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Statistical Modeling of the Kenyan Secondary School Students Enrollment: An Application of the Markov Chain Model

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Abstract: *This paper models the Kenyan Secondary school students' enrollment data using Markov chain model. The 2011 student cohort is observed from time of entry to the time of graduating from the system after the expected four years in Kenya's secondary school level of education. The cohort formed the study target population. The model was used to determine the Country's secondary school completion and dropout rate, retention rate and, the expected duration of schooling by sex. Secondary data from the 2015 Statistical Abstract was used. It was established that completion rates for male students was higher than that of female students and dropout rates for female students was higher than that of male students. The study also revealed that 11 students out of 100 of the students who join secondary schools in Kenya do not complete the secondary education level of schooling. Female students had lower expectation years of schooling compared to male students in Kenya. The study recommended further investigations on the causes of gender disparity in completion rates besides efforts to close the gap.*

Keywords: *Absorbing States, Absorbing Markov Chain, Transition Rates, Dropout Rates, Completion Rates, Fundamental Matrix*

I. Introduction

Mose J. N., Odhiambo E. A., Ojunga O. S., & Prof. Onyango F., 2014 noted that education is widely valued as a central factor in economic, social, and political development of any country. Secondary school education provides a vital link between basic education and further training in tertiary level of education on one hand, and the future world of work, on the other. It is therefore an important sub-sector of education in the preparation of human capital for development and provision of life opportunities. For quality and equality in education to be realized, an understanding of the secondary school students' completion rates, dropout rates, retention rates per class and the expected duration of schooling by sex is needed for planning and proper decision making.

The Markov chain model has been widely used in different fields including education. Mostly it has been applied in a single school, a university or a college but little has been done in applying it to a region. Mose J. N. *et al,* 2014 demonstrated the use of Markov Chain Model to study progress of secondary school students by sex in Kisii Central District, Kisii County, Kenya. According to Musiga, Owino and Weke, 2010 and 2011, education system is comparable to a hierarchical organization in which after an academic year, three possibilities arise in the new status of the students; the student may move to the next higher class, may repeat the same class, or may leave the system successfully as graduate or dropout of the system before attaining the maximum qualification.

In this paper the Kenyan secondary school system is modeled using the Markov chain approach in which proportions of students who leave the system either successfully or dropout of the system are separately grouped into double absorbing states. The education system was partitioned into two states; non-absorbing (transient) states which corresponded to the various classes within the education system and absorbing states which also corresponded to the group of all successful graduates, repeaters and dropouts. Thus students enter permanent states; absorbing states, either as graduates or dropouts. Based on the data from the 2015 Statistical Abstract of Kenya, secondary school retention rates, completion rates, dropout rates, expected duration of schooling by sex in Kenya Secondary school level of education were established.

The findings will help the country's education stakeholders in the distribution of resources to secondary schools when year of study is put into consideration and, planning and budgeting for the expected future enrollment for quality and equality in education. Equally the model will be used to predict number of dropouts by sex at each class of study which will assist interested parties to make informed decision about secondary education management in view to curb dropouts and reduce the gender disparity in secondary schools.

1.1Objectives of the Study

This study sought to model the Kenyan Secondary Education enrollment data using Markov Chain

Model. This was an extension of the study carried out by Mose J. N. *et al*, 2014 that focused on data at district level. Specific objectives were;

1. To establish the secondary school completion rate by sex;
2. To determine the expected duration of schooling by sex;
3. To determine the dropout rate by sex;
4. To determine secondary school retention rates by sex.

1.2 Assumptions of the Study

The following assumptions as modified from Mose J. N. *et al*, 2014's study were taken into consideration for the model to be appropriate;

- The study population is assumed closed
- Admissions takes place only in form one.
- Dropouts are assumed to be uniformly distributed in the period ($x; x + 1$), x being the year of study.
- No class repetition.
- Transition from primary to secondary level of education in Kenya is assumed constant.

