δgb-closed in Y for each regular-closed set A of X.

Definition 2.3 [3] A topological space X is said to be
(xiii) $T_{\delta gb}$-space if every δgb-closed subset of X is closed.
(xiv) $\delta gbT_{1/2}$-space if every δgb-closed subset of X is b-closed
(xv) $\delta T_{\delta gb}$ -space if every δgb-closed subset of X is δ-closed.

III.Some Stronger Forms of Pre-δgb-Continuous Functions.

In this section, the concepts of strongly δgb-continuous and strongly pre-δgb-continuous functions in topological spaces are introduced and some of their properties and characterizations are established.

Definition 3.1 A function f:X→Y is called,
(i)perfectly δgb-continuous if f^{-1}(V) is clopen in X, for each δgb-closed set V in Y.
(ii)strongly pre-δgb-continuous if f^{-1}(V) is b-closed in X, for each δgb-closed set V in Y.

Theorem 3.2 A function f:X→Y is perfectly δgb-continuous(resp,strongly pre-δgb-continuous) if and only if f^{-1}(G) is both open and closed (resp,b-open) in X for every δgb-open set G in Y.

Theorem 3.3(i)Every perfectly δgb-continuous function is strongly pre-δgb-continuous.
(ii)Every strongly pre-δgb-continuous function is b-irresolute.
(iii)Every strongly pre-δgb-continuous function is δgb-irresolute.
Proof: Follows from definitions.

Remark 3.4 The converse of Theorem 3.3 need not be true as seen from the following examples.

Example 3.5 Let X=Y={a,b,c}.Let τ={X,φ,{a},{b},{a,b}} and σ={Y,φ,{a}} be topologies on X and Y respectively. Let f:X→Y be a function defined by f(a)=a and f(b)=f(c)=a. Then f is strongly pre-δgb-continuous but not perfectly δgb-continuous, since {a} is δgb-closed in Y but f^{-1}({a})={a} is not clopen in X.

Example 3.6 Let X=Y={a,b,c,d}.Let τ={X,φ,{a},{b},{a,b},{a,b,c}} and σ={Y,φ,{a},{b},{a,b},{a,c},{a,b,c}} be topologies on X and Y respectively.
Let f:X→Y be a function defined by f(a)=a,f(b)=b,f(c)=c and f(d)=d. Then f is b-irresolute but not strongly pre- δgb-continuous , since {a,b,d} is δgb-closed in Y but f^{-1}{a,b,d})={a,b,d} is not b-closed in X.
If h:X→Y is a function defined by h(a)=h(b)=h(c)=a and h(d)=d. Then h is δgb-irresolute but not strongly pre-δgb-continuous, since {a} is δgb-closed in Y but f^{-1} ({a})={a,b,c} is not b-closed in X.

Remark 3.7 The converse of Theorem 3.3(ii) is true if Y is $\delta gbT_{1/2}$- space.

Theorem 3.8 If f:X→Y is strongly pre-δgb-continuous ,then for each x∈X and for each δgb-open set V in Y with f(x)∈V,there exists a b-open set U in X containing x such that f(U)⊂V.
Proof: Let x∈X and V is an δgb-open set in Y with f(x)∈V, then x∈f^{-1}(V). Since f is strongly δgb-continuous, f^{-1}(V) is b-open in X. Put U=f^{-1}(V), then x∈U and f(U)=f(f^{-1}(V))⊂V.

Theorem 3.9 Let f:X→Y be a function.
(i) If Y is $T_{\delta gb}$-space then f is perfectly δgb-continuous if and only if it is perfectly continuous.
(ii) If Y is $T_{\delta gb}$-space then f is strongly pre-δgb-continuous if and only if it is b-continuous.
Proof:(i). Suppose Y is $T_{\delta gb}$-space and f is perfectly continuous. Let G be a δgb-closed set in Y, then G is closed in Y.Therefore f^{-1}(G) is clopen in X. Hence f is perfectly δgb-continuous.
Converse is obvious,since every closed set is δgb-closed.
(ii) Suppose Y is $T_{\delta gb}$-space and f is b-continuous. Let G be a δgb-closed set in Y, then G is closed in Y. Therefore f^{-1}(G) is b-closed in X. Hence f is strongly pre-δgb-continuous.

