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Heat and Mass Transfer on MHD Free Convection Flow of Visco-Elastic Kuvshinshiki Fluid through Porous Medium past an Infinite Vertical Porous Plate

G.Raju[1*] and R.Siva Prasad[2]
[1, 2] *Department of Mathematics, S.K.University, Anantapur, Andhra Pradesh, India*
Email: rajumaths2010@gmail.com; rsprasad_racharla@yahoo.co.in
(* - *Corresponding author*)

Abstract: *The unsteady flow of an incompressible Mhd free convection flow of Visco-elastic Kuvshinshiki fluid through a porous medium with simultaneous heat and mass transfer near an infinite vertical oscillating porous plate under the influence of uniform transverse magnetic field has been discussed. The governing equations of the flow field are solved by a regular perturbation method for small elastic parameter. The expressions for the velocity, temperature, concentration have been derived analytically and also its behaviour is computationally discussed with reference to different flow parameters with the help of graphs. The skin friction on the boundary, the heat flux in terms of the Nusselt number, and the rate of mass transfer in terms of the Sherwood number are also obtained and their behaviour discussed.*
Keywords: *Heat and mass transfer, Mhd flows, porous medium, unsteady flows and visco-elastic fluids.*

I. INTRODUCTION

The study of hydro magnetic free convection flow finds applications in science and engineering, in areas such as geophysical exploration, solar physics, and astrophysical studies. The convection problem in a porous medium has important applications in geothermal reservoirs and geothermal extractions. The process of heat and mass transfer is encountered in aeronautics, fluid fuel nuclear reactor, chemical process industries and many engineering applications in which the fluid is the working medium. The wide range of technological and industrial applications has stimulated considerable amount of interest in the study of heat and mass transfer in convection flows. Azzam [1] reported a study on the radiation effects on the Magnetohydrodynamic (MHD) mixed free-forced convection flow past a semi-infinite moving vertical plate for high temperature differences. Unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption was reported by Chamkha [4]. A few other works of interest in this area include Kim [8], Makinde [10], Ogulu and Prakash [12]. The object of this study is to consider unsteady hydromagnetic free convection flow for a dissipative and radiating fluid applying a simple perturbation technique. Most of the studies mentioned above have applied one numerical technique or the other, whereas here we have used a much simpler time-saving technique. Free convective flow past a vertical plate has been studied extensively by Ostrach [13]. Siegel [16] investigated the transient free convection from a vertical flat plate. Cheng and Lau [5] and Cheng and Teckchandani [6] obtained numerical solutions for the convective flow in a porous medium bounded by two isothermal parallel plates in the presence of the withdrawal of the fluid. In all the above mentioned studies, the effect of porosity, permeability and the thermal resistance of the medium is ignored or treated as constant. However, porosity measurements by Benenati and Broselow [3] show that porosity is not constant but varies from the surface of the plate to its interior to which as a result permeability also varies. In case of unsteady free convective flow, Soundalgekar [18] studied the effects of viscous dissipation on the flow past an infinite vertical porous plate. The combined effect of buoyancy forces from thermal and mass diffusion on forced convection was studied by Chen et al. [7]. The free convection on a horizontal plate in a saturated porous medium with prescribed heat transfer coefficient was studied by Ramanaiah and Malarvizhi [14]. Bejan and Khair [2] have investigated the vertical free convective boundary layer flow embedded in a porous medium resulting from the combined heat and mass transfer. Lin and Wu [9] analyzed the problem of simultaneous heat and mass transfer with the entire range of buoyancy ratio for most practical and chemical species in dilute and aqueous solutions.

Rushi Kumar and Nagarajan [15] studied the mass transfer effects of MHD free convection flow of incompressible viscous dissipative fluid past an infinite vertical plate. Mass transfer effects on free convection flow of an incompressible viscous dissipative fluid have been studied by Manohar and Nagaragan [11]. MHD Free Convection Flow of Kuvshinshiki Fluid with Heat and Mass Transfer past a Vertical Porous Plate are studied by Gupta et al. [19]. In this study we consider the work of Sivaiah et al. [17] with Kuvshinshiki fluid. Motivated from the above studies, we have discussed the heat and mass transfer on the unsteady MHD free convection flow of visco-elastic Kuvshinshiki fluid past a vertical oscillating porous plate under the influence of uniform transverse magnetic field.

II. FORMULATION AND SOLUTION OF THE PROBLEM

We consider the unsteady flow of an incompressible MHD free convection flow of Visco-elastic Kuvshinshiki fluid through a porous medium with simultaneous heat and mass transfer near infinite vertical oscillating porous plate under the influence of uniform transverse magnetic field. The plate temperature is constant to be maintained. The Visco-elastic and Darcy's resistance terms are taken into account with constant permeability of the porous medium.

