On Relaxed Skolem Mean Labeling For Four Star

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Abstract: *In this paper, we prove the conjecture that the four star* $K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ *is a relaxed skolem mean graph if* $|m-n| \le 8$ *for* $n = 1, 2, 3,...$ *and* $m = 1, 2, 3,....$

Keywords: *Relaxed skolem mean graph and star.*

I.   Introduction

All graphs in this chapter are finite, simple and undirected. Terms not defined here are used in the sense of Harary[10]. In **[3]**, we proved that the three star $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m-n| = 4 + \ell$ for $\ell = 1, 2, 3,...$, $m = 1, 2, 3,...$, $n = \ell + m + 4$ and $\ell \le m < n$; the three star $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m-n| > 4 + \ell$ for $\ell = 1, 2, 3,...$, $m = 1, 2, 3,...$, $n \ge \ell + m + 5$ and $\ell \le m < n$; the four star $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m-n| = 4 + 2\ell$ for $\ell = 2, 3, 4,...$, $m = 2, 3, 4, ...$, $n = 2\ell + m + 4$ and $\ell \le m < n$; the four star $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m-n| > 4 + 2\ell$ for $\ell = 2, 3, 4,...$, $m = 2, 3, 4, ...$, $n \ge 2\ell + m + 5$ and $\ell \le m < n$; the four star $K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m-n| = 7$ for $m = 1, 2, 3,...$, $n = m + 7$ and $1 \le m < n$. Also, the four star $K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m-n| > 7$ for $m = 1, 2, 3,...$, $n \ge m + 8$ and $1 \le m < n$. In **[5]**, If $\ell \le m < n$, the three star $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a relaxed skolem mean graph if $|m-n| \le 6 + \ell$ for $\ell = 1, 2, 3,...; m = 1, 2, 3,...$ and $\ell + m \le n \le \ell + m + 6$. Also, if $\ell \le m < n$, the four star $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a relaxed skolem mean graph if $|m-n| \le 2\ell + 6$ for $\ell = 2, 3, 4, . . .; m = 2, 3, 4 . . .$ and $2\ell + m \le n \le 2\ell + m + 6$. In **[4]**, the necessary condition for a graph to be relaxed skolem mean is that $p \ge q$.

II.   Relaxed Skolem Mean Labeling

Definition 2.1 : The four star is the disjoint union of $K_{1,a}$, $K_{1,b}$, $K_{1,c}$ and $K_{1,d}$. It is denoted by $K_{1,a} \cup K_{1,b} \cup K_{1,c} \cup K_{1,d}$.

Definition 2.2 [4] : A graph $G = (V, E)$ with p vertices and q edges is said to be a skolem mean graph if there exists a function f from the vertex set of G to $\{1, 2, 3,..., p+1\}$ such that the induced map f* from the edge set of G to $\{2, 3, 4,..., p+1\}$ defined by

$$f^*(e=uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if f(u)+f(v) is even} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if f(u)+f(v) is odd, then} \end{cases}$$

the resulting edges get distinct labels from the set $\{2, 3, 4,..., p+1\}$.

Note 2.3: In a relaxed skolem mean graph, $p \ge q$.

Theorem 2.4: The four star $K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ is a relaxed skolem mean graph if $|m-n| \le 8$ for $n = 1, 2, 3, ...$ and $m = 1, 2, 3, ...$.

Proof:

Without loss of generality assume that $m \le n$. Hence $|m-n| \le 8$ implies $n - m \le 8$ and it means $1 \le m \le n \le m + 8$. That is, there are nine cases viz. $n = m + 8$, $n = m + 7$, $n = m + 6$, $n = m + 5$, $n = m + 4$, $n = m + 3$, $n = m + 2$, $n = m + 1$, and $n = m$. Let us prove in each of the cases the graph $G = K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ is a relaxed skolem mean graph.

Case (a): Let $G = K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ when $n = m + 8$.

Let $V(G) = \{u, u_1, v, v_1, w, x\} \cup \{w_i : 1 \le i \le m\} \cup \{x_i : 1 \le i \le n = m + 8\}$ and

$E(G) = \{uu_1, vv_1\} \cup \{ww_i : 1 \le i \le m\} \cup \{xx_i : 1 \le i \le n = m + 8\}$ be respectively the vertex set and edge set of G. G has $m + n + 6 = 2m + 14$ vertices and $m + n + 2 = 2m + 10$ edges.