Converse is obvious,since every b-closed set is δgb-closed.

Theorem 3.10: Let f:X→Y and g:Y→Z be any two functions.
(i)If f and g are perfectly δgb-continuous,then g∗f is perfectly δgb-continuous.
(ii)If f and g are strongly pre-δgb-continuous,then g∗f is strongly pre-δgb-continuous.
(iii)If f is b-continuous and g is perfectly δgb-continuous,then g∗f is strongly pre-δgb-continuous.
(iv)If f is b-irresolute and g is strongly pre-δgb-continuous,then g∗f is strongly pre-δgb-continuous.
(v)If f is perfectly δgb-continuous and g is b-continuous,then g∗f is perfectly continuous.
(vi)If f is δgb-continuous and g is perfectly δgb-continuous,then g∗f is δgb-irresolute.
(vii)If f is strongly pre-δgb-continuous and g is b-irresolute then g∗f is b-irresolute.

Proof: (i)Let h= g∗f and V be a δgb-closed set in Z. Since g is perfectly δgb-continuous,then g^{-1}(V) is clopen in Y. Now f is perfectly δgb-continuous and every clopen set is δgb-closed,implies g^{-1}(V) is δgb-closed in Y and $f^{-1}[g^{-1}(V)]=h^{-1}(V)$ is clopen in X. Hence g∗f is perfectly δgb-continuous.

(vi) Let h= g∗f and V be a δgb-closed set in Z. Since g is perfectly δgb-continuous, g^{-1}(V) is clopen in Y which implies g^{-1}(V) is closed in Y. Now f is δgb-continuous,implies $f^{-1}[g^{-1}(V)]=h^{-1}(V)$ is δgb- closed in X. Hence g∗f is δgb-irresolute.

The proofs of (ii),(iii),(iv),(v) and (vii) are similar to (i) and (vi) with the obvious changes.

Definition 3.11[11] A subset A of a space X is said to be b-regular if A is both b-closed and b-open.

Definition 3.12 A function f:X→Y is called perfectly pre-δgb-continuous if for each δgb-closed set V in Y, f^{-1}(V) is b-regular in X .

Theorem 3.13 : Let f:X→Y be a function. Then ,
(i)If f is perfectly δgb-continuous, then it is perfectly pre-δgb-continuous.
(ii)If f is perfectly pre-δgb-continuous, then it is strongly pre-δgb-continuous.
Proof: Follows from definitions.

Remark 3.14 The converse of Theorem 3.13 need not be true as seen from the following examples.

Example 3.15 Let X={a,b,c}and τ={X,φ,{a},{b},{a,b}} be a topology on X.
Let f:X→X be a function defined by f(a)=a=f(c) and f(b)=b. Then f is perfectly pre-δgb-continuous but not perfectly δgb-continuous, since {b} is δgb-closed in Y but f^{-1}({b})={b} is not clopen in X.

Example 3.16 In Example 3.15,if h:X→X is a identity function.Then f is strongly pre-δgb-continuous but not perfectly pre-δgb-continuous, since {c} is δgb-closed in Y but f^{-1}({c})={c} is not b-regular in X.

Theorem 3.17 Let f:X→Y and g:Y→Z be any two functions.
(i) If f and g are perfectly pre-δgb-continuous,then g∘f is perfectly pre-δgb-continuous.
(ii) If f is perfectly δgb-continuous and g is δgb-irresolute,then g∘f is perfectly δgb-continuous.
(iii)If f is perfectly δgb-continuous and g is δgb-continuous,then g∘f is perfectly continuous.

Proof:(i) Let h=g∘f and V be a δgb-closed set in Z. Since g is perfectly pre-δgb-continuous, g^{-1}(V) is b-closed in Y. Now f is perfectly pre-δgb-continuous and every b-closed set is δgb-closed, implies g^{-1}(V) is δgb-closed in Y and $f^{-1}[g^{-1}(V)]=h^{-1}(V)$ is b-closed in X. Hence g∘f is perfectly pre-δgb-continuous.
(ii)The proof is similar to (i).

