IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 12, Issue 2 Ver. V (Mar. - Apr. 2016), PP 08-24
www.iosrjournals.org

Hall effects on Unsteady MHD Free Convection flow of an incompressible electrically conducting Second grade fluid through a porous medium over an infinite rotating vertical plate fluctuating with Heat Source/Sink and Chemical reaction

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Abstract: *In this paper, we have considered the unsteady MHD free convection flow of an incompressible electrically conducting second grade fluid through porous medium bounded by an infinite vertical porous surface in the presence of heat source and chemical reaction in a rotating system taking hall current into account. The momentum equation for the fluid flow through porous medium is governed by Brinkman's model. In the undisturbed state, both the plate and fluid are in solid body rotation with the same angular velocity about normal to the infinite vertical plane surface. The vertical surface is subjected to the uniform constant suction perpendicular to it and the temperature on the surface varies with time about a non-zero constant mean while the temperature of free stream is taken to be constant. The exact solutions for the velocity, temperature and concentration are obtained analytically making use of perturbation technique. The velocity expression consists steady state and oscillatory state. It reveals that, the steady part of the velocity field has three layer characters while the oscillatory part of the fluid field exhibits a multi layer character. The influence of various governing flow parameters on the velocity, temperature and concentration is analysed graphically. We also discussed computational results for the skin friction, Nusselt number and Sherwood number in the tabular forms.*

Keywords: *Convection flows, Hall effects, heat and mass transfer, MHD flows, infinite vertical plates, porous medium, rotating channels, second grade fluids.*

I. Introduction

Generally fluid solid mixtures are considered to behave like non-Newtonian fluids. This type of fluids occurs in pneumatic and hydraulic transport of solids and thus has many industrial applications. A specific research area in this direction is the use of coal based slurries which requires the analysis of various transport processes in non-Newtonian fluids. In the study of non-Newtonian fluids, it has been mainly motivated to their importance in the problems from applications of engineering and chemical industry. The partial differential equations usually appear in many areas of the natural and physical sciences. They describe different physical systems, ranging from gravitational to fluid dynamics and have been used to solve problems in the chemistry, mathematical biology, solid state physics etc. Due to complexity of non-Newtonian fluids, there is no one model which describes all of their properties. Most of the models for such type of fluids have been proposed. In those of the models, there is a second grade fluid model which is the most popular. This is particularly so due to the fact that one can reasonably hope to obtained the analytic solution of the mathematical model. We also mentioned for the most interesting studies of second grade fluids [2, 6, 11, 12, 15, 24 and 25]. Some of these methods include the tanh method [36], the quotient trigonometric function expansion method [21], F-expansion method [8] and so on. The special class of non-Newtonian fluids for which the exact solution is reasonably possible is the visco-elastic fluids, that were first introduced by Rivlin and Ericksen [29]. Rajagopal [22-23] established the exact solutions for creeping flow and for unidirectional flow. Hayat et al. [14, 16] and Siddiqui et al. [30] extended that idea for the periodic flows. Rajagopal and Gupta [26] also discussed the exact flow between the rotating parallel plates. Veera Krishna.M and S.G. Malashetty [34] discussed unsteady flow of an incompressible electrically conducting second grade fluid through a composite medium in a rotating parallel plate channel and the problem extended for taking the hall currents by Veera Krishna.M and S.G. Malashetty [35].

The rate of heat transfer can be controlled by using the intensity of the magnetic field. The inclusion of magnetic field in the study of second grade fluid flow has many practical applications for example, the cooling of turbine blades. Magnethydrodynamics (MHD) provides a mean of cooling the turbine blade and keeping the structural integrity of the nose cone. Hence, the boundary layer MHD flows of non-Newtonian fluids have drawn the attention of many researchers since the past few decades. Hayat et al. [17] discussed the unsteady flow of an incompressible second grade fluid in a circular duct with a given volume flow rate variation taking the effects of Hall current. Hydro magnetic transport through porous media has received considerable attention owing to applications in materials processing, chemical engineering geophysics, astrophysical flows. Magnetic fields induce many complex phenomena in an electrically conducting flow regime including Hall currents, ion-slip effects, Joule, Alfven waves in plasma flows, etc. [5]. Those types of effects can have a considerable influence on heat and mass transfer and flow dynamics. For ex. in ionized gases with low density subjected to a strong strength of magnetic field, the electrical conductivity perpendicular to the magnetic field is lowered owing to free spiralling of electrons and ions about the magnetic lines of force prior to collisions, a current is thereby induced which is mutually perpendicular to both electrical and magnetic fields, constituting the Hall current effect. Under very high magnetic fields, in ionized plasmas, the diffusion velocity of ions becomes significant and ion-slip effects arise. Hall current effects however tend to be more dominant. In magnetic material fabrication applications, porous media are frequently used to regulate flow regimes. A considerable number of studies, both steady and transient, have appeared examining various hydro magnetic convective flows in Darcian regimes, which are viscous-dominated and in which Reynolds numbers are generally less than 10. Anand Rao [1] investigated the magneto-convective flow through a Darcian porous medium in planar channel. Ram [27] discussed analytically the transient hydro magnetic natural convection flow with Hall current effects in a Darcian regime and this extended to consider the supplementary effects of mass transfer [28]. Takhar and Ram [32] have investigated hall current effects on natural MHD convection flow through a porous medium. Kafoussias [18] has studied the hydro magnetic natural convection flow over an isothermal conical body to a non-homogenous porous regime. Takhar et al. [33] further reported on heat generation and hall currents in hydro magnetic convection flow through porous. Ezzat and Zakaria [10] discussed the oscillating hydro magnetic visco-elastic flow through porous medium making use of the state space technique. Kamel [19] more recently considered the transient one dimensional magneto convective heat and mass transfer through porous medium over an infinite vertical porous plate using the Laplace transform technique and the state space approach. Krishna et al. [20] have investigated hydromagnetic convection boundary layer heat transfer through porous medium in a rotating parallel plate channel, presenting analytical solutions and discussing the structure of the different boundary layers formed. Zakaria [37] discussed on the magneto hydro dynamic transient natural convection flow of a couple stress fluid through porous medium with relaxation effects also using the state space solution approach. Recently, Beg et al. [3] have studied the oscillatory hydro magnetic convection through porous regime using a perturbation method. El-Kabeir et al. [9] investigated the group transformation method to study transient hydro magnetic convection boundary layer flow through porous medium. Joule and viscous dissipation effects on fluid flow can be important in numerous magneto fluid engineering systems. Kinetic energy dissipated in the flow field due to retardation by the magnetic field manifests as Joule or Ohmic heating. Several researchers have been considered on hydro magnetic flows through porous medium in duct or channel with Joule and viscous dissipation effects. El-Amin [7] has studied viscous heating, Joule heating and also inertial porous drag effects on forced magneto convection boundary layers over a non-isothermal horizontal cylinder through porous media. Chen [4] analyzed numerically the magneto hydro dynamic natural convection heat and mass transfer with Joule and viscous heating. Studies of Couette magneto hydro dynamic flows, although without consideration of porous media effects include the analysis by Soundalgekar et al. [31] and more recently the transient model presented by Attia H.A.[13]. Palani and Srikanth [38] have explained the mass transfer effects on MHD flow past a semi infinite vertical plate. Chaudhary and Jain [39] have analyzed the combined heat and mass diffusion in a MHD free convective flow past a surface embedded in a porous medium. Recently, we explore the flow of a Jeffery fluid [40, 41] over a stretched sheet subject to power law temperature in the presence of heat source/sink. Abbasi et al. [42] have studied the peristaltic flow in an asymmetric channel with convective boundary conditions and Joule heating. Mixed convective heat and mass transfer analysis for peristaltic transport in an asymmetric channel with Soret and Dufour effects was investigated by Abbasi et al. [43]. Soret and Dufour effects on the peristaltic transport of a third-order fluid were studied by Hayat et al. [44]. Heat transfer in viscous free convective fluctuating MHD flow through porous media past a vertical porous plate with variable temperature is analyzed by Mishra et al. [45]. Makinde [46] discussed MHD heat and mass transfer over a moving vertical plate with a convective surface boundary condition. Recently, Tripati et al. [47] discussed MHD mixed convection flow of a visco-elastic fluid embedded in a porous medium over a moving vertical plate taking the radiation and mass transfer into account. Veera Krishna.M and G.Dharmaiah [48] discussed Heat Transfer on unsteady MHD Couette flow of a Bingham fluid through a Porous medium in a parallel plate channel with uniform suction and injection under the effect of inclined magnetic field and taking Hall currents. Veera Krishna.M and Devika Rani [49] investigated unsteady MHD mixed convection oscillatory flow of viscous incompressible fluid in a rotating vertical channel with radiation effects. Radiative heat transfer on unsteady MHD oscillatory visco-elastic flow through porous medium in a parallel plate channel was studied by Veera Krishna et al. [50].

Motivated the above studies, the aim of the present study was to analyze the effects on the unsteady MHD free convection flow of an incompressible electrically conducting second grade fluid through porous medium bounded by an infinite vertical porous surface in the presence of heat source and chemical reaction in a rotating system taking hall current into account.