II. Literature Review

Over the years Markov chains have been widely used to model stochastic processes and to evaluate time to event data. Musiga *et al.*, 2010 modeled a hierarchical system with a single absorbing state for an education system where dropouts and graduates were grouped together. Also Musiga *et al.*, 2011, modeled a hierarchical system with double absorbing states for an education system, where graduates were separated from dropouts. Mose *et al*, 2014 basing his argument on the two studies used the Markov Chain model to model secondary school enrollment data at district level. In his study, Mose *et al* 2014, generalized the application of the model to the group of institutions in a district to be specific and took into consideration sex disaggregation of the data. This study therefore models the entire Kenyan secondary education data obtained from the 2015 Statistical Abstract using the Markov Chain approach.

III. Model Development

The model is based on the stochastic process principles in which a system in one state, say, s_i moves to a subsequent state, say, s_j without necessarily depending on the prior states. This movement is referred to a transition or a step from the "Current state" to the "Next state".

We develop a model with N states, $N=t+r$ where t being non-absorbing states and r absorbing states. Non-absorbing states include classes of study while absorbing corresponds to the various education system exit factors.

Note that;

- transition probabilities between absorbing states should be represented by one, hence the use of identity matrix.
- transition from an absorbing state to a non-absorbing state which is impossible, is represented by zero, hence the matrix of zeroes.
- transition from non-absorbing states to absorbing states are possible, likewise transitions between non-absorbing states.

3.1 Absorbing Markov Chain

Definition: A Markov Chain is absorbing if it has at least one absorbing state and if from every state it is possible to go to an absorbing state (not necessarily in one step). In an absorbing Markov Chain, a state which is not absorbing is called transient. If we have an absorbing Markov chain with t transient states and r absorbing states, the transition probability matrix $\boldsymbol{P}$, will take the following canonical form;

$$\boldsymbol{P}=\begin{bmatrix} \boldsymbol{Q} & \boldsymbol{R} \\ \boldsymbol{O} & \boldsymbol{I} \end{bmatrix}$$

Where;

$\boldsymbol{Q}$ is a $t \times t$ matrix, q_{ij} being the probability that a student who is in class i at time $(t-1)$ will be in class j at time $t; i, j = 1, 2, 3 \ldots, t;$

$\boldsymbol{R}$ is a non-zero $t \times r$ matrix, r_{ik} being the probability that a student in class i at time $(t-1)$ will graduate with final education k at time $t; i = 1, 2, 3 \ldots, t \;\&\; k = 1, 2, 3, \ldots, r;$

$\boldsymbol{O}$ is an $r \times t$ zero matrix and;

$\boldsymbol{I}$ is an $r \times r$ identity matrix.

The first t states are transient states and the last r states are absorbing states (Beck and Pauker, 1983).

The ij^{th} entry, $\boldsymbol{P}_{ij}^{n}$ of the matrix $\boldsymbol{P}^n$ gives the probability that the Markov chain, starting in state s_i will be in state s_j after n steps by Chapman-Kolmogorov theorem. The canonical form of the matrix $\boldsymbol{P}^n$ is given as;

$$\boldsymbol{P^n}=\begin{bmatrix} \boldsymbol{Q^n} & \boldsymbol{R^n} \\ \mathbf{0} & \boldsymbol{I} \end{bmatrix}$$

Where;

- $\boldsymbol{Q^n}$ is a $t \times t$ matrix which gives the probability that a student who is in class i will be in class j, n years later; $i; j = 1, 2, 3\ldots, t,$
- $\boldsymbol{R^n} = (\boldsymbol{I}+\boldsymbol{Q}+\boldsymbol{Q}^2+\ldots\ldots+\boldsymbol{Q}^{n-1})\boldsymbol{R}$ is a $t \times r$ matrix which gives the probability that a student who is in class i will graduate with final education k within n years, $i = 1, 2\ldots, t; \ k = 1, \ 2\ldots, r.$ It is also called the completion rate,
- is an $r \times t$ matrix of zeros which gives transition probabilities from absorbing states to non absorbing states in n steps and,
- $\boldsymbol{I}$ is an $r \times r$ identity matrix which gives transition probabilities between absorbing states in n steps.

The probability matrix summarizes transition probability of the cohort while the transient states analysis allows prediction or prognosis for an individual subject, given their starting state, current state and cycle (Urakabe *et al*, 1986 and Silverstein *et al*, 1988).