The vertex labeling $f : V(G) \to \{1, 2, ..., 2m + 15\}$ is defined as follows:

$$f(u) = 1; \quad f(v) = 5;$$

$$f(w) = 3; \quad f(x) = 2m + 14;$$

$$f(u_1) = 11; \quad f(v_1) = 9;$$
$$f(w_i) = 2i + 11 \quad \text{for} \quad 1 \le i \le m;$$

$$f(x_i) = 2i \quad \text{for} \quad 1 \le i \le m + 6$$

$$f(x_{m+7}) = 2m + 13$$

$$f(x_{m+8}) = 2m + 15$$

The corresponding edge labels are as follows:

The edge label of uu_1 is 6; vv_1 is 7; (edge labels are 6, 7); ww_i is $i + 7$ for $1 \le i \le m$ (edge labels are 8, 9, …, m + 7) and xx_i is $m + 7 + i$ for $1 \le j \le m + 6$; xx_{m+7} is 2m + 14 and xx_{m+8} is 2m + 15 (edge labels are m + 8, m + 9, …, 2m + 13, 2m +14, 2m + 15). The edge labels hence are 6, 7, 8, …, m + 7, m + 8, …, 2m + 13, 2m +14, 2m + 15.

Hence the induced edge labels of G are distinct.

Hence the graph G is relaxed skolem mean graph.

Case (b): Let $G = K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ when $n = m + 7$

Let $V(G) = \{u, u_1, v, v_1, w, x\} \cup \{w_i : 1 \le i \le m\} \cup \{x_i : 1 \le i \le n = m + 7\}$ and

$E(G) = \{uu_1, vv_1\} \cup \{ww_i : 1 \le i \le m\} \cup \{xx_i : 1 \le i \le n = m + 7\}$ be respectively the vertex set and edge set of G. G has $m + n + 6 = 2m + 13$ vertices and $m + n + 2 = 2m + 9$ edges.

The vertex labeling $f : V(G) \to \{1, 2, ..., 2m + 14\}$ is defined as follows:

$$f(u) = 1; \quad f(v) = 2;$$

$$f(w) = 4; \quad f(x) = 2m + 13;$$

$$f(u_1) = 6; \quad f(v_1) = 8;$$

$$f\left(w_i\right) = 2i+8 \quad \text{for } 1 \le i \le m;$$

$$f\left(x_i\right) = 2i+1 \quad \text{for } 1 \le i \le m+5$$

$$f\left(x_{m+6}\right) = 2m+12$$

$$f\left(x_{m+7}\right) = 2m+14$$

The corresponding edge labels are as follows:

The edge label of uu_1 is 4; vv_1 is 5; (edge labels are 4, 5); ww_i is $i+6$ for $1 \le i \le m$ (edge labels are 7, 8, …, m + 6) and xx_i is $m+7+i$ for $1 \le j \le m+5$; xx_{m+6} is 2m + 13 and xx_{m+7} is 2m + 14 (edge labels are m + 8, m + 9, …, 2m + 12, 2m +13, 2m + 14). The edge labels hence are 4, 5, 7, 8, …, m + 6, m + 8, m + 9, …, 2m + 12, 2m + 13, 2m + 14.
Hence the induced edge labels of G are distinct.
Hence the graph G is relaxed skolem mean graph.

Case (c): Let $G = K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ when n = m + 6.

Let $V(G) = \left\{u, u_1, v, v_1, w, x\right\} \cup \left\{w_i : 1 \le i \le m\right\} \cup \left\{x_i : 1 \le i \le n = m+6\right\}$ and
$E(G) = \left\{uu_1, vv_1\right\} \cup \left\{ww_i : 1 \le i \le m\right\} \cup \left\{xx_i : 1 \le i \le n = m+6\right\}$ be respectively the vertex set and edge set of G. G has $m+n+6 = 2m+12$ vertices and $m+n+2 = 2m+8$ edges.