II. FORMULATION AND SOLUTION OF THE PROBLEM

We consider the unsteady MHD free convection flow of an electrically conducting viscous incompressible second grade fluid bounded by a vertical porous surface in a rotating system in the presence of heat source and chemical reaction subjected to a uniform transverse magnetic field of strength B_0 normal to plate and taking hall current into account. The temperature on the surface varies with the time about a non-zero constant mean while the temperature of free stream is taken to be constant. We consider that the vertical infinite porous plate rotates with the constant angular velocity about an axis is perpendicular to the vertical plane surface. The physical configuration of the problem is as shown in Fig. 1.

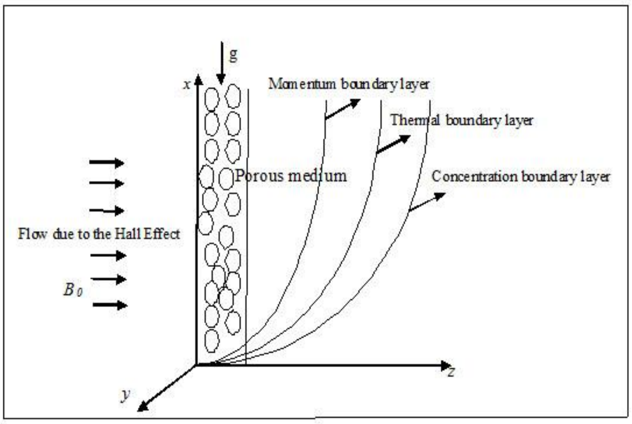


Figure 1: Physical configuration of the problem

We choose a Cartesian co-ordinate system $O(x, y, z)$ such that *x*, *y* axes respectively are in the vertical upward and perpendicular directions on the plane of the vertical porous surface $z = 0$, while z-axis normal to it. The interaction of Coriolis force with the free convection sets up a secondary flow in addition to primary flow and hence the flow becomes three dimensional. With the above frame of reference and assumptions, all the physical variables are functions of *z* and *t* alone. In the equation of motion, along *x*-direction the *x*-component current density $B_0 J_y$ and the *y*-component current density $-B_0 J_x$.

The constitutive equation for the stress T in an incompressible fluid of second grade is given by

$$T(t) = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1 \tag{2.1}$$

Where, μ is the dynamic viscosity α_1, α_2 are the normal stress moduli and the kinematical tensors A_1 and A_2 are defined through [Rivlin et al. (29)].

$$A_1 = (gradV) + (gradV)^T, \; A_2 = \frac{DA_1}{Dt} + A_1 (gradV) + (gradV)^T A_1 \tag{2.2}$$

Where, *V* is the velocity, *grad* the gradient operator and *D/Dt* the material time derivative.

The unsteady hydro magnetic flow in a rotating co-ordinate system is governed by the equation of motion, continuity equation and the Maxwell equations in the form.

$$\rho\left(\frac{\partial V}{\partial t} + (V.\nabla)V + 2\Omega \times V + \Omega \times (\Omega \times r)\right) = \nabla.T + J \times B \tag{2.3}$$

$$\nabla.V = 0 \tag{2.4}$$

$$\nabla.B = 0 \tag{2.5}$$

$$\nabla \times B = \mu_m J \tag{2.6}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{2.7}$$

Where, J is the current density, B is the total magnetic field, E is the total electric field, μ_m is the magnetic permeability and r is radial co-ordinate given by $r^2 = x^2 + y^2$. When the strength of the magnetic field is very large, the generalized ohm's law is modified to include the hall current so that

$$J + \frac{\omega_e \tau_e}{B_0}\left(J \times B\right) = \sigma\left[E + V \times B + \frac{1}{e\eta_e}\nabla P_e\right] \tag{2.8}$$

Where, ω_e is the cyclotron frequency of the electrons, τ_e is the electron collision time, σ is the electrical conductivity, e is the electron charge and P_e is the electron pressure. The ion-slip and thermo electric effects are not included in equation (2.8). Further it is assumed that $\omega_e \tau_e \sim 0$ (1) and $\omega_i \tau_i << 1$, where ω_i and τ_i are the cyclotron frequency and collision time for ions respectively. The unsteady hydro magnetic flow in a rotating system is governed by the equation of motion for momentum, the conservation of mass, energy and the equation of mass transfer, under usual Boussinesq approximation, are given by

$$\frac{\partial w}{\partial z} = 0 \tag{2.9}$$

$$\frac{\partial u}{\partial t} + w\frac{\partial u}{\partial z} - 2\Omega v = \nu\frac{\partial^2 u}{\partial z^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 u}{\partial z^2 \partial t} + B_0 J_y - \frac{\nu}{K_1}u + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) \tag{2.10}$$

$$\frac{\partial v}{\partial t} + w\frac{\partial v}{\partial z} + 2\Omega u = \nu\frac{\partial^2 v}{\partial z^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 v}{\partial z^2 \partial t} - B_0 J_x - \frac{\nu}{K_1}v \tag{2.11}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z} - \frac{\nu}{k}w \tag{2.12}$$

$$\frac{\partial T}{\partial t} + w\frac{\partial T}{\partial z} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial z^2} + S_1(T - T_\infty) \tag{2.13}$$

$$\frac{\partial C}{\partial t} + w\frac{\partial C}{\partial z} = D\frac{\partial^2 C}{\partial z^2} - K_c(C - C_\infty) \tag{2.14}$$

where, (u, v) is the velocity components along x and y directions, T is the temperature of the fluid, C is the species concentration, α_1 is the normal stress modulus, ρ is the density of the fluid, σ is the electrical conductivity of the fluid, K_1 is the permeability of the porous medium, B_0 is the uniform magnetic field of strength, ν is the coefficient of kinematic viscosity, k is the thermal conductivity of the fluid, C_p is the specific heat of the fluid at constant pressure, β is the volumetric coefficient of the thermal expansion, β^* is the volumetric coefficient of the thermal expansion with concentration, g is the acceleration due to gravity, D is the thermal diffusivity of the fluid, S_1 is the heat source/sink parameter and K_c is the chemical reaction parameter. In equation (2.8) the electron pressure gradient, the ion-slip and thermo-electric effects are neglected. We also assume that the electric field $E = 0$ under assumptions reduces to

$$J_x + m\ J_y = \sigma B_0 v \tag{2.15}$$

$$J_y - m\ J_x = -\sigma B_0 u \tag{2.16}$$

Where $m = \tau_e \omega_e$ is the hall parameter.

On solving equations (2.15) and (2.16) we obtain

$$J_x = \frac{\sigma B_0}{1 + m^2}(v + mu) \tag{2.17}$$

$$J_y = \frac{\sigma B_0}{1 + m^2}(mv - u) \tag{2.18}$$

Using the equations (2.17) and (2.18), the equations of motion with reference to a rotating frame are given by

$$\frac{\partial u}{\partial t} + w\frac{\partial u}{\partial z} - 2\Omega v = \nu\frac{\partial^2 u}{\partial z^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 u}{\partial z^2 \partial t} + \frac{\sigma B_0^2}{1 + m^2}(mv - u) - \frac{\nu}{K_1}u +$$
$$g\beta(T - T_\infty) + g\beta^*(C - C_\infty) \tag{2.19}$$

$$\frac{\partial v}{\partial t} + w\frac{\partial v}{\partial z} + 2\Omega u = \nu\frac{\partial^2 v}{\partial z^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 v}{\partial z^2 \partial t} - \frac{\sigma B_0^2}{1 + m^2}(v + mu) - \frac{\nu}{K_1}v \tag{2.20}$$

The corresponding boundary conditions are

$$u = v = 0,\ T = T_w + \varepsilon(T_w - T_\infty)e^{i\omega t}, C = C_w + \varepsilon(C_w - C_\infty)e^{i\omega t} \quad \text{at} \quad z = 0 \tag{2.21}$$

$$u = v = 0, T = T_\infty, C = C_\infty \quad \text{at} \quad z = \infty \tag{2.22}$$

Where $\varepsilon << 1$ and ω is the frequency of oscillation. There will be always some fluctuation in the temperature, the plate temperature is assumed to vary harmonically with time. It varies from $T_w \pm \varepsilon(T_w - T_\infty)$ as t varies from 0 to $\pi / 2\omega$. Now there may also occur some variation in suction at the plate due to the variation of the temperature, here we assume that, the frequency of suction and temperature variation are same.