(iii) Let h=g∘f and V be a closed set in Z.Since g is δgb-continuous, then g^{-1}(V) is δgb-closed in Y. Now f is perfectly δgb-continuous, implies $f^{-1}[g^{-1}(V)]=h^{-1}(V)$ is clopen in X. Hence g∘f is perfectly continuous.

IV.Some Stronger And Weaker Forms Of δgb-Closed Functions.

In this section, the notions of of strongly δgb-closed,quasi δgb-closed and regular δgb-closed functions in topological spaces are introduced and discuss some of their properties.

Definition 4.1 A function f:X→Y is said to be strongly δgb-closed (resp,strongly δgb-open) if f(B) is δgb-closed (resp,strongly δgb-open) in Y for each δgb-closed (resp,strongly δgb-open) set B of X.

Theorem 4.2 Every strongly δgb-closed function is b-δgb-closed. But converse need not be true in general.

Example 4.3 Let X=Y={a,b,c}.Let τ={X,φ,{a},{b},{a,b},{a,c}} and σ={X,φ,{a},{b},{a,b}} be topologies on X and Y respectively. Let f:X→Y be a identity function. Then f is b-δgb-closed but not strongly δgb-closed, since {a,b} is δgb-closed in X but f({a,b})={a,b} is not δgb-closed in Y.

Remark 4.4 The following examples show that strongly δgb-closed and strongly b-closed functions are independent

Example 4.5 In Example 4.3, f is strongly b-closed but not strongly δgb- closed.

Example 4.6 Let X=Y={a,b,c}Let τ={X,φ,{a},{b},{a,b}}and σ={X,φ,{a},{b},{a,b},{a,c}} be topologies on X and Y respectively. Let f:X→Y be a identity function ,then f is strongly δgb-closed but not strongly b-closed, since {a} is b-closed in X but f({a})={a} is not b-closed in Y.

Theorem 4.7 If f:X →Y is b-δgb-closed and X is $\delta gbT_{1/2}$ –space,then f is strongly δgb-closed.

Lemma 4.8 A surjective function f:X→Y is strongly δgb-closed if and only if for each subset M of Y and each δgb-open set U containing f^{-1}(M), there exists a δgb-open set G of Y such that M⊂G and f^{-1}(G)⊂U.

Theorem 4.9 [2] A subset A of a topological space X is δgb-open if and only if M⊆bint(A) whenever M is δ-closed and M⊆A.

Corollary 4.10 If f:X→Y is strongly δgb-closed,then each δ-closed set K of Y and each δgb-open set U containing f^{-1}(K),there exists a b-open set V containing K such that f^{-1}(V)⊂U.

Proof: Suppose that f:X→Y is strongly δgb-closed.Let K be any δ-closed set of Y and U be any δgb-open set containing f^{-1}(K).By Lemma 4.8, there exists a δgb-open set H of Y such that K⊂H and f^{-1}(H)⊂U. Since K is δ-closed, then by Theorem 4.9, K⊂bint(H).Put bint(H)=V, then V is a b-open set such that K⊂V and f^{-1}(V)⊂U.

Definition 4.11 A function f:X→Y is said to be quasi δgb-closed (resp, quasi δgb-open) if f(A) is closed (resp, open) in Y for each δgb-closed (resp,δgb-open) set A of X.

Theorem 4.12 Every quasi δgb-closed function is strongly δgb-closed. But converse need not be true in general.

Example 4.13 Let X=Y={a,b,c}.Let τ={X,φ,{a},{b},{a,b}} and σ={X,φ,{a},{b},{a,b},{a,c}} be topologies on X and Y respectively. Let f:X→Y be a identity function, then f is strongly δgb-closed but not quasi δgb-closed, since {a} is δgb-closed in X but f({a})={a} is not closed in Y.