3.2 The Fundamental matrix

In an absorbing Markov chain model, the matrix $\boldsymbol{N}$ is the fundamental matrix where;

$$\boldsymbol{N}= (\boldsymbol{I}-\boldsymbol{Q})^{-1} = \boldsymbol{I}+\boldsymbol{Q}+\boldsymbol{Q}^2+\ldots$$

with entries n_{ij} being the expected number of times the process is in state s_j given that it started in the transient state s_i. The fundamental matrix gives the average number of cycles that a subject resides in transient states before absorption, given a specified starting state.

The transition matrix, $\boldsymbol{P}=((P_{ij}));\ i,j= 1, 2, \ldots, N$ with p_{ij} being the probability that a student in class i at time t-1 will be in class j at time t.

In this study the non-absorbing states (transient states) were four represented by values *1, 2, 3,* and *4* hence the $\boldsymbol{Q}$ component of the transition matrix $\boldsymbol{P}$ is a 4×4 matrix.

The absorbing states were two represented by states *5* and *6*.

The absorbing state *5* represents graduation from the system after attaining the maximum qualification and state *6* represents dropping out of the system before attaining the maximum qualification. Hence the $\boldsymbol{R}$ component of matrix $\boldsymbol{P}$ was a 4×2 matrix.

The purpose of the transition matrix is to represent the probability of movement between states in a single time period. In this case, it was the probability that a student will reach a particular state by the end of the year of study.

3.3 Initial Transition Matrix

The initial transition matrix has probabilities $\mathbf{P}_{ij}= n_{ij}(t)/n_i(t-1)$, where; *i, j = 1, 2…, t: assuming the multinomial distribution*

This was the proportion of students who were in class i at time $(t-1)$ who ended up being in class j at time t.

3.4 The n-step Transition Matrix

The *n*-step transition probability matrix takes the canonical form below as per Beck and Pauker, 1983, Musiga *et al.*, 2011, and Mose *et al.*, 2014. This is proved by use of the Chapman Kolmogorov theorem;

$$\boldsymbol{P^n}=\begin{bmatrix} \boldsymbol{Q^n} & \boldsymbol{R^n} \\ \mathbf{0} & \boldsymbol{I} \end{bmatrix}$$

The solution to this *n*-step transition matrix gives the state of a student *n*-steps (years later). The elements of the *n*-step transition probability matrix represents the probabilities that an object in a given state will be in the next state *n*-steps later.

3.5 Completion rates

Musiga *et al.*, 2011 defined the dropout rate from class *i, n* years later by;

$$r_{ik}^{(n)} = \sum_{j=1}^{s} q_{ij}^{(n-1)} r_{ik}, \ \ i, j = 1; 2\ldots, s$$

Where;

$p_{ij}^{(n-1)}$ is the probability that a student in class i will be in class j, n- 1 years later and r_{ik} is the probability that a student in class i at time $t - 1$ graduates with final education k at time t. It is the $(i, k)^{th}$ element of the product $\boldsymbol{Q^{n-1}R}$.

Therefore the cumulative dropout rate within y years from class i will be given by;

$r_{ik}^{(y)} = \sum_{n=1}^{y} r_{ik}^{(n)}$ $i=1,2,\ldots,t$ and $k=1, 2, \ldots, r.$

Where, $r_{ik}^{(y)}$ is the $(i,k)^{th}$ element of $(\boldsymbol{I}+\boldsymbol{Q}+\boldsymbol{Q}^2+\ldots\ldots+\boldsymbol{Q}^{y-1})\boldsymbol{R}$.

3.6 Absorbing rates

Assuming that students will remain in the system indefinitely, then the absorbing rate is given by;

$$r_{i1}^{(\infty)} = \sum_{n=1}^{\infty} r_{i1}^{(n)}$$
$$= (\mathbf{I}+\mathbf{Q}+\mathbf{Q}^2+\ldots\ldots)\mathbf{R}$$

$$= (\mathbf{I}-\mathbf{Q})^{-1}\mathbf{R}$$

The solution to this gives the absorbing rate under double absorbing states system (Uche, 1980, Musiga *et al.*, 2011, and Mose *et al*, 2014).