Integrating the equation (2.9), we get

$$w(t) = -w_0(1 + \varepsilon A e^{i\omega t}) \tag{2.13}$$

Where, A is the suction parameter, w_0 is the constant suction velocity and ε is the small positive number such that $\varepsilon A \leq 1.$ The equation (2.12) determines the pressure distribution along the axis of rotation and the absence of $\partial p / \partial y$ in the equation (2.11) implies that there is a net cross flow in the $y-$ direction. We choose, $q = u + iv$ and taking into consideration (2.23), the momentum equation (2.19) and (2.20) can be written as

$$\frac{\partial q}{\partial t} - w_0(1 + \varepsilon A e^{i\omega t})\frac{\partial q}{\partial z} + 2i\Omega q = \nu \frac{\partial^2 q}{\partial z^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 q}{\partial z^2 \partial t} - \frac{\sigma B_0^2}{\rho(1 - im)} q - \frac{\nu}{K_1} q$$

$$g\beta(T - T_\infty) + g\beta^*(C - C_\infty) \tag{2.24}$$

Introducing the following non-dimensional quantities:

$$z^* = \frac{w_0 z}{\nu}, q^* = \frac{q}{w_0}, T^* = \frac{T - T_\infty}{T_w - T_\infty}, C^* = \frac{C - C_\infty}{C_w - C_\infty}, \omega^* = \frac{\nu\omega}{w_0^2}, t^* = \frac{t w_0^2}{\nu}$$

Making use of non-dimensional quantities (dropping asterisks), the equation (2.24), (2.13) and (2.14) can be written as

$$\frac{\partial q}{\partial t} - (1 + \varepsilon A e^{i\omega t})\frac{\partial q}{\partial z} + 2iRq = \frac{\partial^2 q}{\partial z^2} + \alpha \frac{\partial^3 q}{\partial z^2 \partial t} - \left(\frac{M^2}{1 - im} + \frac{1}{K}\right) q + \mathrm{Gr}T + \mathrm{Gm}C \tag{2.25}$$

$$\frac{\partial T}{\partial t} - (1 + \varepsilon A e^{i\omega t})\frac{\partial T}{\partial z} = \frac{1}{\mathrm{Pr}}\frac{\partial^2 T}{\partial z^2} + ST \tag{2.26}$$

$$\frac{\partial C}{\partial t} - (1 + \varepsilon A e^{i\omega t})\frac{\partial C}{\partial z} = \frac{1}{\mathrm{Sc}}\frac{\partial^2 C}{\partial z^2} - \mathrm{Kc}C \tag{2.27}$$

Where, $M^2 = \dfrac{\sigma B_0^2 \nu}{\rho w_0^2}$ is the Hartmann number (Magnetic field parameter), $K = \dfrac{K_1 w_0^2}{\nu^2}$ is the Porosity parameter, $R = \dfrac{\Omega \nu}{w_0^2}$ is the Rotation parameter, $\alpha = \dfrac{\alpha_1 w_0^2}{\rho \nu^2}$ is the second grade fluid parameter, $\mathrm{Gr} = \dfrac{g\beta\nu(T_w - T_\infty)}{w_0^3}$ is the thermal Grashof number, $\mathrm{Gm} = \dfrac{g\beta^*\nu(C_w - C_\infty)}{w_0^3}$ is the mass Grashof number, $\mathrm{Pr} = \dfrac{\rho\nu C_p}{k}$ is Prandtl parameter, $S = \dfrac{S_1\nu}{w_0}$ is the Source parameter, $\mathrm{Kc} = \dfrac{K_c \nu}{w_0^2}$ chemical reaction parameter, $m = \tau_e \omega_e$ is the hall parameter and $\mathrm{Sc} = \dfrac{\nu}{D}$ is the Schmidt number.

The corresponding non-dimensional boundary conditions

$$q = 0, T = 1 + \varepsilon e^{i\omega t}, C = 1 + \varepsilon e^{i\omega t} \quad \text{at} \quad z = 0 \tag{2.28}$$

$$q = T = C = 0 \quad \text{at} \quad z = \infty \tag{2.29}$$

In order to reduce the system of partial differential equations (2.25) – (2.27) under their boundary conditions (2.28) and (2.29), to a system of ordinary differential equations in the non-dimensional form, In view of the equation (2.23) and oscillating plate temperature T, The solution form of the equations (2.25), (2.26) and (2.27) are,

$$q(z,t) = q_0(z) + \varepsilon q_1(z) e^{i\omega t} \tag{2.30}$$

$$T(z,t) = T_0(z) + \varepsilon T_1(z) e^{i\omega t} \tag{2.31}$$

$$C(z,t) = C_0(z) + \varepsilon C_1(z) e^{i\omega t} \tag{2.32}$$

These equations (2.30) – (2.32) are valid for small amplitude of oscillation. Substituting from (2.30) to (2.32) into the system of equations (2.25) – (2.27) respectively, and equating the harmonic and non-harmonic terms, we get

$$\frac{d^2 q_0}{dz^2} + \frac{dq_0}{dz} - \left(2iR + \frac{M^2}{1-im} + \frac{1}{K} \right) q_0 = -\mathrm{Gr} T_0 - \mathrm{Gm} C_0 \tag{2.33}$$

$$(1+\alpha i\omega)\frac{d^2 q_1}{dz^2} + \frac{dq_1}{dz} - \left((2R+\omega)i + \frac{M^2}{1-im} + \frac{1}{K} \right) q_1 = -\mathrm{Gr} T_1 - \mathrm{Gm} C_1 - A\frac{dq_0}{dz} \tag{2.34}$$

$$\frac{d^2 T_0}{dz^2} + \Pr\frac{dT_0}{dz} + S\Pr T_0 = 0 \tag{2.35}$$

$$\frac{d^2 T_1}{dz^2} + \Pr\frac{dT_1}{dz} - (i\omega - S)\Pr T_1 = -A\Pr\frac{dT_0}{dz} \tag{2.36}$$

$$\frac{d^2 C_0}{dz^2} + \mathrm{Sc}\frac{dC_0}{dz} - \mathrm{Sc}\,\mathrm{Kc}\, C_0 = 0 \tag{2.37}$$

$$\frac{d^2 C_1}{dz^2} + \mathrm{Sc}\frac{dC_1}{dz} - (i\omega + \mathrm{Kc})\mathrm{Sc} C_1 = -A\mathrm{Sc}\frac{dC_0}{dz} \tag{2.38}$$

The corresponding boundary conditions

$$\left. \begin{array}{l} q_0 = 0, T_0 = 1, C_0 = 1 \\ q_1 = 0, T_1 = 1, C_1 = 1 \end{array} \right\} \quad \text{at} \quad z = 0 \tag{2.39}$$

$$\left. \begin{array}{l} q_0 = T_0 = C_0 = 0 \\ q_1 = T_1 = C_1 = 0 \end{array} \right\} \quad \text{at} \quad z = \infty \tag{2.40}$$

The solutions of the equations (2.35) and (2.36) using the boundary conditions (2.39) and (2.40), we obtain T_0 and T_1 , the equation (2.31) becomes,

$$T(z,t) = e^{C_5 z} + \varepsilon \left(e^{a_2 z} + \frac{A\Pr C_5}{C_6}\left(e^{a_2 z} - e^{C_5 z}\right) \right) e^{i\omega t} \tag{2.41}$$

The solutions of the equations (2.37) and (2.38) using the boundary conditions (2.39) and (2.40), we obtain C_0 and C_1 , the equation (2.32) becomes,

$$C(z,t) = e^{C_2 z} + \varepsilon \left(e^{a_4 z} + \frac{A\mathrm{Sc} C_2}{C_3}\left(e^{a_4 z} - e^{C_2 z}\right) \right) e^{i\omega t} \tag{2.42}$$

The solutions of the equations (2.33) and (2.34) using the boundary conditions (2.39) and (2.40), we obtain q_0 and q_1, the equation (2.30) becomes,

$$\begin{aligned} q(z,t) &= b_1 e^{C_5 z} + b_2 e^{C_2 z} + b_3 e^{a_6 z} + \\ &\varepsilon \left(C_{17} e^{a_8 z} + C_{12} e^{a_2 z} + C_{13} e^{C_5 z} + C_{14} e^{a_4 z} + C_{15} e^{C_2 z} + C_{16} e^{a_6 z} \right) e^{i\omega t} \end{aligned} \tag{2.43}$$

The equation (2.43) reveals that the steady part of the velocity field has three layer character while the oscillatory part of the fluid field exhibits a multilayer character. From equations (2.41) and (2.42), we observe that in case of considerably slow motion of the fluid. i.e., when the viscous dissipation term is neglected, the temperature profiles are mainly affected by Prandtl number (Pr) and Source parameter (S): and the concentration profiles are affected by Schimdt number (Sc) and chemical reaction parameter (K_C) of the fluid respectively. Considering

$$q_0 = u_0 + iv_0 \;\text{ and }\; q_1 = u_1 + iv_1$$

Now it is convenient to write the primary and secondary velocity fields in terms of the fluctuating parts, separating the real and imaginary parts from the equation (2.43) and taking only the real parts as they have physical significance. The velocity distribution of the flow field can be expressed as in fluctuating parts,

$$q(z,t) = q_0(z) + \varepsilon q_1(z) e^{i\omega t}$$

$$u + iv = u_0 + iv_0 + \varepsilon u_1 \cos\omega t + i\varepsilon u_1 \sin\omega t + i\varepsilon v_1 \cos\omega t - \varepsilon v_1 \sin\omega t$$

Comparing real and imaginary parts,

$$u(z,t) = w_0(u_0(z) + \varepsilon(u_1\cos\omega t - v_1\sin\omega t)) \tag{2.44}$$

$$v(z,t) = w_0(v_0(z) + \varepsilon(u_1\sin\omega t + v_1\cos\omega t)) \tag{2.45}$$