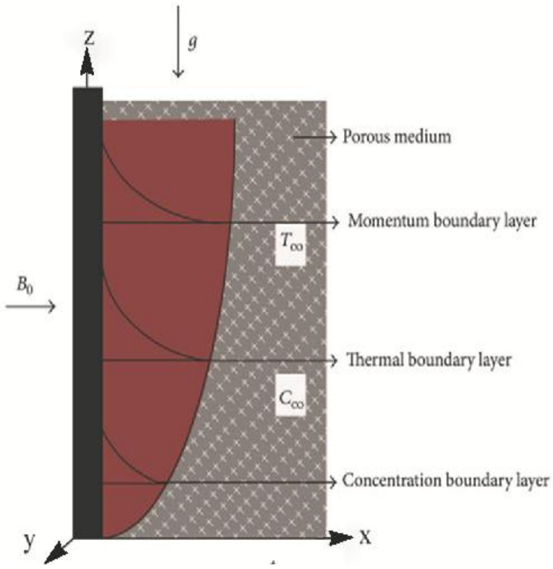

Figure 1: Physical configuration of the Problem

We consider the Cartesian co-ordinate system such that $z = 0$ on the plate. The suction velocity normal to the plate is a constant and may be written as, $w = -W_0$. All the fluid properties considered constant except that the influence of the density variation with temperature. The influence of the density variation in other terms of the momentum and the energy equation and the variation of the expansion coefficient with temperature is negligible. This is the well-known Boussinesq approximation. Under these conditions, the unsteady hydromagnetic flow through porous medium under the influence of uniform transverse magnetic field is governed by the following system of Equations

Equation of continuity:

$$\frac{\partial w}{\partial z} = 0 \tag{2.1}$$

Equation of Momentum:

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} + w\frac{\partial u}{\partial z} = \nu\frac{\partial^2 u}{\partial z^2} - \left(1+\lambda\frac{\partial}{\partial t}\right)\left(\frac{\sigma B_0^2}{\rho} + \frac{\nu}{k}\right)u + g\beta(T-T_\infty) + g\beta^*(C-C_\infty) \tag{2.2}$$

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial v}{\partial t} + w\frac{\partial v}{\partial z} = \nu\frac{\partial^2 v}{\partial z^2} - \left(1+\lambda\frac{\partial}{\partial t}\right)\left(\frac{\sigma B_0^2}{\rho} + \frac{\nu}{k}\right)v \tag{2.3}$$

Equation of Energy:

$$\frac{\partial T}{\partial t} + w\frac{\partial T}{\partial z} = \alpha\frac{\partial^2 T}{\partial z^2} - \frac{\partial q_r}{\partial z} \tag{2.4}$$

Equation of Concentration:

$$\frac{\partial C}{\partial t} + w\frac{\partial C}{\partial z} = D\frac{\partial^2 C}{\partial z^2} \tag{2.5}$$

Where, *u*, *v* are the velocity components along *x* and *y* directions; *T* and *C* are the temperature and concentration components, ν is the kinematic viscosity. ρ is the density, σ is the electric conductivity, B_0 is the magnetic induction, *k* is the permeability of the porous medium, g is the acceleration due to gravity, β is the thermal expansion co-efficient, β^* is the concentration expansion co-efficient, α is the thermal conductivity and *D* is the concentration diffusivity q_r is the radiation heat flux. Using Rosseland approximation for radiation,

$$\frac{\partial q_r}{\partial z} = 4\alpha^2 (T - T_\infty) \tag{2.6}$$

The corresponding boundary conditions are

$$u = v = 0, T = T_w, C = C_w \qquad \text{at} \quad z = 0 \tag{2.7}$$

$$u = v = 0, T = T_\infty, C = C_\infty \qquad \text{at} \quad z \to \infty \tag{2.8}$$

Combining the equations (2.2) and (2.3), we obtain

$$\left(1 + \lambda \frac{\partial}{\partial t}\right)\frac{\partial q}{\partial t} + w\frac{\partial q}{\partial z} = \nu \frac{\partial^2 q}{\partial z^2} - \left(1 + \lambda \frac{\partial}{\partial t}\right)\left(\frac{\sigma B_0^2}{\rho} + \frac{\nu}{k}\right) q + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) \tag{2.9}$$

We introduce the non-dimensional variables,

$$u^* = \frac{u}{W_0}, v^* = \frac{v}{W_0}, t^* = \frac{tW_0^2}{\nu}, z^* = \frac{zW_0}{\nu}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, C^* = \frac{C - C_\infty}{C_w - C_\infty}$$

Making use of non-dimensional variables, the governing equations reduces to (Dropping asterisks)

$$\left(1 + \lambda \frac{\partial}{\partial t}\right)\frac{\partial q}{\partial t} - \frac{\partial q}{\partial z} = \frac{\partial^2 q}{\partial z^2} - \left(1 + \lambda \frac{\partial}{\partial t}\right)\left(M^2 + \frac{1}{K}\right) q + Gr(\theta + \phi C) \tag{2.10}$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial z} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial z^2} - R\theta \tag{2.11}$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial z} = \frac{1}{Sc}\frac{\partial^2 C}{\partial z^2} \tag{2.12}$$