3.7 Retention rates

According to (Dworkin, 2005 and Mose *et al.*, 2014), retention rates are of interest not only to learning institutions hopeful of maintaining or increasing enrollment, but act as a social economic indicator of well being for the community as a whole. It is therefore, considered to be in the best public interest to maintain high retention rates.

IV. Model Fitting

4.1 Initial transition Probabilities

From Mose *et al.*,2014 publication $n_{ij}(t)$ represent the number of students in class i at time $(t - 1)$ who will be in class j at time t, and $n_i(t-1)$ represents the number of students in class i at time $(t - 1)$. By assuming the multinomial distribution, the transition probabilities can be estimated by;

$$P_{ij}= n_{ij}(t)/ n_i(t-1),\ i,j= 1, 2, .., N$$

P_{ij} is the proportion of students who are in class i at time $(t- 1)$ who ends up being in class j at time t.

4.1.1 Important Notations

Some of the notations used in the subsequent sections are defined below;

- subscript ***a*** represents total student enrollment
- subscript ***m*** represents male students
- subscript ***f*** represents female students

Such that $\boldsymbol{P}_a$ =Secondary school transition probability matrix, $\boldsymbol{P}_m$ =male students transition probability matrix, and $\boldsymbol{P}_f$ =female students transition probability matrix, etc

4.2 The Initial Secondary School Transition Probability Matrix

The Secondary school enrollment data in Form I through Form IV for the years 2011 to 2014 were extracted from the Kenya National Bureau of Statistics, 2015 Statistical Abstract and tabulated as shown in Table I.

The initial dropout proportions for Forms *1, 2, 3* and *4* were computed as follows; *(7,663/521,601)=0.014691, (17,848=513,938) = 0.034728, (29,391/496,090)=0.059245 and (2,975/466,699)=0.006375,* respectively. These proportions and the proportion that graduated successfully from the system forms the ***R*** component of the initial Secondary school transition matrix ***P***.

The initial transition rates for forms 1, 2, 3, and 4 will therefore be given as 1-0.014691=0.985309, 0.965272, 0.940755, and 0.993625 respectively which forms the ***Q*** component of the ***P*** matrix whose states are transient states.

Thus, assuming time homogeneity, the initial Kenyan secondary school transition probability matrix $\boldsymbol{P_a}$, is;

$$\boldsymbol{P}_a=\begin{bmatrix} 0.000000 & 0.985309 & 0.000000 & 0.000000 & 0.000000 & 0.014691 \\ 0.000000 & 0.000000 & 0.965272 & 0.000000 & 0.000000 & 0.034728 \\ 0.000000 & 0.000000 & 0.000000 & 0.940755 & 0.000000 & 0.059245 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.993625 & 0.006375 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 1.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

Implying that the ***Q*** component is;

$$\boldsymbol{Q}_a=\begin{bmatrix} 0.000000 & 0.985309 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.965272 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 & 0.940755 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 \end{bmatrix}$$

and the ***R*** component is;

$$\boldsymbol{R}_a=\begin{bmatrix} 0.000000 & 0.014691 \\ 0.000000 & 0.034728 \\ 0.000000 & 0.059245 \\ 0.993625 & 0.006375 \end{bmatrix}$$

4.3 The Initial Transition Probability matrix by sex

The initial transition probability matrix computed by sex yields the following matrices;

The ***P*** for male students is;

$$=\begin{bmatrix} 0.000000 & 0.989999 & 0.000000 & 0.000000 & 0.000000 & 0.010001 \\ 0.000000 & 0.000000 & 0.974566 & 0.000000 & 0.000000 & 0.025434 \\ 0.000000 & 0.000000 & 0.000000 & 0.936438 & 0.000000 & 0.063562 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.993626 & 0.006374 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 1.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

The ***Q*** component for the male students is;

$$\boldsymbol{Q}_m=\begin{bmatrix} 0.000000 & 0.989999 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.974566 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 & 0.936438 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 \end{bmatrix}$$

and the ***R*** component is;

$$\boldsymbol{R}_m=\begin{bmatrix} 0.000000 & 0.010001 \\ 0.000000 & 0.025434 \\ 0.000000 & 0.063562 \\ 0.993626 & 0.006374 \end{bmatrix}$$