Hence the expression for the transient velocity profiles for
$\omega t = \pi / 2$ are given by

$$u\left(z, \frac{\pi}{2\omega} \right) = w_0 \left(u_0(z) - \varepsilon v_1(z) \right) \tag{2.46}$$

$$v\left(z, \frac{\pi}{2\omega} \right) = w_0 \left(v_0(z) + \varepsilon u_1(z) \right) \tag{2.47}$$

Skin friction:
The non-dimensional skin friction at the plate $z = 0$ in term of amplitude and phase angle is given by

$$\tau = \left(\frac{dq}{dz} \right)_{z=0} = \left(\frac{dq_0}{dz} \right)_{z=0} + \varepsilon \left(\frac{dq_1}{dz} \right)_{z=0} e^{i\omega t}$$

$$= C_5 b_1 + C_2 b_2 + C_3 a_6 + \varepsilon \left(a_8 C_{17} + a_2 C_{12} + C_5 C_{13} + a_4 C_{14} + C_2 C_{15} + a_6 C_{16} \right) e^{i\omega t} \tag{2.48}$$

The τ_{xz} and τ_{yz} components of skin friction at the plate are given by

$$\tau_{xz} = \left(\frac{du_0}{dz} \right)_{z=0} - \varepsilon \left(\frac{dv_1}{dz} \right)_{z=0} \quad \text{and} \quad \tau_{yz} = \left(\frac{dv_0}{dz} \right)_{z=0} + \varepsilon \left(\frac{du_1}{dz} \right)_{z=0}$$

Rate of heat transfer (Nusselt number):
The rate of heat transfer co-efficient at the plate $z = 0$ in term of amplitude and phase angle is given by

$$Nu = \left(\frac{dT}{dz} \right)_{z=0} = \left(\frac{dT_0}{dz} \right)_{z=0} + \varepsilon \left(\frac{dT_1}{dz} \right)_{z=0} e^{i\omega t} = C_5 + \varepsilon \left(a_2 + \frac{A \Pr C_5}{C_6} (a_2 - C_5) \right) e^{i\omega t} \tag{2.49}$$

Rate of mass transfer (Sherwood number):
The rate of mass transfer co-efficient at the plate $z = 0$ in term of amplitude and phase angle is given by

$$Sh = \left(\frac{dC}{dz} \right)_{z=0} = \left(\frac{dC_0}{dz} \right)_{z=0} + \varepsilon \left(\frac{dC_1}{dz} \right)_{z=0} e^{i\omega t} = C_2 + \varepsilon \left(a_4 + \frac{A \mathrm{Sc} C_2}{C_3} (a_4 - C_2) \right) e^{i\omega t} \tag{2.50}$$

III. Results and Discussion

We discussed the unsteady magnetohydrodynamic free convection flow of an incompressible electrically conducting second grade fluid bounded by an infinite vertical porous surface in a rotating system taking hall current into account under the presence of heat source and chemical reaction. The closed form solutions for the velocity $q = u + iv$, temperature θ and concentration C are obtained making use of perturbation technique. The velocity expression consists of steady state and oscillatory state. It reveals that, the steady part of the velocity field has three layer characters while the oscillatory part of the fluid field exhibits a multi layer character. For computational purpose we are fixing the values $A = 0.05$; $\omega = 5\pi / 2$; $\varepsilon = 0.001$.

The Figures (2-13) shows the effects of non-dimensional parameters on velocity such as *M* the Hartmann number, α the second grade fluid parameter, *K* permeability parameter, *m* hall parameter, *R* rotation parameter, *S* heat source parameter, Gr Grashof number, Gm mass Grashof number, Kc chemical reaction parameter, Pr the Prandtl number and *t* time; the Figure (5) exhibit the temperature distribution with different variations in the governing parameters *S*, Pr, the frequency of oscillation ω and time *t*; and the Figure (6) depicts the concentration profiles with variations in Schmidt number Sc and chemical reaction parameter Kc, the frequency of oscillation ω and time *t*.

It is noticed that, from the Figures (2-5) the magnitude of the velocity *u* reduces with increasing the intensity of the magnetic field (Hartmann number *M*) while it enhances with increasing second grade fluid parameter α or permeability of porous medium *K* or hall parameter *m* throughout the fluid region. The magnitude of the velocity component *v* enhances with increasing *M* or second grade fluid parameter α or permeability of porous medium *K* or hall parameter *m*. The application of the transverse magnetic field plays the important role of a resistive type force (Lorentz force) similar to drag force (that acts in the opposite direction of the fluid motion) which tends to resist the flow thereby reducing its velocity. The resultant velocity q enhances with increasing α, *K* and *m*; and reduces with increasing *M*. We observed that lower the permeability of porous medium lesser the fluid speed in the entire fluid region.

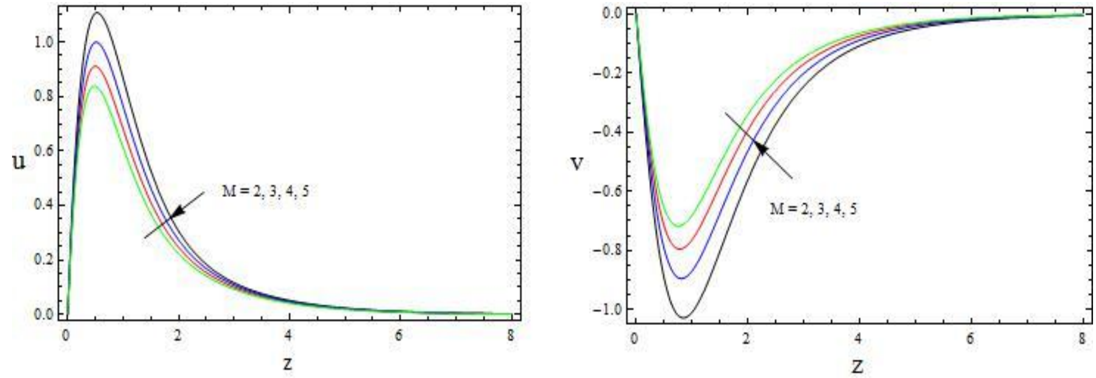


Figure 2. The velocity Profiles for *u* and *v* against *M*

$\alpha = 1, K = 1, m = 1, \mathrm{R} = 1.2, \mathrm{S} = 2, \mathrm{Gr} = 5, \mathrm{Gm} = 10, \mathrm{Sc} = 0.22, \mathrm{Kc} = 2, \mathrm{Pr} = 0.71, t = 0.2$

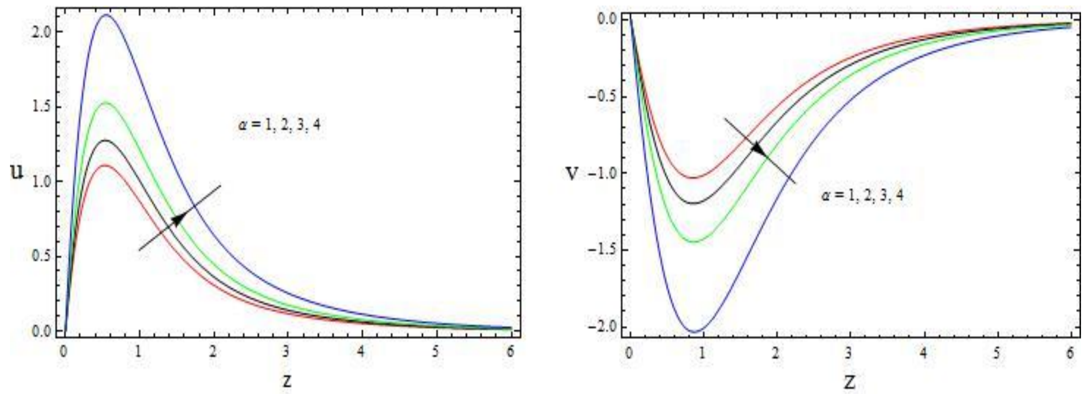


Figure 3. The velocity Profiles for *u* and *v* against α

$M = 2, K = 1, m = 1, \mathrm{R} = 1.2, \mathrm{S} = 2, \mathrm{Gr} = 5, \mathrm{Gm} = 10, \mathrm{Sc} = 0.22, \mathrm{Kc} = 2, \mathrm{Pr} = 0.71, t = 0.2$

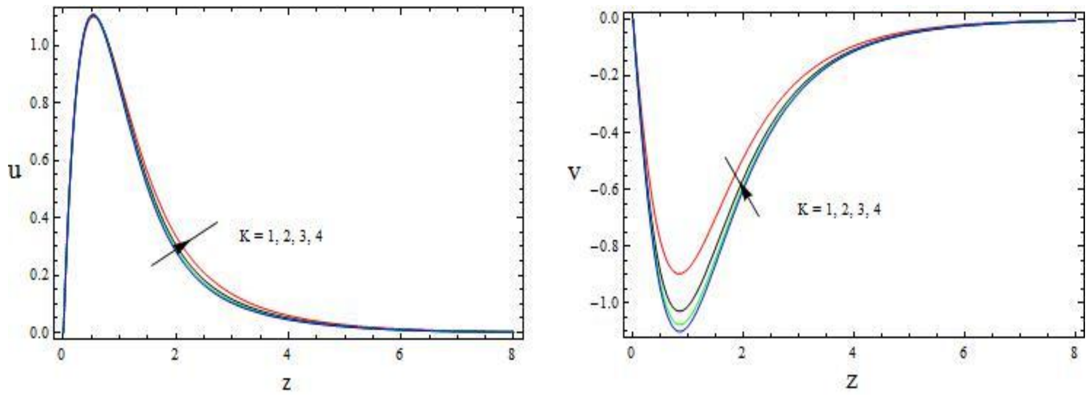


Figure 4. The velocity Profiles for *u* and *v* against K

$M = 2, \alpha = 1, m = 1, \mathrm{R} = 1.2, \mathrm{S} = 2, \mathrm{Gr} = 5, \mathrm{Gm} = 10, \mathrm{Sc} = 0.22, \mathrm{Kc} = 2, \mathrm{Pr} = 0.71, t = 0.2$

From the Figures (6-9) depicts the velocity component *u* reduces with increasing the rotation parameter *R* while it enhances with increasing source parameter *S*, Grashof number Gr and mass Grashof number Gm. The profiles show the magnitude of the velocity component *v* reverse trend whenever there is increasing rotation parameter *R* or source parameter *S* or Gr or Gm. The resultant velocity q increases with increasing *R* or Gr or Gm; and reduces with increasing *S*.