Theorem 4.14 If f:X→Y is strongly δgb-closed and Y is $T_{\delta gb}$-space, then f is quasi δgb-closed.

Definition 4.15[5] A space X is said to be b-normal if for any pair of disjoint closed sets A and B, there exist disjoint b-open sets U and V such that $A\subset U$ and $B\subset V$.

Corollary 4.16[4] If $f:X\rightarrow Y$ is almost δgb-closed, then each δ-closed set K of Y and each regular-open set U containing $f^{-1}(K)$,there exists a b-open set V containing K such that $f^{-1}(V)\subset U$.

Theorem 4.17 Let $f:X\rightarrow Y$ be a continuous almost δgb-closed surjection and Y is $\delta T_{\delta gb}$-space. If X is normal, then Y is b-normal.

Proof: Let N_1 and N_2 be any two disjoint closed sets of Y.Since f is continuous, then $f^{-1}(N_1)$ and $f^{-1}(N_2)$ are disjoint closed sets of X. By the normality X, there exist disjoint open sets M_1 and M_2 such that $f^{-1}(N_i)\subset M_i$, where i=1,2. Now put $int(cl(M_i))=U_i$, for i=1,2, then $U_i \in RO(X)$, $f^{-1}(N_i)\subset M_i\subset U_i$ and $U_1\cap U_2=\varphi$. Since every closed set is δgb-closed and Y is $\delta T_{\delta gb}$-space, then N_1 and N_2 are disjoint δ-closed sets of Y. By Corollary 4.16, there exists $V_i\in bO(Y)$ such that $N_i\subset V_i$ and $f^{-1}(V_i)\subset U_i$,where i=1,2. Since $U_1\cap U_2=\varphi$ and f is surjective, then $V_1\cap V_2=\varphi$.

Corollary 4.18[4] If $f:X\rightarrow Y$ is b-δgb-closed,then each δ-closed set K of Y and each b-open set U containing $f^{-1}(K)$,there exists a b-open set V containing K such that $f^{-1}(V)\subset U$.

Theorem 4.19 Let $f:X\rightarrow Y$ be a continuous b-δgb-closed surjection and Y is $\delta T_{\delta gb}$–space. If X is b-normal, then Y is b-normal.

Proof: Let H_1 and H_2 be any disjoint closed sets of Y.Since f is continuous, then $f^{-1}(H_1)$ and $f^{-1}(H_2)$ are disjoint closed sets of X.By the b-normality X ,there exist disjoint b-open sets N_1 and N_2 such that $f^{-1}(H_i)\subset N_i$, where i=1,2. Since every closed set is δgb-closed and Y is $\delta T_{\delta gb}$-space, then H_1 and H_2 are disjoint δ-closed sets of Y. By Corollary 4.18, there exists $V_i\in bO(Y)$ such that $H_i\subset V_i$ and $f^{-1}(V_i)\subset N_i$ where i=1,2. Since $N_1\cap N_2=\varphi$ and f is surjective, then $V_1\cap V_2=\varphi$.

Theorem 4.20 Let $f:X\rightarrow Y$ be a closed pre-δgb-continuous injection and X is $\delta T_{\delta gb}$–space. If Y is b-normal,then X is b-normal.

Proof: Let K_1 and K_2 be any disjoint closed sets of X. Since f is a closed injection, then $f(K_1)$ and $f(K_2)$ are disjoint closed sets of Y. By the b-normality Y, there exist disjoint b-open sets N_1 and N_2 in Y such that $f(K_i)\subset N_i$,where i=1,2. Since f is pre-δgb-coninuous, then $f^{-1}(N_1)$ and $f^{-1}(N_2)$ are disjoint δgb- open sets of X and $K_i\subset f^{-1}(N_i)$,for i=1,2.Since every closed set is δgb-closed and X is $\delta T_{\delta gb}$-space, then K_1 and K_2 are disjoint δ-closed sets of X. Therefore by Theorem 4.9, $K_i\subset bint(f^{-1}N_i)$,for i=1,2.Put $bint(f^{-1}N_i)=U_i$ then $U_i\in bO(X)$ and $K_i\subset U_i$,for i=1,2 and $U_1\cap U_2=\varphi$.