Similarly, the initial transition probability matrix ***P*** for female students, with the double absorbing states, assuming time homogeneity is;

$$\boldsymbol{P}_f=\begin{bmatrix} 0.000000 & 0.979999 & 0.000000 & 0.000000 & 0.000000 & 0.020001 \\ 0.000000 & 0.000000 & 0.954643 & 0.000000 & 0.000000 & 0.045357 \\ 0.000000 & 0.000000 & 0.000000 & 0.945794 & 0.000000 & 0.054206 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.993625 & 0.006375 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 1.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

Its ***Q*** component is;

$$\boldsymbol{Q}_f=\begin{bmatrix} 0.000000 & 0.979999 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.954643 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 & 0.945794 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 \end{bmatrix}$$

and the ***R*** component is;

$$\boldsymbol{R}_f=\begin{bmatrix} 0.000000 & 0.020001 \\ 0.000000 & 0.045357 \\ 0.000000 & 0.054206 \\ 0.993625 & 0.006375 \end{bmatrix}$$

4.4 Completion rates

The secondary education system has been modeled such that there are two possible ways of getting out of the system; drop out at any given stage or complete the four year course successfully.

Completion rate at time x is the $(i, k)^{th}$ element of $(\boldsymbol{I} + \boldsymbol{Q} + \boldsymbol{Q}^2 + \ldots + \boldsymbol{Q}^{(x-1)})\boldsymbol{R}$:

Thus within one year of study the secondary school completion rate will be given by;

$$(1+\boldsymbol{Q}_a)\ \boldsymbol{R}_a=\begin{bmatrix} 0.000000 & 0.048909 \\ 0.000000 & 0.091916 \\ 0.934758 & 0.065242 \\ 0.993625 & 0.006375 \end{bmatrix}$$

Within two years it will be;

$$(\boldsymbol{I}+\boldsymbol{Q}_a+\boldsymbol{Q}_a^2)\boldsymbol{R}_a=\begin{bmatrix} 0.000000 & 0.105256 \\ 0.902295 & 0.097705 \\ 0.934758 & 0.065242 \\ 0.993625 & 0.006375 \end{bmatrix}$$

Completion rates within x years of schooling using the absorbing states model is as in Table II in the Appendix. From Table II it can be concluded that within three years of a study **90.2295%** of the students who were in Form II two years ago will graduate from the system after attaining the maximum qualification. Similarly, **93.4758%** of the students in Form 3 graduates from the system within two years of study.

Therefore, a student who joins form one in Kenya has **0.889040** chance of completing the secondary school level of education successfully.

4.5 Completion rates by sex

Computations were done by sex and yielded the following results for female students;

$$(\boldsymbol{I} + \boldsymbol{Q}_f)\boldsymbol{R}_f = \begin{bmatrix} 0.000000 & 0.064451 \\ 0.000000 & 0.097104 \\ 0.939765 & 0.060235 \\ 0.993625 & 0.006375 \end{bmatrix}$$

and for male students were;

$$(\boldsymbol{I} + \boldsymbol{Q}_m)\boldsymbol{R}_m = \begin{bmatrix} 0.000000 & 0.035181 \\ 0.000000 & 0.087379 \\ 0.930469 & 0.069531 \\ 0.993626 & 0.006374 \end{bmatrix}$$

Table II and Table III in the Appendix presents the completion rates for female and male students within x years respectively. The completion rate for female students who join secondary education level of schooling is **87.9196%** after completing successfully the four year training while that of male students is **89.77735%.** A female student in Form II has **0.897140** chance of graduating successfully from the secondary school education system compared to **0.906804** for male students.

Generally, a student in Form I has a probability of **0.110960** of not completing the secondary education level of education compared to **0.006375** of a student in Form IV. A female student in Form I has a higher probability **(0.120804)** of dropping out of the system compared to that of male counterparts **(0.102265).**

4.6 The expected duration of study

Consider the fundamental matrix $\boldsymbol{N}$ that gives the number of cycles that a subject resides in transient states before absorption, given a specified starting state.