The corresponding non-dimensional boundary conditions are

$$q = 0, \theta = 1, C = 1 \qquad \text{at} \quad z = 0 \tag{2.13}$$

$$q = 0, \theta = 0, C = 0 \qquad \text{at} \quad z \to \infty \tag{2.14}$$

Where

$M^2 = \frac{\sigma B_0^2 \nu}{\rho W_0^2}$ is the Hartmann number (Magnetic field parameter), $K = \frac{kW_0^2}{\nu^2}$ is the permeability parameter (Darcy parameter), $R = \frac{4\alpha^2 W_0^2}{\nu}$ is the Radiation parameter, $\Pr = \frac{\nu}{\alpha}$ is the Prandtl number, $\mathrm{Sc} = \frac{\nu}{D}$ is the Schmidt number, $\lambda = \frac{\lambda W_0^2}{\nu}$ is the Visco-elastic parameter and $\phi = \frac{\beta^*(C_w - C_\infty)}{\beta(T_w - T_\infty)}$ is the Buoyancy ratio.

We assume the solutions of the equations (2.10) to (2.12) as,

$$q(z,t) = q_0(t)e^{i\omega t}, \ \theta(z,t) = \theta_0(t)e^{i\omega t}, \ C(z,t) = C_0(t)e^{i\omega t} \tag{2.15}$$

Using the equations (2.15), the equations (2.10) to (2.12) reduces to

$$\frac{d^2 q_0}{dz^2} + \frac{dq_0}{dz} - \left(\left(M^2 + \frac{1}{K} - i\omega\right)(1 - i\omega\lambda)\right) q_0 = -Gr(\theta_0 + \phi C_0) \tag{2.16}$$

$$\frac{d^2 \theta_0}{dz^2} + \Pr\frac{d\theta_0}{dz} + \Pr(i\omega - R)\theta_0 = 0 \tag{2.17}$$

$$\frac{d^2 C_0}{dz^2} + \mathrm{Sc}\frac{dC_0}{dz} + \mathrm{Sc}\, i\omega C_0 = 0 \tag{2.18}$$

The corresponding boundary conditions are

$$q_0 = 0, \theta_0 = 1, C_0 = 1 \qquad \text{at} \qquad z = 0 \tag{2.19}$$

$$q_0 = 0, \theta_0 = 0, C_0 = 0 \qquad \text{at} \qquad z \to \infty \tag{2.20}$$

Solving the equations (2.16) to (2.18) making use of the boundary conditions (2.19) and (2.20),

$$q_0 = (a_1 + a_2)\,\mathrm{e}^{-m_1 z} - a_1 \mathrm{e}^{-m_2 z} - a_2 \mathrm{e}^{-m_3 z} \tag{2.21}$$

$$\theta_0 = \mathrm{e}^{-m_2 z} \tag{2.22}$$

$$C_0 = \mathrm{e}^{-m_3 z} \tag{2.23}$$

Substituting the equations (2.21) to (2.20) in the equations (2.15), we obtained the velocity, temperature and concentration distributions.

Skin friction:

$$\tau = \left(\frac{\partial q}{\partial z}\right)_{z=0} = -\left(m_1(a_1 + a_2) + a_1 m_2 + a_2 m_3\right) e^{i\omega t} \tag{2.24}$$

Nusselt number:

$$Nu = -\left(\frac{\partial \theta}{\partial z}\right)_{z=0} = m_2\, e^{i\omega t} \tag{2.25}$$

Sherwood number:

$$Sh = -\left(\frac{\partial C}{\partial z}\right)_{z=0} = m_3\, e^{i\omega t} \tag{2.26}$$

IV. RESULTS AND DISCUSSION

We have considered the unsteady flow of an incompressible MHD free convection flow of Visco-elastic Kuvshinshiki fluid through a porous medium with simultaneous heat and mass transfer near infinite vertical oscillating porous plate under the influence of uniform transverse magnetic field. The plate temperature is constant to be maintained. The visco-elastic and Darcy's resistance terms are taken into account with constant permeability of the porous medium. The governing equations of the flow field are solved by a regular perturbation method for small elastic parameter. The expressions for the velocity, temperature, concentration have been derived analytically and also its behaviour is computationally discussed with reference to different flow parameters as shown in the graphs (Fig. 2-13) using Mathematica.

From the Figures (2, 8-9), we noticed that the magnitude of the velocity components *u* and *v* reduces with increasing the intensity of the magnetic field or Prandtl number Pr or Schimdt number Sc. The similar behaviour is observed for the resultant velocity with increasing M, Pr and Sc. The velocity components *u*, *v* and the resultant velocity enhance with increasing Grashof number Gr or Buoyancy ratio ϕ (Figures 5 & 6). The Figures (3-4 & 10) depicts the velocity components *u* enhances and *v* reduces with increasing the permeability parameter K throughout the fluid region. Lower the permeability of the porous medium lesser the fluid speed in the entire region. The reversal behaviour is observed with increasing visco-elastic parameter λ or the frequency of oscillation ω ω (Figures 4 & 10). The magnitude of the velocity component *u* enhances and the experiences retardation in the flow field with increasing radiation parameter R and reverse trend is observed with increasing time, whereas velocity component *v* increases with increasing R and *t* (Figure 7 and 11).

