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Bulk Viscous Fluid Bianchi Type - I String Cosmological Model in General Relativity

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Abstract: *In this paper we have investigated Bulk viscous fluid Bianchi Type - I string cosmological model in general relativity. To get a deterministic model, it is assumed that* $\xi\boldsymbol{\theta} = M\ (constant)$, *where* ξ *is the coefficient of bulk viscosity is,* $\boldsymbol{\theta}$ *is the scalar of expansion and a relation between metric potential* $B = A^n$ *.The physical and geometrical aspects of the model are also discussed.*
Key words: *Bianchi Type-I models, bulk viscosity, scalar of expansion.*

I. Introduction

It is a challenging problem to determine the exact physical situation at the very early stages of the formation of our universe. String cosmological models are widely studied in recent times because of their prime role in the description of the evolution of the early phase of universe. These arise during the phase transition after the big bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories [1, 2, 3]. It is believed that cosmic strings may act as gravitational lenses and these objects are considered as possible seeds for formation of galaxies. The general relativistic treatment of string was initiated by Letelier[4] and Stachel [5]. Letelier [4] has obtained the solution of Einstein's field equation for a cloud string with spherical, plane and cylindrical symmetry. Then in 1983, he solved Einstein's field equation for cloud massive string and obtains cosmological models in Bianchi Type-I and Kantowski-Sachs sphase time.

Tikekar and patel [6,7]. Bali and Tyagi [8] have obtained a cylindrically symmetric inhomogeneous cosmological model with electromagnetic field for perfect fluid distribution. Benerjee et al. [9] have investigated an axially symmetric Bianchi Type I string dust cosmological model in presence and absence of magnetic field. Bali et al.[10,11,12] have investigated Bianchi Type I magnetized string cosmological models. pawar et al. [13] have investigated about different aspects of plane symmetric Bulk viscous fluid string dust magnetized cosmological model in general relativity. Saha et al. [14] and Saha [15] have studied Bianchi Type I cosmological model in presence of magnetic flux in different contexts. Recently Tyagi et al. [16] investigated Bulk viscous fluid plane symmetric string dust magnetized cosmological model in general relativity.

In this paper, we have investigated Bulk viscous fluid Bianchi Type - I String cosmological model in general relativity. An equation $\xi\theta = M(constant)$, where ξ is the coefficient of bulk viscosity, θ is the scalar of expansion and a relation between metric potentials $B = A^n$ are assumed. The physical and geometrical aspects of the model are also discussed.

Metric And Field Equation

We consider Bianchi Type - I metric of the form

$$ds^2 = -dt^2 + A^2(dx^2 + dy^2) + B^2 dz^2 \quad \text{... (1)}$$

where A and B are function of time only. The energy momentum tensor for a cloud of string with bulk viscosity is

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi\theta(\rho u_i u_j + g_{ij}) \quad \text{... (2)}$$

where $\rho = \rho_\rho + \lambda$ is the rest energy density of the cloud of string with particles attached to them with ρ_ρ is the rest energy density of particle and λ is the tension density of the cloud of strings, $\theta = u^i_{;i}$ is the scalar of expansion and ξ is the coefficient of bulk viscosity. The vector u^i describes the flow velocity vector and x^i represent a direction of anisotropy i.e. the direction of strings, satisfy the standard relation.

$$u^i u_i = -x^i x_i = -1, u^i x_i = 0 \quad \text{... (3)}$$

The expression for scalar of expansion θ and shear scalar σ are

$$\theta = u^i_{;i} = \frac{2\dot{A}}{A} + \frac{\dot{B}}{B} \qquad \text{... (4)}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left(\frac{2\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2}\right) - \frac{\theta^2}{6} \qquad \text{... (5)}$$

The Einstein's field equation (in gravitational units c = 1, 8πG = 1) for a system of string

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \qquad \text{... (6)}$$

For the metric (1), Einstein's field equation's can be written as

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = \xi\theta \qquad \text{... (7)}$$

$$\frac{2\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} = \lambda + \xi\theta \qquad \text{... (8)}$$

$$\frac{\dot{A}^2}{A^2} + \frac{2\dot{A}\dot{B}}{AB} = \rho \qquad \text{... (9)}$$

Where an over dot stands for the first and double dot for the second derivative with respect to cosmic time t.

Solution Of The Field Equations

The field equations (7) to (9) are three equations in five unknowns parameters A, B, ξ, ρ and λ. In order to obtain a determinate solution, we have assumed two conditions:

$$(i)\ \xi\theta = M\ (constant) \qquad \text{… (10)}$$

i.e. the coefficient of bulk viscosity is inversely proportional to expansion (θ).

(ii) $B = A^n$ … (11)

relation between metric potentials A and B,
Where n is constant.
From equation (7), (10) and (11), we get

$$\frac{\ddot{A}}{A} + \frac{n^2}{n+1}\frac{\dot{A}^2}{A^2} = \frac{M}{n+1}$$

This leads to

$$\ddot{A} + \frac{n^2}{n+1}\frac{\dot{A}^2}{A} = \frac{M}{n+1}A \qquad \text{... (12)}$$

Let us assume

$$\dot{A} = f(A) \qquad \text{… (13)}$$

$$\ddot{A} = f\frac{df}{dA} = ff', \quad f' = \frac{df}{dA} \qquad \text{… (14)}$$

Using (13) and (14) in equation (12), we get

$$\frac{df^2}{dA} + \frac{2n^2}{n+1}\frac{f^2}{A} = \frac{2M}{n+1}A \qquad \text{... (15)}$$

After integration, we get

$$f^2 = \frac{M}{n^2+n+1}A^2 + KA^{-\left(\frac{2n^2}{n+1}\right)}$$

Where K is the constant of integration.