$\boldsymbol{N} = \boldsymbol{I} + \boldsymbol{Q} + \boldsymbol{Q}^2 + \ldots = (\boldsymbol{I}-\boldsymbol{Q})^{-1}$

$$\boldsymbol{N}=(\boldsymbol{I} - \boldsymbol{Q}_a)^{-1}= \begin{bmatrix} 1.000000 & 0.985309 & 0.951091 & 0.894744 \\ 0.000000 & 1.000000 & 0.965272 & 0.908084 \\ 0.000000 & 0.000000 & 1.000000 & 0.940755 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

The expected duration of study according to Beck and Pauker, 1983 and Mose *et al*, 2014 is given by;

$$(\boldsymbol{I} - \boldsymbol{Q}_a)^{-1}(1111)^{\mathrm{T}} = \begin{bmatrix} 3.831144 \\ 2.873356 \\ 1.940755 \\ 1.000000 \end{bmatrix}$$

This implies that a student in Form I has **3.831144** years of being in the secondary education system while that in Form III has **1.940755** years of schooling.

4.7 The expected duration of study by sex

Using the same approach as above we get that **N** for male students is given as;

$$\boldsymbol{N}=(\boldsymbol{I} - \boldsymbol{Q}_m)^{-1}= \begin{bmatrix} 1.000000 & 0.989999 & 0.964819 & 0.903494 \\ 0.00000 & 1.000000 & 0.974566 & 0.912621 \\ 0.000000 & 0.000000 & 1.000000 & 0.936438 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

while that for female students is;

$$(\boldsymbol{I} - \boldsymbol{Q}_f)^{-1}= \begin{bmatrix} 1.000000 & 0.979999 & 0.935549 & 0.884837 \\ 0.00000 & 1.000000 & 0.954643 & 0.902896 \\ 0.000000 & 0.000000 & 1.000000 & 0.945794 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

The expected duration of schooling for male students is given by;

$$(\boldsymbol{I} - \boldsymbol{Q}_m)^{-1}(1111)^{\mathrm{T}}= \begin{bmatrix} 3.858312 \\ 2.887187 \\ 1.936438 \\ 1.000000 \end{bmatrix}$$

Similarly that for female students will be;

$$(\boldsymbol{I} - \boldsymbol{Q}_f)^{-1}(1111)^{\mathrm{T}} = \begin{bmatrix} 3.800385 \\ 2.857539 \\ 1.945794 \\ 1.000000 \end{bmatrix}$$

From the above findings the expected duration in secondary school for male students is generally higher compared to that of the female students except in Form III. This implies that male students have a higher chance of staying in the secondary school system compared to the female students.

4.8 Absorbing rates

A student in secondary school has two ways of exiting the system when using this model; dropping out at any level or completing the four years successfully by graduating. Thus a student may graduate from the

system or drop at any level of schooling.
The absorbing rates are given by a solution to;
(I – Q)$^{-1}$***R***
Therefore, the secondary school absorbing rates were established to be;

$$(\boldsymbol{I} - \boldsymbol{Q}_a)^{-1}\boldsymbol{R}_a == \begin{bmatrix} 0.889040 & 0.110960 \\ 0.902295 & 0.097705 \\ 0.934758 & 0.065242 \\ 0.993625 & 0.006375 \end{bmatrix}$$

This implies that in the Kenyan secondary school level of education a student in Form I has **0.889040** chance of exiting the system by completing successfully and **0.110960** chance of not completing the system. That implies that out of **100** students who join secondary level of education in Kenya, **89** complete the system while **11** do not complete.

4.9 Absorbing rates by sex

The absorbing rates for the female students are given as;

$$(\boldsymbol{I} - \boldsymbol{Q}_f)^{-1}\boldsymbol{R}_f = \begin{bmatrix} 0.879196 & 0.120804 \\ 0.897140 & 0.102860 \\ 0.939765 & 0.060235 \\ 0.993625 & 0.006375 \end{bmatrix}$$

while that for male students was;

$$(\boldsymbol{I} - \boldsymbol{Q}_m)^{-1}\boldsymbol{R}_m = \begin{bmatrix} 0.897735 & 0.102265 \\ 0.906804 & 0.093196 \\ 0.930469 & 0.069531 \\ 0.993626 & 0.006374 \end{bmatrix}$$

The exit rates for male students are higher compared to those for female students in all the years of study except in Form III where male students have **0.930469** chance of exiting the system by graduation compared to **0.939765** for the female students.