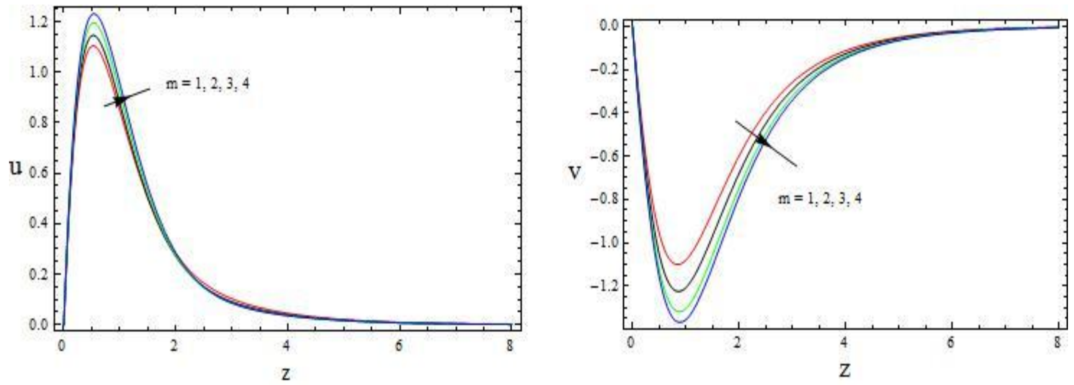


Figure 5. The velocity Profiles for *u* and *v* against m

$M = 2, \alpha = 1, K = 1, \mathrm{R} = 1.2, \mathrm{S} = 2, \mathrm{Gr} = 5, \mathrm{Gm} = 10, \mathrm{Sc} = 0.22, \mathrm{Kc} = 2, \mathrm{Pr} = 0.71, t = 0.2$

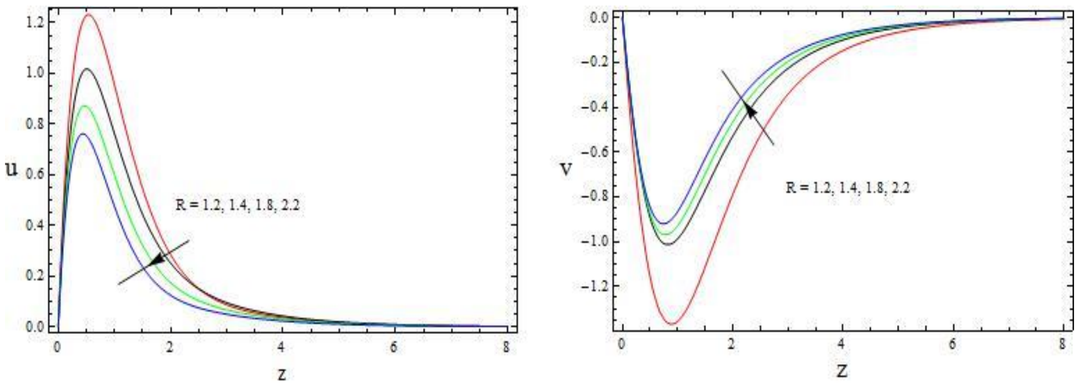


Figure 6. The velocity Profiles for *u* and *v* against R

$M = 2, \alpha = 1, m = 1, K = 1, \mathrm{S} = 2, \mathrm{Gr} = 5, \mathrm{Gm} = 10, \mathrm{Sc} = 0.22, \mathrm{Kc} = 2, \mathrm{Pr} = 0.71, t = 0.2$

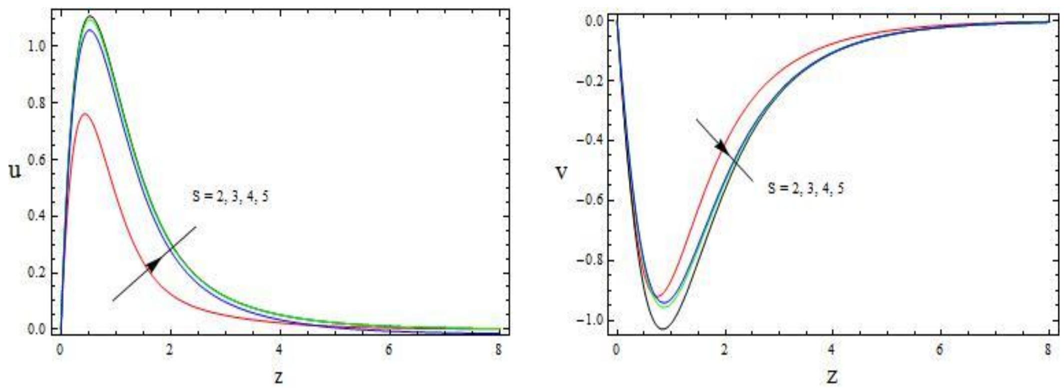


Figure 7. The velocity Profiles for *u* and *v* against *S*

$M = 2, \alpha = 1, m = 1, K = 1, \mathrm{R} = 1.2, \mathrm{Gr} = 5, \mathrm{Gm} = 10, \mathrm{Sc} = 0.22, \mathrm{Kc} = 2, \mathrm{Pr} = 0.71, t = 0.2$

Further, it is to observed that from Figures (10-13) the velocity *u* reduces and *v* enhances with increasing Schmidt number Sc, first the velocity *u* increases and then experiences retardation where as *v* reduces in the entire fluid region with increasing chemical reaction parameter Kc. With increasing Prandtl number Pr the velocity *u* reduces and *v* enhances in the complete flow field. This implies that an increase in Prandtl number Pr leads to fall the thermal boundary layer flow. This is because fluids with large have low thermal diffusivity which causes low heat penetration resulting in reduced thermal boundary layer.

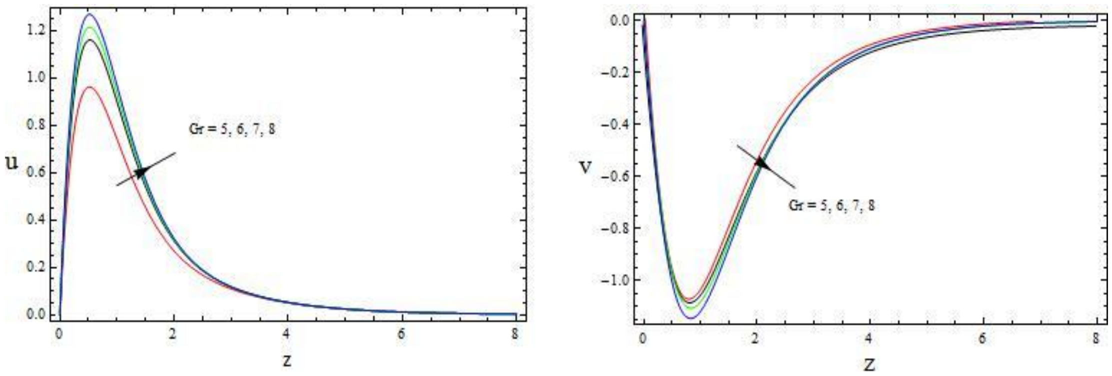


Figure 8. The velocity Profiles for *u* and *v* against Gr
$M = 2, \alpha = 1, m = 1, K = 1, \mathrm{R} = 1.2, \mathrm{S} = 2, \mathrm{Gm} = 10, \mathrm{Sc} = 0.22, \mathrm{Kc} = 2, \mathrm{Pr} = 0.71, t = 0.2$