The vertex labeling $f : V(G) \to \left\{1, 2,..., 2m+13\right\}$ is defined as follows:

$$f(u) = 2; \quad f(v) = 3;$$

$$f(w) = 5; \quad f(x) = 2m+12;$$

$$f(u_1) = 7; \quad f(v_1) = 9;$$

$$f\left(w_i\right) = 2i+9 \quad \text{for } 1 \le i \le m;$$

$$f\left(x_i\right) = 2i+2 \quad \text{for } 1 \le i \le m+4$$

$$f\left(x_{m+5}\right) = 2m+11$$

$$f\left(x_{m+6}\right) = 2m+13$$

The corresponding edge labels are as follows:

The edge label of uu_1 is 5; vv_1 is 6; (edge labels are 5, 6); ww_i is $i+7$ for $1 \le i \le m$ (edge labels are 8, 9, …, m + 7) and xx_i is $m+7+i$ for $1 \le j \le m+4$; xx_{m+5} is 2m + 12 and xx_{m+6} is 2m + 13(edge labels are m + 8, m + 9, …, 2m + 11, 2m +12, 2m + 13). The edge labels hence are 5, 6, 8, 9, …, m + 7, m + 8, m + 9, …, 2m + 11, 2m +12, 2m + 13.
Hence the induced edge labels of G are distinct.
Hence the graph G is relaxed skolem mean graph.

Case (d): Let $G = K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ when n = m + 5.

Let $V(G) = \left\{u, u_1, v, v_1, w, x\right\} \cup \left\{w_i : 1 \le i \le m\right\} \cup \left\{x_i : 1 \le i \le n = m+5\right\}$ and
$E(G) = \left\{uu_1, vv_1\right\} \cup \left\{ww_i : 1 \le i \le m\right\} \cup \left\{xx_i : 1 \le i \le n = m+5\right\}$ be respectively the vertex set and edge set of G. G has $m+n+6 = 2m+11$ vertices and $m+n+2 = 2m+7$ edges.

The labeling $f : V(G) \to \{1, 2, ..., 2m+12\}$ is defined as follows:

$$f(u) = 3; \quad f(v) = 4;$$

$$f(w) = 6; \quad f(x) = 2m+11;$$

$$f(u_1) = 8; \quad f(v_1) = 10;$$

$$f(w_i) \quad = 2i+8 \quad \text{for } 1 \le i \le m;$$

$$f(x_i) \quad = 2i+3 \quad \text{for } 1 \le i \le m+3$$

$$f(x_{m+4}) = 2m+10$$

$$f(x_{m+5}) = 2m+12$$

The corresponding edge labels are as follows:

The edge label of uu_1 is 6; vv_1 is 7; (edge labels are 6, 7); ww_i is $i+7$ for $1 \le i \le m$ (edge labels are 8, 9, …, m + 7) and xx_i is $m+7+i$ for $1 \le j \le m+3$; xx_{m+4} is 2m + 12 and xx_{m+5} is 2m + 13(edge labels are m + 8, m + 9, …, 2m + 10, 2m +11, 2m + 12). The edge labels hence are 5, 6, 8, 9, …, m + 7, m + 8, m + 9, …, 2m + 10, 2m +11, 2m + 12.

Hence the induced edge labels of G are distinct.

Hence the graph G is relaxed skolem mean graph.

Case (e): Let $G = K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ when n = m + 4.

Let $V(G) = \{u, u_1, v, v_1, w, x\} \cup \{w_i : 1 \le i \le m\} \cup \{x_i : 1 \le i \le n = m+4\}$ and

$E(G) = \{uu_1, vv_1\} \cup \{ww_i : 1 \le i \le m\} \cup \{xx_i : 1 \le i \le n = m+4\}$ be respectively the vertex set and edge set of G. G has $m+n+6 = 2m+10$ vertices and $m+n+2 = 2m+6$ edges.

The vertex labeling $f : V(G) \to \{1, 2, ..., 2m+11\}$ is defined as follows:

$$f(u) = 1; \quad f(v) = 2;$$

$$f(w) = 4; \quad f(x) = 2m+11;$$

$$f(u_1) = 6; \quad f(v_1) = 8;$$

$$f(w_i) \quad = 2i+8 \quad \text{for } 1 \le i \le m;$$

$$f(x_i) \quad = 2i+3 \quad \text{for } 1 \le i \le m+3$$

$$f(x_{m+4}) = 2m+10$$

The corresponding edge labels are as follows:

The edge label of uu_1 is 4; vv_1 is 5; (edge labels are 4, 5); ww_i is $i+6$ for $1 \le i \le m$ (edge labels are 7, 8, …, m + 6) and xx_i is $m+7+i$ for $1 \le j \le m+3$; xx_{m+4} is 2m + 11 (edge labels are m + 8, m + 9, …, 2m + 10, 2m+11). The edge labels hence are 4, 5, 7, 8, …, m + 8, m + 9, …, 2m + 10, 2m +11.