4.10 Retention rates

In this study retention rates are actually the transition probabilities. The retention rates for Forms 1, 2, 3 and 4 were **0.985309, 0.965272, 0.940755** and **0.993625** respectively. Form III had the lowest retention rate. Retention rates by sex were computed and for male students they were; **0.989999, 0.9745666, 0.936438,** and **0.993626** while those for female students were **0.979999, 0.954643, 0.945794,** and **0.993625** respectively.

V. Conclusion

The following conclusions were drawn from the above analysis and discussions;

- In every **100** students joining secondary education system in Kenya, only **88** students graduate from the system successfully;
- The completion rate for a male student joining secondary school education system is higher than that of a female student in Kenya;
- The expected duration of schooling is higher for male students compared to the female students in Kenya. This finding concurs with Mose *et al,* 2014 study in Kisii district;
- The dropout rates for female students is higher than that of male students in Kenya. However, the male students dropout rates were found to be higher than those of female students in Form III;
- Form III has the least retention rate in the Secondary school level of education in Kenya. However in the earlier study in Kisii District, Mose *et al,* 2014 found out that Form II had the least retention rate.

VI. Recommendations

From the above conclusions, the following recommendations were made;

- Sensitization should be done to close the completion gap by sex
- Research should be done to determine the causes of the gender disparity in expectation of schooling in secondary schools
- Research should be done to determine the root cause for high dropout rates in Form III.
- This model can be customized to study the school enrollment trends at County level of Government.

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Appendix: Tables

Table I: Secondary School Student Enrollment 2011-2014

Class	Enrollment			Drop outs		
	Male	Female	Total	Male	Female	Total
Form 1	276,965	244,636	521,601	2,770	4,893	7,663
Form 2	274,195	239,743	513,938	6,974	10,874	17,848
Form 3	267,221	228,869	496,090	16,985	12,406	29,391
Form 4	250,236	216,463	466,699	1,595	1,380	2,975

Source:2015 Statistical Abstract, KNBS

Table II: Secondary school completion rates within x years.

Yrs(x)	Year 1		Year 2		Year 3		Year 4	
	Completion	Drop	Completion	Drop	Completion	Drop	Completion	drop
FM 1	0	0.014691	0	0.048909	0	0.105256	0.889040	0.110960
FM 2	0	0.034728	0	0.091916	0.902295	0.097705	0.902295	0.097705
FM 3	0	0.059245	0.934758	0.065242	0.934758	0.065242	0.934758	0.065242
FM 4	0.993625	0.006375	0.993625	0.006375	0.993625	0.006375	0.993625	0.006375

Table III: Secondary School Female Students completion rates within x years.

Year(x)	Year 1		Year 2		Year 3		Year 4	
	Completion	Dropouts	Completion	Dropouts	Completion	Dropouts	Completion	Dropouts
FM 1	0	0.020001	0	0.064451	0	0.115163	0.879196	0.120804
FM 2	0	0.045357	0	0.097104	0.897140	0.102860	0.897140	0.102860
FM 3	0	0.054206	0.939765	0.060235	0.939765	0.060235	0.939765	0.060235
FM 4	0.993625	0.006375	0.993625	0.006375	0.993625	0.006375	0.993625	0.006375

Table IV: Secondary School Male students Completion Rates within x years.

Year(x)	Year 1		Year 2		Year 3		Year 4	
	Completion	Dropouts	Completion	Dropouts	Completion	Dropouts	Completion	Dropouts
FM 1	0	0.010001	0	0.035181	0	0.096506	0.897735	0.102265
FM 2	0	0.025434	0	0.087379	0.906804	0.093196	0.906804	0.093196
FM 3	0	0.063562	0.930469	0.069531	0.930469	0.069531	0.930469	0.069531
FM 4	0.993626	0.006374	0.993626	0.006374	0.993626	0.006374	0.993626	0.006374