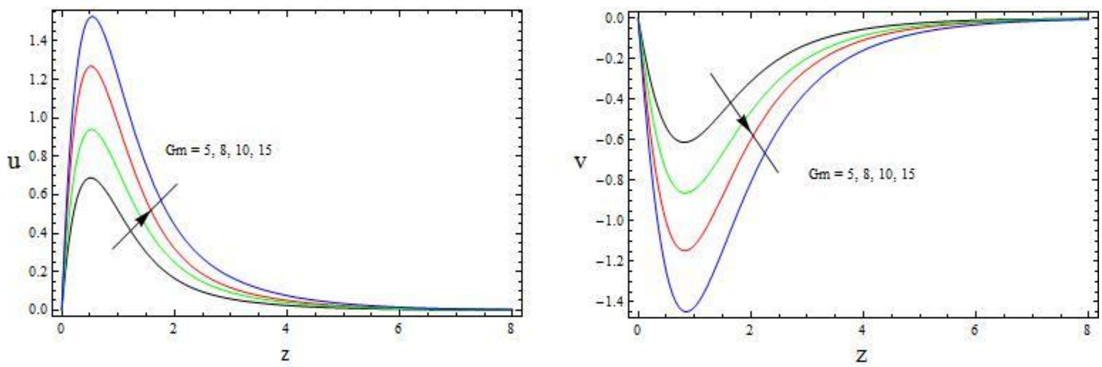


Figure 9. The velocity Profiles for *u* and *v* against Gm
$M = 2, \alpha = 1, m = 1, K = 1, \mathrm{R} = 1.2, \mathrm{S} = 2, \mathrm{Gr} = 5, \mathrm{Sc} = 0.22, \mathrm{Kc} = 2, \mathrm{Pr} = 0.71, t = 0.2$

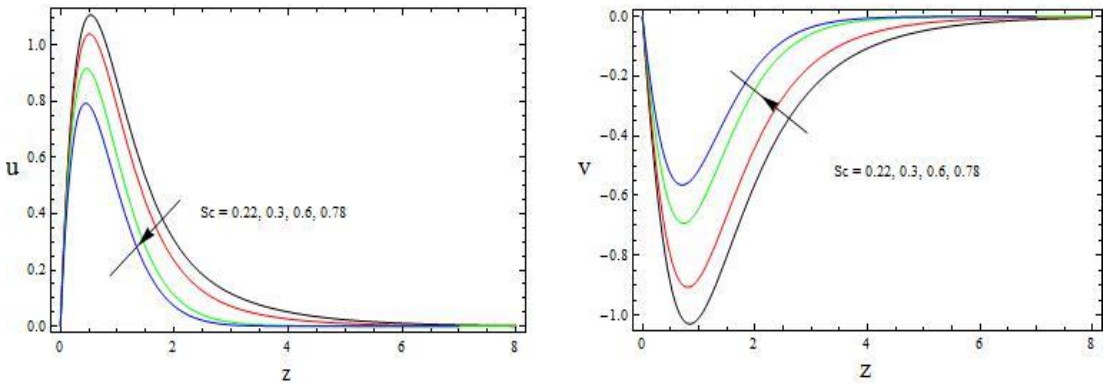


Figure 10. The velocity Profiles for *u* and *v* against Sc
$M = 2, \alpha = 1, m = 1, K = 1, \mathrm{R} = 1.2, \mathrm{S} = 2, \mathrm{Gr} = 5, \mathrm{Gm} = 10, \mathrm{Kc} = 2, \mathrm{Pr} = 0.71, t = 0.2$

Likewise the velocity *u* enhances and *v* decreases with increasing the frequency of oscillation ω and time *t*. The resultant velocity reduces with increasing Kc or Sc and increases with increasing Pr and time *t*.

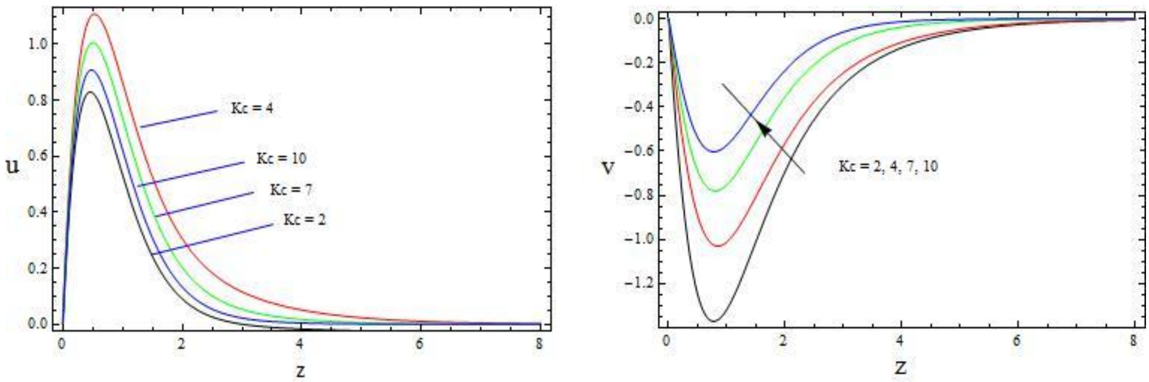


Figure 11. The velocity Profiles for *u* and *v* against Kc
$M = 2, \alpha = 1, m = 1, K = 1, \mathrm{R} = 1.2, \mathrm{S} = 2, \mathrm{Gr} = 5, \mathrm{Gm} = 10, \mathrm{Sc} = 0.22, \mathrm{Pr} = 0.71, t = 0.2$

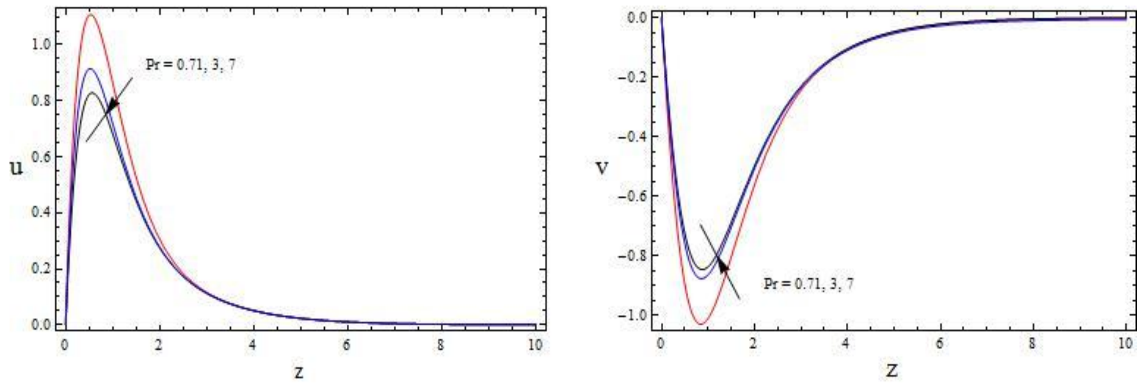


Figure 12. The velocity Profiles for *u* and *v* against Pr
$M = 2, \alpha = 1, m = 1, K = 1, \mathrm{R} = 1.2, \mathrm{S} = 2, \mathrm{Gr} = 5, \mathrm{Gm} = 10, \mathrm{Sc} = 0.22, \mathrm{Kc} = 2, t = 0.2$