Definition 4.21 A space X is said to be b^*-normal if for any pair of disjoint δ-closed sets A and B, there exist disjoint b-open sets U and V such that $A\subset U$ and $B\subset V$.

Remark 4.22 Every b-normal space is b^*-normal but not conversely in general.

Example 4.23 Let $X=\{a,b,c,d\}$ and $\tau=\{X,\varphi,\{a\},\{c\},\{a,c\},\{b,c\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$ be a topology on X. Then X is b^*-normal but not b-normal.

Theorem 4.24 For a space X the following statements are equivalent:

(i) X is b^*-normal.

(ii)For every pair of δ-open sets M and N whose union is X, there exist b-closed sets A and B such that A⊂M ,B⊂N and A∪B=X.
(iii)For every δ-closed set H and every δ-open set K containing H,there exists a b-open set U such that H⊂U⊂b-cl(U)⊂K.

Theorem 4.25 :For a space X the following statements are equivalent:

(i) X is b^*-normal.
(ii)For every pair of disjoint δ-closed sets A and B,there exist disjoint δgb-open sets U and V such that A⊂U and B⊂V
(iii)For every δ-closed set H and every δ-open set K containing H,there exists a δgb-open set M such that H⊂M⊂δgbcl(M)⊂K.
Proof:(i)→(ii):Obvious,since every b-open set is δgb-open.
(ii)→(iii):Let H be a δ-closed set and K be an δ-open set containing H. Then H and X-K are disjoint δ-closed sets .Then by (ii),there exist disjoint δgb-open sets M and N such that H⊂M and X-K⊂N. Now M∩N=φ, implies M⊂X-N. Therefore H⊂M⊂X-N⊂K.As X-N is δgb-closed , we have δgbcl(M)⊂X-N and H⊂M⊂δgb-cl(M)⊂K.
(iii)→(i):Let K_1 and K_2 be any two disjoint δ-closed sets of X. Put $X-K_2=H$, then $K_2\cap H=\varphi$ and $K_1\subset H$. Therefore by (iii),there exists a δgb-open set M such that $K_1\subset M\subset$δgbcl(M)⊂H. It follows that $K_2\subset X$-δgbcl(M)=N,then N is δgb-open and M∩N=φ. Therefore by Theorem 4.9, $K_1\subset$bint(M)=U and $K_2\subset$bint(N)=V and U∩V=φ. So K_1 and K_2 are separated by b-open sets U and V. Hence X is b^*- normal.

Theorem 4.26 Let f:X→Y be a δ-continuous b-δgb-closed surjection. If X is b^*-normal,then Y is b^*-normal.
Proof: Let K_1 and K_2 be any disjoint δ-closed sets of Y.Since f is δ-continuous,then $f^{-1}(H_1)$ and $f^{-1}(H_2)$ are disjoint δ-closed sets of X. By the b^*-normality X, there exist disjoint b-open sets N_1 and N_2 such that $f^{-1}(K_i)\subset N_i$, where i=1,2.and f is b-δgb-closed .Therefore by Corollary 4.18, there exists $V_i\in$bO(Y) such that $K_i\subset V_i$ and $f^{-1}(V_i)\subset N_i$,where i=1,2.Since $N_1\cap N_2=\varphi$ and f is surjective, then $V_1\cap V_2=\varphi$.

Theorem 4.27 Let f:X→Y be a δ-closed pre-δgb-continuous injection. If Y is b^*-normal, then X is b^*-normal.
Proof: Let M_1 and M_2 be any disjoint δ-closed sets of X. Since f is a δ- closed injection, then $f(K_1)$ and $f(K_2)$ are disjoint δ-closed sets of Y. By the b^*-normality Y, there exist disjoint b-open sets K_1 and K_2 in Y such that $f(M_i)\subset K_i$,where i=1,2. Since f is pre-δgb-coninuous, then $f^{-1}(K_1)$ and $f^{-1}(K_2)$ are disjoint δgb-open sets of X and $M_i\subset f^{-1}(K_i)$,for i=1,2.Therefore by Theorem 4.9, $M_i\subset$bint($f^{-1}K_i$),for i=1,2. Put bint($f^{-1}K_i$)=U_i, then $U_i\in$bO(X) and $K_i\subset U_i$, for i=1,2 and $U_1\cap U_2=\varphi$.