Hence the induced edge labels of G are distinct.

Hence the graph G is relaxed skolem mean graph.

Case (f): Let $G = K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ when $n = m + 3$.

Let $V(G) = \{u, u_1, v, v_1, w, x\} \cup \{w_i : 1 \le i \le m\} \cup \{x_i : 1 \le i \le n = m + 3\}$ and

$E(G) = \{uu_1, vv_1\} \cup \{ww_i : 1 \le i \le m\} \cup \{xx_i : 1 \le i \le n = m + 3\}$ be respectively the vertex set and edge set of G. G has $m + n + 6 = 2m + 9$ vertices and $m + n + 2 = 2m + 5$ edges.

The vertex labeling $f : V(G) \rightarrow \{1, 2, ..., 2m + 10\}$ is defined as follows:

$$f(u) = 2; \quad f(v) = 4;$$

$$f(w) = 5; \quad f(x) = 2m + 10;$$

$$f(u_1) = 3; \quad f(v_1) = 7;$$

$$f(w_i) = 2i + 7 \quad \text{for } 1 \le i \le m;$$

$$f(x_i) = 2i + 4 \quad \text{for } 1 \le i \le m + 2$$

$$f(x_{m+3}) = 2m + 9$$

The corresponding edge labels are as follows:

The edge label of uu_1 is 3; vv_1 is 6; (edge labels are 3, 6); ww_i is $i + 6$ for $1 \le i \le m$ (edge labels are 7, 8, …, m + 6) and xx_i is $m + 7 + i$ for $1 \le j \le m + 2$; xx_{m+3} is 2m + 10 (edge labels are m + 8, m + 9, …, 2m + 9, 2m +10). The edge labels hence are 3, 6, 7, 8, …, m + 8,   m + 9, …, 2m + 9, 2m +10.
Hence the induced edge labels of G are distinct.
Hence the graph G is relaxed skolem mean graph.

Case (g): Let $G = K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ when $n = m + 2$.

Let $V(G) = \{u, u_1, v, v_1, w, x\} \cup \{w_i : 1 \le i \le m\} \cup \{x_i : 1 \le i \le n = m + 2\}$ and

$E(G) = \{uu_1, vv_1\} \cup \{ww_i : 1 \le i \le m\} \cup \{xx_i : 1 \le i \le n = m + 2\}$ be respectively the vertex set and edge set of G. G has $m + n + 6 = 2m + 8$ vertices and $m + n + 2 = 2m + 4$ edges.

The vertex labeling $f : V(G) \rightarrow \{1, 2, ..., 2m + 9\}$ is defined as follows:

$$f(u) = 2; \quad f(v) = 4;$$

$$f(w) = 5; \quad f(x) = 2m + 9;$$

$$f(u_1) = 3; \quad f(v_1) = 7;$$
$$f(w_i) = 2i + 7 \quad \text{for } 1 \le i \le m;$$

$$f(x_i) = 2i + 4 \quad \text{for } 1 \le i \le m + 2$$

The corresponding edge labels are as follows:

The edge label of uu_1 is 3; vv_1 is 6; (edge labels are 3, 6); ww_i is $i + 6$ for $1 \le i \le m$ (edge labels are 7, 8, 9, …, m + 6) and xx_i is $m + 7 + i$ for $1 \le j \le m + 2$ (edge labels are m + 8, m + 9, …, 2m + 9). The edge labels hence are 3, 6, 7, 8, …, m + 6, m + 8, m + 9, …, 2m + 9.
Hence the induced edge labels of G are distinct.
Hence the graph G is a relaxed skolem mean graph.

Case (h): Let $G = K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ when $n = m + 1$.

Let $V(G) = \{u, u_1, v, v_1, w, x\} \cup \{w_i : 1 \le i \le m\} \cup \{x_i : 1 \le i \le n = m + 1\}$ and

$E(G) = \{uu_1, vv_1\} \cup \{ww_i : 1 \le i \le m\} \cup \{xx_i : 1 \le i \le n = m + 1\}$ be respectively the vertex set and edge set of G. G has $m + n + 6 = 2m + 7$ vertices and $m + n + 2 = 2m + 3$ edges.