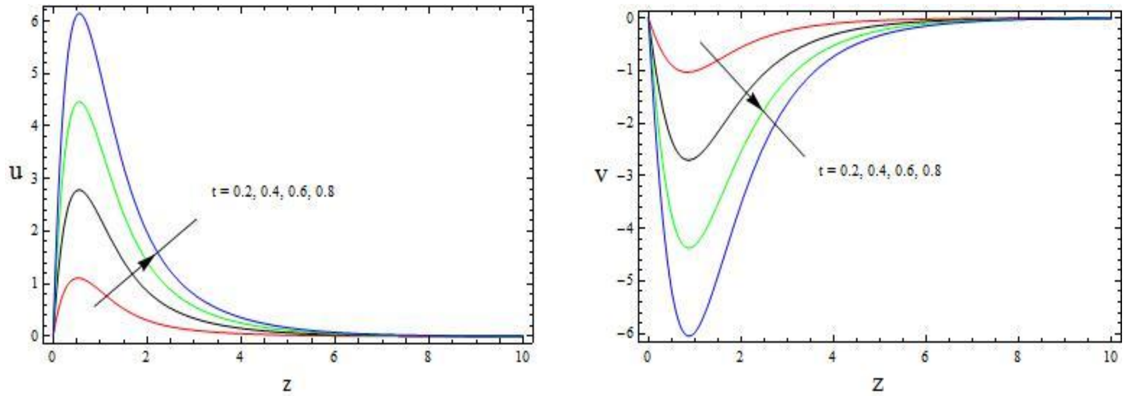
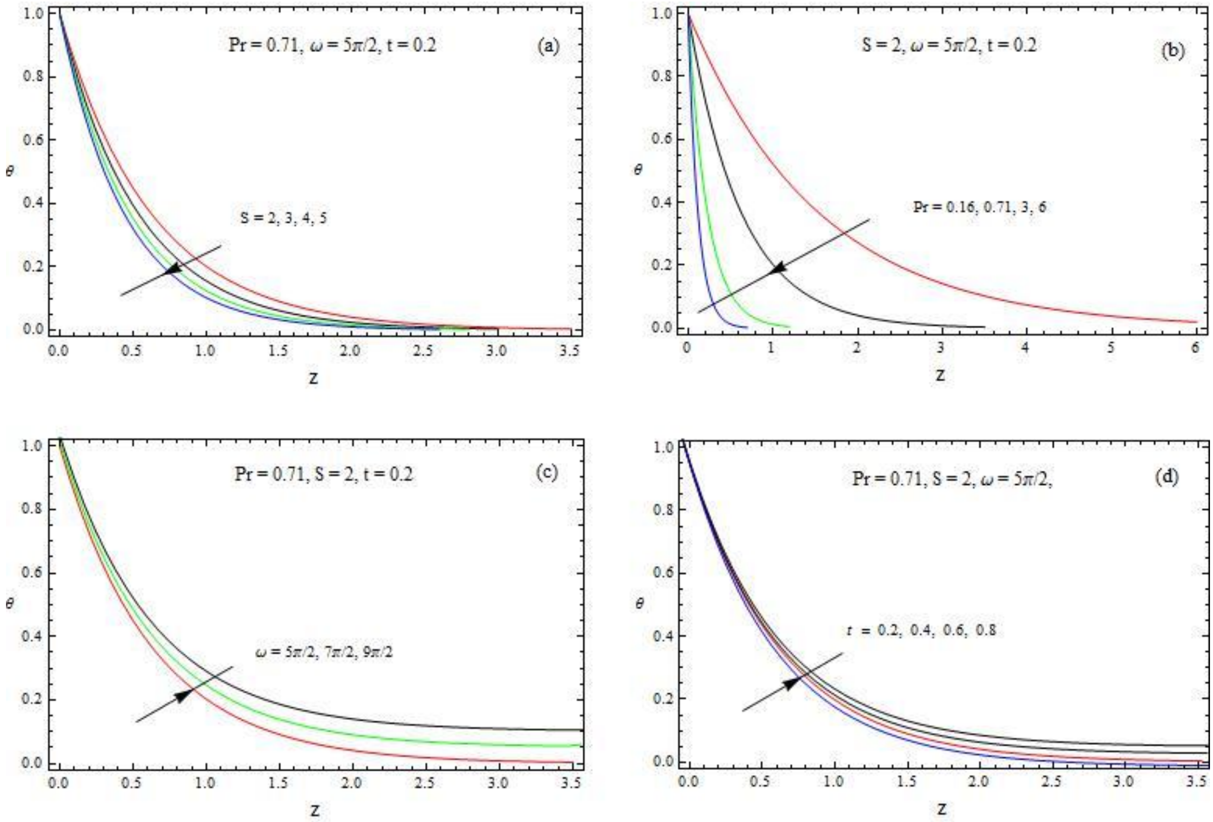
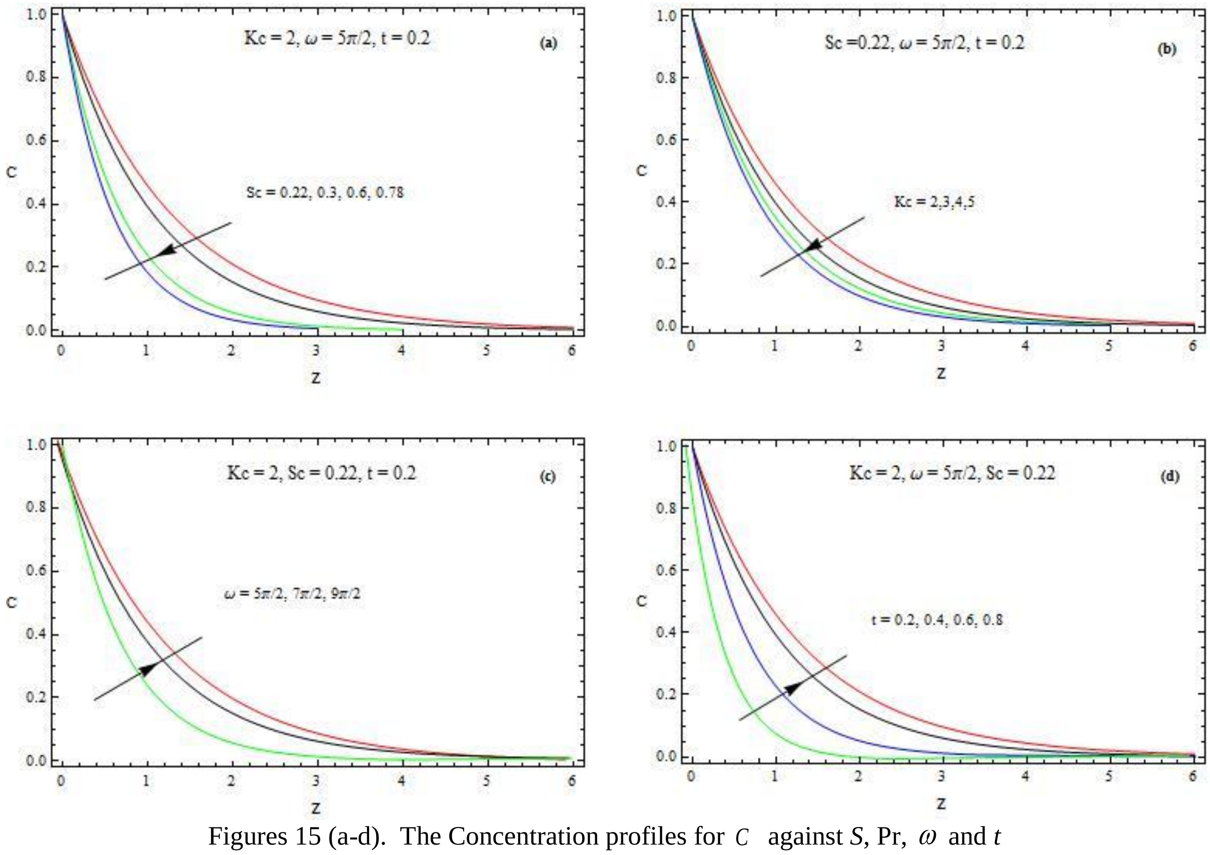


Figure 13. The velocity Profiles for *u* and *v* against t
$M = 2, \alpha = 1, m = 1, K = 1, \mathrm{R} = 1.2, \mathrm{S} = 2, \mathrm{Gr} = 5, \mathrm{Gm} = 10, \mathrm{Sc} = 0.22, \mathrm{Kc} = 2, \mathrm{Pr} = 0.71$

The temperature profiles exhibit in the Figures 14(a-d) for different variations in source parameter *S*, Prandtl number Pr, the frequency of oscillation ω and time *t*. It is observed that Prandtl number Pr leads to decrease the temperature uniformly in all layers being the heat source parameter fixed. It is found that the temperature decreases in all layers with increase in the heat source parameter *S*. It is concluded that the heat source parameter *S* and Prandtl number Pr reduces the temperature in all layers. The temperature increases with increasing the frequency of oscillation ω and time *t*. The concentration profiles are shown in the Figures 15 (a-d) for different variations in Schmidt number Sc, the chemical reaction parameter Kc, the frequency of oscillation ω and time *t*. It is noticed that the concentration decreases at all layers of the flow for heavier species such as CO_2, H_2O and NH_3 having Schmidt number 0.3, 0.6 and 0.78 respectively.



Figures 14 (a-d). The temperature profiles for θ against *S*, Pr, ω and *t*



Figures 15 (a-d). The Concentration profiles for C against *S*, Pr, ω and *t*

It is observed that for heavier diffusing foreign species, i.e., the velocity reduces with increasing Schmidt number Sc in both magnitude and extent and thinning of thermal boundary layer occurs. Likewise, the concentration profiles decrease with increase in chemical reaction parameter Kc. It is concluded that the Schmidt number and the chemical reaction parameter reduces the concentration in all layers. The concentration increases with increasing the frequency of oscillation ω and time *t*.

Table. 1. Skin Friction

M	α	**K**	**m**	**R**	**S**	**Gr**	**Gm**	**Sc**	**Kc**	**Pr**	τ_{xz}	τ_{yz}
2	**1**	**2**	**1**	**1.2**	**2**	**5**	**10**	**0.22**	**2**	**0.71**	5.620268	-2.685635
3	1	2	1	1.2	2	5	10	0.22	2	0.71	5.280022	-2.431979
4	1	2	1	1.2	2	5	10	0.22	2	0.71	4.994062	-2.238832
2	**2**	2	1	1.2	2	5	10	0.22	2	0.71	5.619835	-2.675965
2	**3**	2	1	1.2	2	5	10	0.22	2	0.71	5.519604	-2.646093
2	1	**3**	1	1.2	2	5	10	0.22	2	0.71	5.630642	-2.798579
2	1	**4**	1	1.2	2	5	10	0.22	2	0.71	5.633692	-2.856412
2	1	2	**2**	1.2	2	5	10	0.22	2	0.71	5.781484	-3.117423
2	1	2	**3**	1.2	2	5	10	0.22	2	0.71	5.936172	-3.295582
2	1	2	1	**1.4**	2	5	10	0.22	2	0.71	5.368144	-2.707081
2	1	2	1	**1.8**	2	5	10	0.22	2	0.71	4.939473	-2.707612
2	1	2	1	1.2	**3**	5	10	0.22	2	0.71	5.513113	-2.599534
2	1	2	1	1.2	**4**	5	10	0.22	2	0.71	5.431000	-2.539932
2	1	2	1	1.2	2	**6**	10	0.22	2	0.71	5.938664	-2.802592
2	1	2	1	1.2	2	**7**	10	0.22	2	0.71	6.257066	-2.919556
2	1	2	1	1.2	2	5	**5**	0.22	2	0.71	3.606121	-1.635212
2	1	2	1	1.2	2	5	**8**	0.22	2	0.71	4.814612	-2.265465
2	1	2	1	1.2	2	5	10	**0.3**	2	0.71	5.438912	-2.441874
2	1	2	1	1.2	2	5	10	**0.6**	2	0.71	4.923213	-1.885492
2	1	2	1	1.2	2	5	10	0.22	**4**	0.71	5.310184	-2.286522
2	1	2	1	1.2	2	5	10	0.22	**7**	0.71	4.999061	-1.957933
2	1	2	1	1.2	2	5	10	0.22	2	**3**	4.900980	-2.261319
2	1	2	1	1.2	2	5	10	0.22	2	7	4.533414	-2.153403