Figures 12 showed the effect of Radiation parameter R, the Prandtl number Pr, and the frequency of oscillation ω and time *t* on the temperature of the flow field. We noted that the temperature of the flow field diminishes as the Prandtl number increases. This is consistent with the fact that the thermal boundary layer thickness decreases with increasing Prandtl number. With increasing radiation parameter reduces the temperature of the flow field. This may happen due the elastic property of the fluid. It is observed that temperature of the flow field diminishes as the time parameter or the frequency of oscillation increases.

Figures 13 depict the effect of the Schmidt number Sc and the frequency of oscillation ω on concentration distribution. The concentration distribution decreases at all points of the flow field with the increase in the Schmidt number Sc. This shows that the heavier diffusing species have a greater retarding effect on the concentration distribution of the flow field. Also, it is observed that presence of the frequency of oscillation ω reduces the concentration distribution.

$$K = 1, \lambda = 1, \omega = \frac{\pi}{4}, R = 1, \ Pr = 0.71, Sc = 0.78, Gr = 10, \ \phi = 0.2, t = 0.1$$

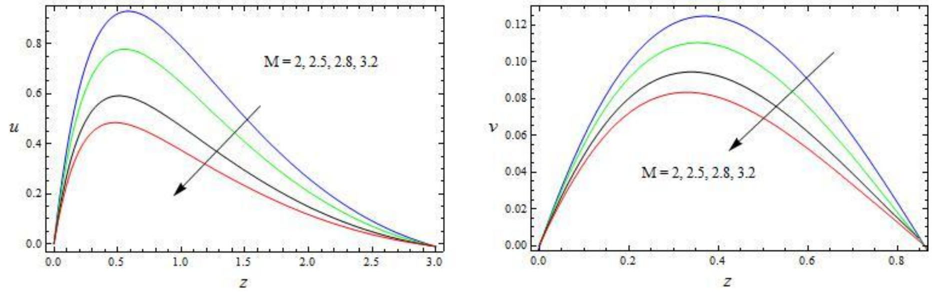

Fig. 2: The velocity Profiles for *u* and *v* against M

$$M = 2, \lambda = 1, \omega = \frac{\pi}{4}, R = 1, \ Gr = 10, \ Pr = 0.71, Sc = 0.78, \ \phi = 0.2, t = 0.1$$

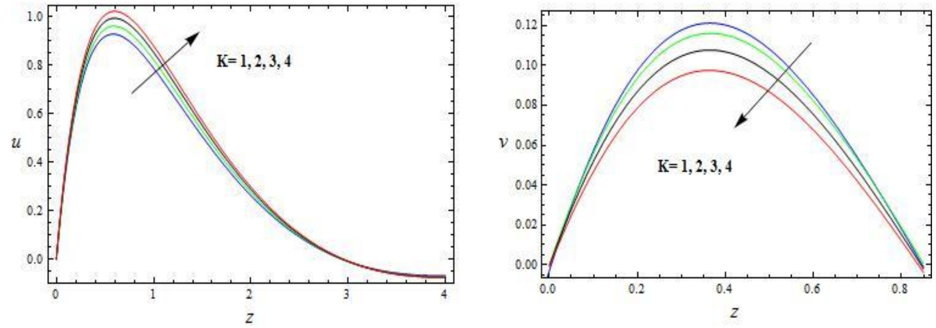

Fig. 3: The velocity Profiles for *u* and *v* against K

$$K = 1, \ M = 2, \ \omega = \frac{\pi}{4}, R = 1, \ Pr = 0.71, Sc = 0.78, Gr = 10, \ \phi = 0.2, t = 0.1$$

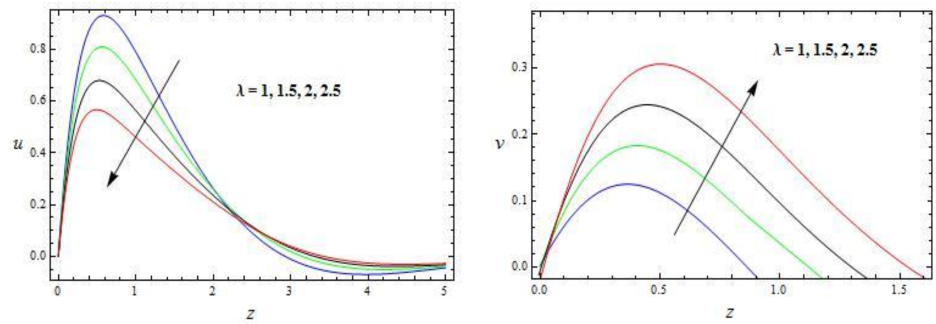

Fig. 4: The velocity Profiles for *u* and *v* against λ

$$K = 1, \lambda = 1, \omega = \frac{\pi}{4}, R = 1, \ Pr = 0.71, Sc = 0.78, M = 2, \ \phi = 0.2, t = 0.1$$