The vertex labeling $f : V(G) \to \{1, 2, ..., 2m + 8\}$ is defined as follows:

$$f(u) = 2; \quad f(v) = 4;$$

$$f(w) = 5; \quad f(x) = 2m + 8;$$

$$f(u_1) = 3; \quad f(v_1) = 6;$$

$$f(w_i) = 2i + 6 \quad \text{for } 1 \le i \le m;$$

$$f(x_i) = 2i + 5 \quad \text{for } 1 \le i \le m + 1$$

The corresponding edge labels are as follows:

The edge label of uu_1 is 3; vv_1 is 5; (edge labels are 3, 5); ww_i is $i + 6$ for $1 \le i \le m$ (edge labels are 7, 8, 9, …, m + 6) and xx_i is $m + 7 + i$ for $1 \le j \le m + 1$ (edge labels are m + 8, m + 9, …, 2m + 8). The edge labels hence are 3, 5, 7, 8, 9, …, m + 6, m + 8, m + 9, …, 2m + 8.
Hence the induced edge labels of G are distinct.
Hence the graph G is a relaxed skolem mean graph.

Case (i): Let $G = K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ when $n = m$.

Let $V(G) = \{u, u_1, v, v_1, w, x\} \cup \{w_i : 1 \le i \le m\} \cup \{x_i : 1 \le i \le n = m\}$ and

$E(G) = \{uu_1, vv_1\} \cup \{ww_i : 1 \le i \le m\} \cup \{xx_i : 1 \le i \le n = m\}$ be respectively the vertex set and edge set of G. G has $m + n + 6 = 2m + 6$ vertices and $m + n + 2 = 2m + 2$ edges.

The vertex labeling $f : V(G) \to \{1, 2, ..., 2m + 7\}$ is defined as follows:

$$f(u) = 2; \quad f(v) = 4;$$

$$f(w) = 6; \quad f(x) = 2m + 7;$$

$$f(u_1) = 3; \quad f(v_1) = 5;$$

$$f(w_i) = 2i + 5 \quad \text{for } 1 \le i \le m;$$

$$f(x_i) = 2i + 6 \quad \text{for } 1 \le i \le m.$$

The corresponding edge labels are as follows:

The edge label of uu_1 is 3; vv_1 is 5; (edge labels are 3, 5); ww_i is $i + 6$ for $1 \le i \le m$ (edge labels are 7, 8, …, m + 6) and xx_i is $m + 7 + i$ for $1 \le j \le m$ (edge labels are m + 8, m + 9, …, 2m + 7). The edge labels hence are 3, 5, 7, 8,…, m + 6, m + 8, m + 9, …, 2m + 7.
Hence the induced edge labels of G are distinct.
Hence the graph G is relaxed skolem mean graph.

Application of Graph Labeling in Communication Networks

The Graph Theory plays a vital role in various fields. One of the important area is Graph (Relaxed Skolem mean) Labeling, used in many applications like coding theory, X - ray crystallography, radar, astronomy, circuit design, communication network addressing and data base management. Applications of labeling (Relaxed Skolem Mean) of graphs extends to heterogeneous fields but here we mainly focus on the communication networks. Communication network is of two types 'Wired Communication' and 'Wireless Communication'. Day by day wireless networks have been developed to ease communication between any two systems, results more efficient communication. To explore the role of labeling in expanding the utility of this channel assignment process in communication networks. Also, graph labeling has been observed and identified its usage towards communication networks. We address how the concept of graph labeling can be applied to network security, network addressing, channel assignment process and social networks.

Network representations play an important role in many domains of computer science, ranging from data structures and graph algorithms, to parallel and communication networks. Geometric representation of the graph structure imposed on these data sets provides a powerful aid to visualizing and understanding the data. The graph labeling is one of the most widely used labeling methods of graphs. While the labeling of graphs is perceived to be a primarily theoretical subject in the field of graph theory and discrete mathematics, it serves as models in a wide range of applications as listed below.

The coding theory.
The x-ray crystallography.
The communication network addressing.
 Fast Communication in Sensor Networks Using Graph Labeling.
Automatic Channel Allocation for Small Wireless Local Area Network.
Graph Labeling in Communication Relevant to Adhoc Networks.
Effective Communication in Social Networks by Using Graphs.
Secure Communication in Graphs.

III.   Conclusion

Researchers may get some information related to graph labeling and its applications in communication field and work on some ideas related to their field of research.

Acknowledgement

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