$$f=\left[\frac{M}{n^2+n+1}A^2+KA^{-\left(\frac{2n^2}{n+1}\right)}\right]^{\frac{1}{2}} \quad \text{... (16)}$$

$$\dot{A}=\left[\frac{M}{n^2+n+1}A^2+KA^{-\left(\frac{2n^2}{n+1}\right)}\right]^{\frac{1}{2}} \quad \text{... (17)}$$

Equation (17) leads to

$$dt=\left[\frac{M}{n^2+n+1}A^2+KA^{-\left(\frac{2n^2}{n+1}\right)}\right]^{-\frac{1}{2}}dA$$

Hence the model (1) is reduce to

$$ds^2=-\left[\frac{M}{n^2+n+1}A^2+KA^{-\left(\frac{2n^2}{n+1}\right)}\right]^{-1}dA^2+A^2\left(dx^2+dy^2\right)+A^{2n}dz^2 \quad \text{... (18)}$$

After using a suitable transformation of coordinates the model (18) reduce to

$$ds^2=-\left[\frac{M}{n^2+n+1}T^2+KT^{-\left(\frac{2n^2}{n+1}\right)}\right]^{-1}dT^2+T^2\left(dX^2+dY^2\right)+T^{2n}dZ^2 \quad \text{... (19)}$$

Where A=T, x = X, y = Y, z = Z

Some Physical And Geometrical Aspects Of The Model

For the model of equation (19), the other physical and geometrical parameters can be easily obtained. The energy densityρ, the string tension density λ, the scalar of expansion θ, the coefficient of bulk viscosity ξ, and the shear scalar σ are respectively given by

$$\rho=(2n+1)\left[\frac{M}{n^2+n+1}+KT^{-2\left(\frac{n^2+n+1}{n+1}\right)}\right] \quad \text{... (20)}$$

$$\lambda=\left[\frac{M\left(2-n-n^2\right)}{n^2+n+1}+\frac{k\left(n+1-2n^2\right)}{n+1}T^{-2\left(\frac{n^2+n+1}{n+1}\right)}\right] \quad \text{... (21)}$$

$$\theta=(n+2)\left[\frac{M}{n^2+n+1}+KT^{-2\left(\frac{n^2+n+1}{n+1}\right)}\right]^{\frac{1}{2}} \quad \text{... (22)}$$

$$\xi=\frac{M}{(n+2)}\left[\frac{M}{n^2+n+1}+KT^{-2\left(\frac{n^2+n+1}{n+1}\right)}\right]^{-\frac{1}{2}} \quad \text{... (23)}$$

$$\sigma=\frac{(n-1)}{\sqrt{6}}\left[\frac{M}{n^2+n+1}+KT^{-2\left(\frac{n^2+n+1}{n+1}\right)}\right]^{\frac{1}{2}} \quad \text{... (24)}$$

$$\frac{\sigma}{\theta}=\frac{(n-1)}{\sqrt{6}(n+2)}=consant \qquad \text{... (25)}$$

In the absence of bulk viscosity i.e. when M→0,then line element (1) reduce to

$$ds^2=-\left[KT^{-\frac{2n^2}{n+1}}\right]^{-1}dT^2+T^2\left(dX^2+dY^2\right)+T^{2n}dZ^2 \qquad \text{… (26)}$$

The energy densityρ, the string tension densityλ, the scalar of expansionθ, the coefficient of bulk viscosityξ, and the shear scalar σ are respectively given by

$$\rho=(2n+1)\left[KT^{-2\left(\frac{n^2+n+1}{n+1}\right)}\right] \qquad \text{... (27)}$$

$$\lambda=\left[\frac{k\left(n+1-2n^2\right)}{n+1}T^{-2\left(\frac{n^2+n+1}{n+1}\right)}\right] \qquad \text{... (28)}$$

$$\theta=(n+2)\left[KT^{-2\left(\frac{n^2+n+1}{n+1}\right)}\right]^{\frac{1}{2}} \qquad \text{... (29)}$$

$$\xi=0 \qquad \text{… (30)}$$

$$\sigma=\frac{(n-1)}{\sqrt{6}}\left[KT^{-2\left(\frac{n^2+n+1}{n+1}\right)}\right]^{\frac{1}{2}} \qquad \text{... (31)}$$

$$\frac{\sigma}{\theta}=\frac{(n-1)}{\sqrt{6}(n+2)}=consant \qquad \text{... (32)}$$

II. Conclusion

The energy condition $\rho \geq 0$ in the presence of bulk viscous fluid leads to

$$(2n+1)\left[\frac{M}{n^2+n+1}+KT^{-2\left(\frac{n^2+n+1}{n+1}\right)}\right]\geq 0 \qquad \text{… (33)}$$

Thus the model (19) exists during the time span given by (33).

When $T\to\infty$, $\rho=\left[\frac{(2n+1)M}{n^2+n+1}\right]$ and $\theta=(n+2)\left[\frac{M}{n^2+n+1}\right]^{\frac{1}{2}}$

due to presence of bulk viscous fluid.

Since $Lim(T\to\infty)\frac{\sigma}{\theta}\neq 0$

Hence model does not isotropize for large value of T.

As $T \to 0$ then $\rho\to\infty$, $\theta\to\infty$, the model starts with a big bang at T=0.

The energy condition $\rho \geq 0$ for the model (26) leads to

$$\left(2n+1\right)\left[KT^{-2\left(\frac{n^2+n+1}{n+1}\right)}\right] \geq 0 \qquad \ldots (34)$$

Thus the model (26) exists during the time span given by (34). The model starts big bang at T=0 and the expansion in the model decreases as time increases $Lim\left(T \to \infty\right)\frac{\sigma}{\theta} \neq 0$,hence model does not isotropize for large value of T.

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