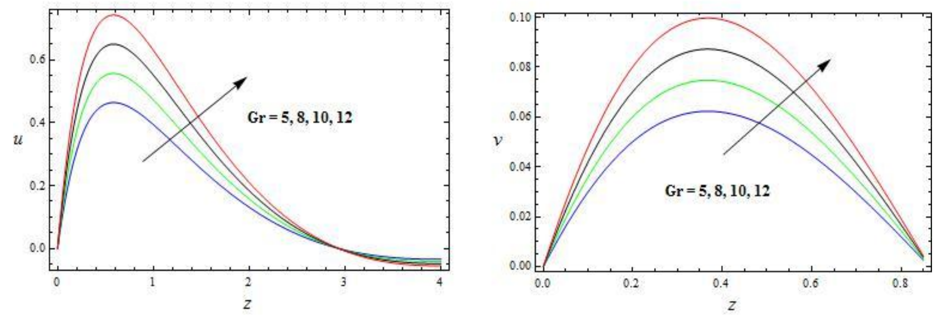

Fig. 5: The velocity Profiles for *u* and *v* against Gr

$K = 1, \lambda = 1, \omega = \frac{\pi}{4}, R = 1, Pr = 0.71, Sc = 0.78, M = 2, Gr = 10, t = 0.1$

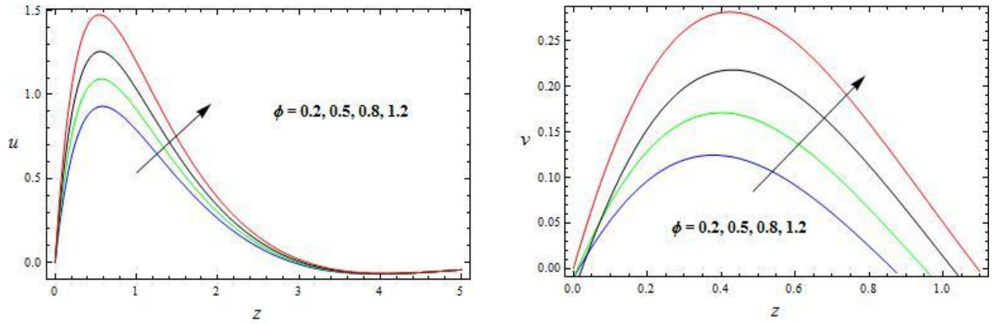

Fig. 6: The velocity Profiles for *u* and *v* against ϕ

$K = 1, \lambda = 1, \omega = \frac{\pi}{4}, M = 2, Pr = 0.71, Sc = 0.78, Gr = 10, \phi = 0.2, t = 0.1$

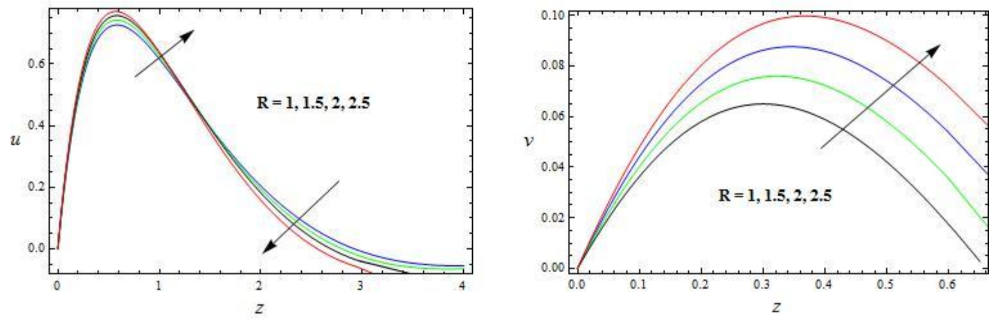

Fig. 7: The velocity Profiles for *u* and *v* against R

$K = 1, \lambda = 1, \omega = \frac{\pi}{4}, R = 1, M = 2, Sc = 1, Gr = 10, \phi = 0.2, t = 0.1$