Table. 2. Nusselt Number

S	**Pr**	ω	t	**Nu**
2	**0.71**	$5\pi/2$	**0.2**	-1.59653
3	0.71	$5\pi/2$	0.2	-1.85503
4	0.71	$5\pi/2$	0.2	-2.07512
2	**3**	$5\pi/2$	0.2	-4.36861
2	**7**	$5\pi/2$	0.2	-8.61827
2	0.71	$7\pi/2$	0.2	-1.59538
2	0.71	$9\pi/2$	0.2	-1.59431
2	0.71	$5\pi/2$	**0.4**	-1.59854
2	0.71	$5\pi/2$	**0.6**	-1.60026

Table. 3. Sherwood Number

Sc	**Kc**	ω	t	**Sh**
2	**0.22**	$5\pi/2$	**0.2**	-0.781334
3	0.22	$5\pi/2$	0.2	-0.928700
4	0.22	$5\pi/2$	0.2	-1.053333
2	**0.3**	$5\pi/2$	0.2	-0.937762
2	**0.6**	$5\pi/2$	0.2	-1.434060
2	0.22	$7\pi/2$	0.2	-0.780754
2	0.22	$9\pi/2$	0.2	-0.778487
2	0.22	$5\pi/2$	**0.4**	-0.782446
2	0.22	$5\pi/2$	**0.6**	-0.783434

It is noted from the table 1 that the magnitudes of both the skin friction components τ_{xz} and τ_{yz} increase with increase in permeability parameter K, hall parameter *m*, thermal Grashof number Gr and mass Grashof number Gm, and where as it reduces with increase in Hartmann number *M*, second grade fluid parameter α, heat source parameter *S*, Schmidt number *Sc*, chemical reaction parameter *Kc* and Prandtl number Pr. Likewise the rotation parameter *R* enhances skin friction component τ_{xz} and reduces skin friction component τ_{yz}.

From the table 2 that the magnitude of the Nusselt number Nu increases for the parameters heat source parameter *S* and Prandtl number Pr or time *t*, and it reduces with the frequency of oscillation ω. Also from the table 3, the similar behaviour is observed. The magnitude of the Sherwood number *Sh* increases for increasing the parameters Schmidt number *Sc* and chemical reaction parameter Kc or time *t* and reduce with increasing the frequency of oscillation ω.

IV. CONCLUSIONS

We have considered the unsteady MHD free convection flow of an incompressible electrically conducting second grade fluid through porous medium bounded by an infinite vertical porous surface in the presence of heat source and chemical reaction in a rotating system taking hall current into account. The conclusions are made as follows

1. The resultant velocity enhances with increasing α, K, *m*, *R*, Gr, Gm, Pr and time *t*; and reduces with increasing *M*, *S*, Kc and Sc.
2. Lower the permeability of porous medium lesser the fluid speed in the entire fluid region.
3. The parameters *S* and Pr reduce the temperature in all layers. The temperature increases with increasing ω and time.
4. The Schmidt number and Kc reduce the concentration in all layers. The concentration increases with increasing ω and time.
5. The skin friction components τ_{xz} and τ_{yz} increase with increase in K, *m*, Gr and Gm, and where as it reduces with increase in *M*, α, *S*, Sc, Kc and Pr. The rotation parameter *R* enhances skin friction component τ_{xz} and reduces τ_{yz}.
6. The heat transfer coefficient increases with increasing *S* and Pr or time span, and it reduces with ω.
7. The Sherwood number enhances for increasing the parameters Schmidt number Sc and chemical reaction parameter Kc or time span *t* and reduces with increasing ω.

ACKNOWLEDGEMENTS

The authors are thankful to Prof. R. Siva Prasad, Department of Mathematics, Sri Krishnadevaraya University, Anantapur, Andhra pradesh, India, and Department of Mathematics, Rayalaseema University, Kurnool, Andhra pradesh, India, provided me for the computational facilities throughout our work, and ISOR Journal for the support to develop this document.

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APPENDIX

$$a_1 = \frac{-\Pr + \sqrt{\Pr^2 + 4\Pr(i\omega - S)}}{2}, \quad a_2 = \frac{-\Pr - \sqrt{\Pr^2 + 4\Pr(i\omega - S)}}{2},$$

$$a_3 = \frac{-\mathrm{Sc} + \sqrt{\mathrm{Sc}^2 + 4\mathrm{Sc}(i\omega - \mathrm{Kc})}}{2}, \quad a_4 = \frac{-\mathrm{Sc} - \sqrt{\mathrm{Sc}^2 + 4\mathrm{Sc}(i\omega - \mathrm{Kc})}}{2}$$

$$a_5 = \frac{-1 + \sqrt{1 + 4\left(2iR + \frac{M^2}{1-im} + \frac{1}{K}\right)}}{2}, \quad a_6 = \frac{-1 - \sqrt{1 + 4\left(2iR + \frac{M^2}{1-im} + \frac{1}{K}\right)}}{2},$$

$$a_7 = \frac{-1 + \sqrt{1 + 4(1+\alpha i\omega)\left(i(2R+\omega) + \frac{M^2}{1-im} + \frac{1}{K}\right)}}{2},$$

$$a_8 = \frac{-1 - \sqrt{1 + 4(1+\alpha i\omega)\left(i(2R+\omega) + \frac{M^2}{1-im} + \frac{1}{K}\right)}}{2},$$

$$b_1 = \frac{-\mathrm{Gr}}{C_5^2 + C_5 - \left(2iR + \frac{M^2}{1-im} + \frac{1}{K}\right)}, \quad b_2 = \frac{-\mathrm{Gm}}{C_2^2 + C_2 - \left(2iR + \frac{M^2}{1-im} + \frac{1}{K}\right)},$$

$$b_3 = -(b_1 + b_2), \quad C_1 = \frac{-\mathrm{Sc} + \sqrt{\mathrm{Sc}^2 + 4\mathrm{Sc}\,\mathrm{Kc}}}{2}, \quad C_2 = \frac{-\mathrm{Sc} - \sqrt{\mathrm{Sc}^2 + 4\mathrm{Sc}\,\mathrm{Kc}}}{2},$$

$$C_3 = C_2^2 + \mathrm{Sc}\,C_2 - \mathrm{Sc}(i\omega + \mathrm{Kc}), \quad C_4 = \frac{-\Pr + \sqrt{\Pr^2 - 4S\Pr}}{2}, \quad C_5 = \frac{-\Pr - \sqrt{\Pr^2 - 4S\Pr}}{2}$$

$$C_6 = C_5^2 + \Pr C_5 - \Pr(i\omega - S), \quad C_7 = -\mathrm{Gr}\left(1 + \frac{A\Pr C_5}{C_6}\right), \quad C_8 = \mathrm{Gr}\frac{A\Pr C_5}{C_6} - AC_5 b_1,$$

$$C_9 = -\mathrm{Gm}\left(1 + \frac{A\mathrm{Sc}\,C_2}{C_3}\right), \quad C_{10} = \mathrm{Gm}\frac{A\mathrm{Sc}\,C_2}{C_3} - AC_2 b_2, \quad C_{11} = Aa_6 b_3,$$

$$C_{12} = \frac{C_7}{(1+\alpha i\omega)a_2^2 + a_2 - (1+\alpha i\omega)\left(i(2R+\omega) + \frac{M^2}{1-im} + \frac{1}{K}\right)},$$

$$C_{13} = \frac{C_8}{(1+\alpha i\omega)C_5^2 + C_5 - (1+\alpha i\omega)\left(i(2R+\omega) + \frac{M^2}{1-im} + \frac{1}{K}\right)},$$

$$C_{14} = \frac{C_9}{(1+\alpha i\omega)a_4^2 + a_4 - (1+\alpha i\omega)\left(i(2R+\omega) + \frac{M^2}{1-im} + \frac{1}{K}\right)},$$

$$C_{15} = \frac{C_{10}}{(1+\alpha i\omega)C_2^2 + C_2 - (1+\alpha i\omega)\left(i(2R+\omega) + \frac{M^2}{1-im} + \frac{1}{K}\right)},$$

$$C_{16} = \frac{C_{11}}{(1+\alpha i\omega)\,a_6^2 + a_6 - (1+\alpha i\omega)\left(i(2R+\omega) + \frac{M^2}{1-im} + \frac{1}{K} \right)},$$

$$C_{17} = -(C_{12} + C_{13} + C_{14} + C_{15} + C_{16})$$