Definition 4.28 A function f:X→Y is said to be regular δgb-closed if f(A) is δgb-closed in Y for each b-regular set A of X.

Theorem 4.29 (i)If a function f:X→Y is b-δgb-closed, then it is regular δgb-closed .
(ii)If a function f:X→Y is regular δgb-closed , then it is almost δgb-closed.

Remark 4.30 The converse of Theorem 4.29 need not be true as seen from the following examples.

Example 4.31 Let X=Y={a,b,c}. Let τ={X,φ,{a},{b},{a,b},{a,c}} and σ={X,φ,{a},{b},{a,b}}. Let f:X→Y be a function defined by f(a)=c,f(b)=a and f(c)=b.Then f is regular δgb-closed but not b-δgb-closed, since the set {b,c} is b-closed in X but f({b,c})={a,b} is not δgb-closed in Y.

Example 4.32 Let X={a,b,c,d} and τ ={X,φ,{a},{b},{a,b},{a,c},{a,b,c}}. Let f:X→X be a identity function ,then f is almost δgb-closed but not regular δgb-closed, since {a,c} is b-regular in X but f({a,c})={a,b} is not δgb-closed in Y.

Remark 4.33 The following examples show that regular δgb-closed function is independent of δgb-closed function and δgb^*-closed function.

Example 4.34 Let X={a,b,c,d} and τ ={X,φ,{a},{b},{a,b}} be a topology on X.
Let f:X→X be a function defined by f(a)=c,f(b)=d,f(c)=a and f(d)=b.Then f is regular δgb-closed but ot δgb^*-closed and hence not δgb-closed, since the set {c,d} is δ-closed in X but f({c,d})={a,b} is not δgb-closed in Y.

Example 4.35 In Example 4.34, If h:X→Y is a function defined by h(a)=a, h(b)=c,h(c)=b and h(d)=d.Then h is δgb-closed and hence δgb^*-closed but not regular δgb-closed.,since the set {a,c} is b-regular in X but h({a,c})={a,b} is not δgb-closed in Y.

Lemma 4.36 A function f:X→Y is regular δgb-closed if and only if for each subset B of Y and each b-regular set U containing f^{-1}(B),there exists a δgb-open set G of Y such that B⊂G and $f^{-1}(G)\subset U$.

Corollary 4.37 If f:X→Y is regular δgb-closed,then each δ-closed set K of Y and each b-regular set U containing f^{-1}(K),there exists a b-open set V containing K such that $f^{-1}(V)\subset U$.

Theorem 4.38[9] Let A be a subset of a space X. Then A is b-open if and only if bcl(A) is b-regular.

Theorem 4.39 Let f:X→Y be a δ-continuous regular δgb-closed surjec- tion. If X is b^*-normal,then Y is b^*-normal.

Proof: Let K_1 and K_2 be any two disjoint δ-closed sets of Y. Since f is δ- continuous, then $f^{-1}(H_1)$ and $f^{-1}(H_2)$ are disjoint δ-closed sets of X.By the b^*-normality X,there exist disjoint b-open sets N_1 and N_2 such that $f^{-1}(K_i)\subset N_i$, for i=1,2. Now put $bcl(N_i)=G_i$,then by Theorem 4.38 ,G_i is b-regular in X,$f^{-1}(K_i)\subset G_i$ and $G_1\cap G_2=\varphi$ and f is regular δgb-closed. Therefore by Corollary 4.37, there exists $V_i\in bO(Y)$ such that $K_i\subset V_i$ and $f^{-1}(V_i)\subset G_i$, where i=1,2.Since $G_1\cap G_2=\varphi$ and f is surjective, we have $V_1\cap V_2=\varphi$.

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