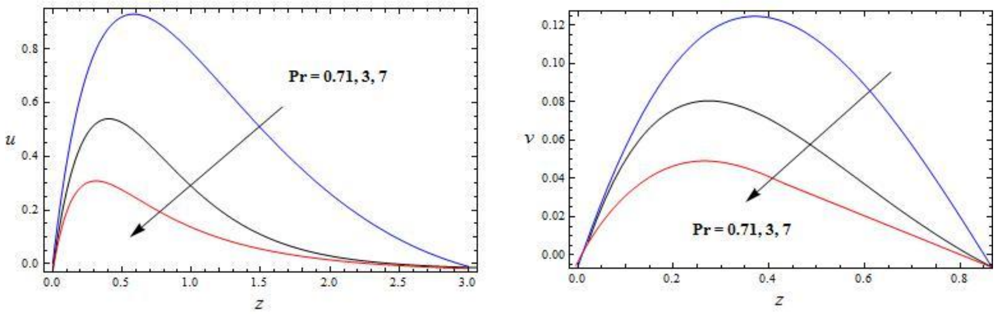

Fig. 8: The velocity Profiles for *u* and *v* against Pr

$K = 1, \lambda = 1, \omega = \frac{\pi}{4}, R = 1, M = 2, Pr = 0.71, Gr = 10, \phi = 0.2, t = 0.1$

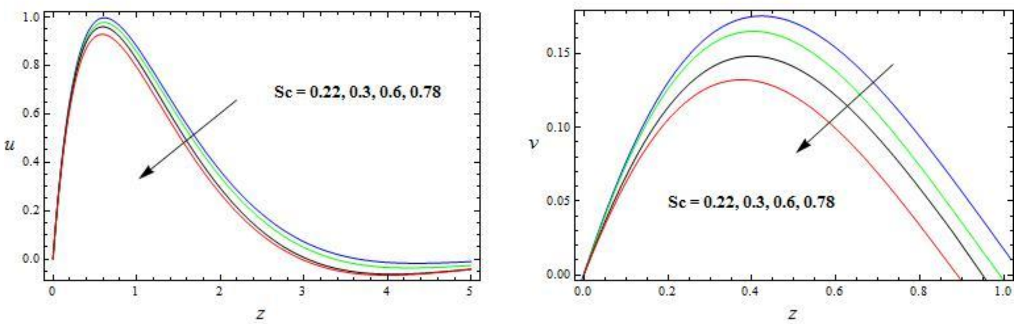

Fig. 9: The velocity Profiles for *u* and *v* against Sc

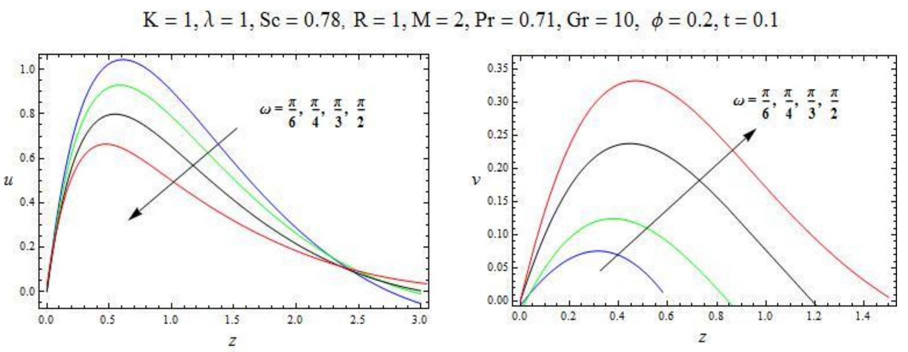

Fig. 10: The velocity Profiles for *u* and *v* against ω

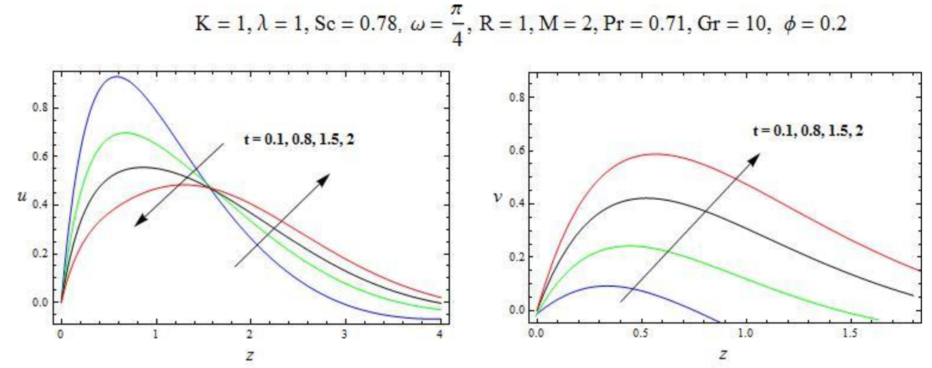

Fig. 11: The velocity Profiles for *u* and *v* against time *t*

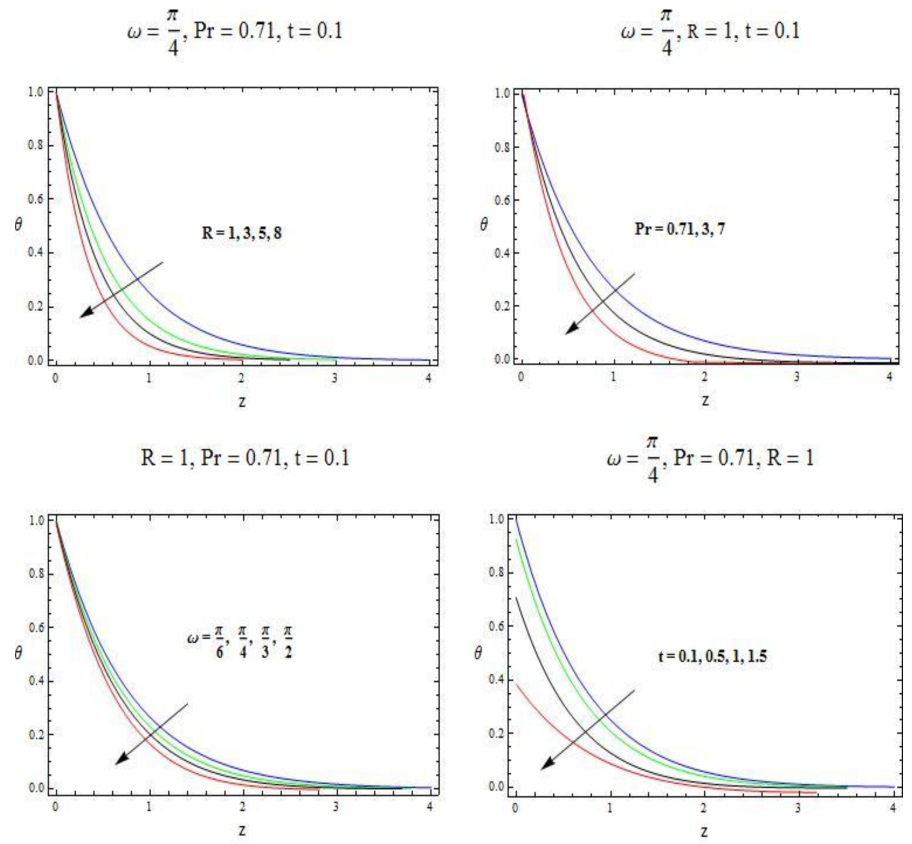

Fig. 12. The Temperature Profiles for θ with R, Pr, ω and *t*

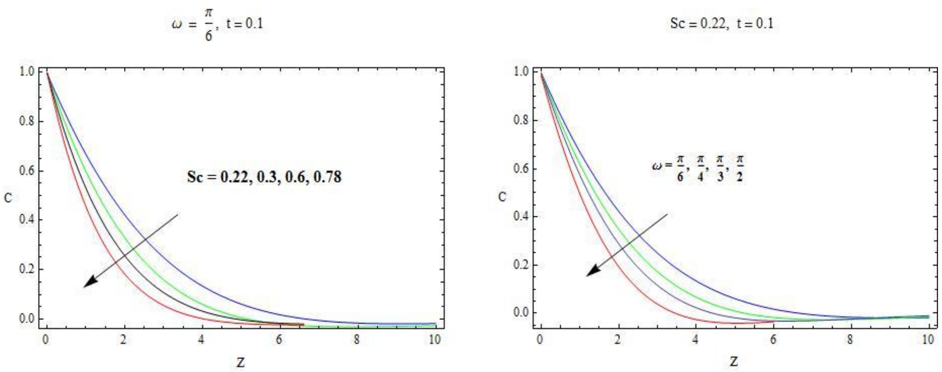

Fig.13. The Concentration Profiles for C with Sc and ω

Table 1: Skin Friction

M	K	λ	R	Pr	Gr	ϕ	Sc	ω	t	τ
2	**1**	**1**	**1**	**0.71**	**10**	**0.2**	**0.22**	$\pi/4$	**0.1**	4.636126
2.5	1	1	1	0.71	10	0.2	0.22	$\pi/4$	0.1	3.856703
3	1	1	1	0.71	10	0.2	0.22	$\pi/4$	0.1	3.280120
2	**2**	1	1	0.71	10	0.2	0.22	$\pi/4$	0.1	4.880755
2	**3**	1	1	0.71	10	0.2	0.22	$\pi/4$	0.1	4.971002
2	1	**1.5**	1	0.71	10	0.2	0.22	$\pi/4$	0.1	4.221541
2	1	**2**	1	0.71	10	0.2	0.22	$\pi/4$	0.1	3.790972
2	1	1	**2**	0.71	10	0.2	0.22	$\pi/4$	0.1	4.822254
2	1	1	**3**	0.71	10	0.2	0.22	$\pi/4$	0.1	4.821654
2	1	1	1	**3**	10	0.2	0.22	$\pi/4$	0.1	3.097385
2	1	1	1	**7**	10	0.2	0.22	$\pi/4$	0.1	2.038196
2	1	1	1	0.71	**15**	0.2	0.22	$\pi/4$	0.1	6.954190
2	1	1	1	0.71	**20**	0.2	0.22	$\pi/4$	0.1	9.272253
2	1	1	1	0.71	10	**0.5**	0.22	$\pi/4$	0.1	5.800466
2	1	1	1	0.71	10	**0.7**	0.22	$\pi/4$	0.1	6.576692
2	1	1	1	0.71	10	0.2	**0.6**	$\pi/4$	0.1	4.556214
2	1	1	1	0.71	10	0.2	**0.78**	$\pi/4$	0.1	4.520928
2	1	1	1	0.71	10	0.2	0.22	$\pi/3$	0.1	4.225156
2	1	1	1	0.71	10	0.2	0.22	$\pi/2$	0.1	3.434511
2	1	1	1	0.71	10	0.2	0.22	$\pi/4$	0.5	4.144350
2	1	1	1	0.71	10	0.2	0.22	$\pi/4$	0.8	3.505097

Table 2: Nusselt number

R	Pr	ω	t	Nu
1	**0.71**	$\pi/4$	**0.1**	1.333165
2	0.71	$\pi/4$	0.1	1.630177
3	0.71	$\pi/4$	0.1	1.876962
1	**3**	$\pi/4$	0.1	3.873236
1	**7**	$\pi/4$	0.1	7.955346
1	0.71	$\pi/3$	0.1	1.375590
1	0.71	$\pi/2$	0.1	1.476144
1	0.71	$\pi/4$	**0.8**	1.234201
1	0.71	$\pi/4$	**1.2**	1.007703

The skin friction is significant phenomenon which characterizes the frictional drag force at the solid surface. From Table 1, it is observed that the skin friction increases with the increase in all the forcing forces, but it is interesting to note that the skin friction decreases with the increase in Hartmann number M. From Table 2, it is to note that all the entries are positive. It is seen that Radiation parameter R, the Prandtl number Pr and the frequency of oscillations ω increase the rate of heat transfer (Nusselt number Nu) at the surface of the plate, the Nusselt number Nu reduces with increasing time *t*. From Table 3 it is to note that all the entries are positive. It is observed that Schmidt number Sc, the frequency of oscillations ω and time *t* increase the rate of mass transfer at the surface of the plate.

Table 3: Sherwood number

Sc	ω	t	Sh
0.22	$\pi / 4$	**0.1**	0.435384
0.3	$\pi / 4$	0.1	0.534097
0.6	$\pi / 4$	0.1	0.865805
0.78	$\pi / 4$	0.1	1.051093
0.22	$\pi / 3$	0.1	0.490472
0.22	$\pi / 2$	0.1	0.590352
0.22	$\pi / 4$	**0.5**	0.491463
0.22	$\pi / 4$	**0.8**	0.502078
0.22	$\pi / 4$	**1.2**	0.473191

IV. CONCLUSIONS

The unsteady flow of an incompressible MHD free convection flow of Visco-elastic Kuvshinshiki fluid through a porous medium with simultaneous heat and mass transfer near an infinite vertical oscillating porous plate under the influence of uniform transverse magnetic field has been discussed. The conclusions are made as the following.

1. The magnitude of the resultant velocity reduces with increasing the intensity of the magnetic field or Prandtl number Pr or Schimdt number Sc.
2. the resultant velocity enhance with increasing Grashof number Gr or Buoyancy ratio ϕ .
3. The resultant velocity enhances with increasing the permeability parameter K throughout the fluid region. Lower the permeability of the porous medium lesser the fluid speed in the entire region.
4. The reversal behaviour is observed with increasing visco-elastic parameter λ or the frequency of oscillation ω .
5. The magnitude of the resultant velocity enhances and the experiences retardation in the flow field with increasing radiation parameter R and reverse trend is observed with increasing time, whereas velocity component *v* increases with increasing R and *t*.
6. The magnitude of the temperature of the flow field diminishes as the Prandtl number or time or the frequency of oscillation.
7. The concentration reduces at all points of the flow field with the increase in the Schmidt number Sc, and presence of the frequency of oscillation ω reduces the concentration distribution.
8. The skin friction increases with the increase in all the forcing forces and decreases with the increase in Hartmann number M.
9. The rate of heat transfer (Nusselt number Nu) at the surface of the plate increase Radiation parameter R, the Prandtl number Pr and the frequency of oscillations ω reduces with increasing time *t*.
10. Schmidt number Sc, the frequency of oscillations ω and time *t* increase the rate of mass transfer at the surface of the plate.

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Appendix:

$$m_1 = \frac{1+\sqrt{1+4\left(M^2+\frac{1}{K}-i\omega\right)(1-i\omega\lambda)}}{2},$$

$$m_2 = \frac{\Pr+\sqrt{\Pr^2-4\Pr(i\omega-R)}}{2}, \quad m_3 = \frac{Sc+\sqrt{Sc^2-4Sc\,i\omega}}{2},$$

$$a_1 = \frac{\mathrm{Gr}}{m_2^2-m_2-\left(M^2+\frac{1}{K}-i\omega\right)(1-i\omega\lambda)}, \quad a_2 = \frac{\mathrm{Gr}\,\phi}{m_3^2-m_3-\left(M^2+\frac{1}{K}-i\omega\right)(1-i\omega\lambda)}.$$