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An Interior Griffith – Crack Opened By Stresses At Crack Faces

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ABSTRACT
In this paper we obtained closed form expressions for crack shape and stress-intensity factors for single crack. These are very important parameters for fracture designing of structures. The problem is first reduced to dual series equations . The dual series equations is reduced to Fredholm-integral equation of second kind and obtained by along with other known functions, in terms of geometrical parameters.

KEYWORDS: *crack axis , composite plate , Griffith-Crack, Strain , stress , laminated plate , plane band*

I. INTRODUCTION

The crack opening in buckled isotropic plate has been discussed by with different boundary conditions. In this paper we are solving the problem of an interior Griffith-crack y=0, $0 \leq x < b$ in a rectangle of length 2a and width 2δ Geometrical symmetry and material symmetry are assumed. The surface of crack are stress-free.

The boundary conditions of the problem are, at $x = \pm a$

$$w(\pm a, y) = 0, u^0(\pm a, y) = 0, 0 \leq |y| \leq \delta \quad (1.1)$$

$$M_{xy}(\pm a, y) = 0, M_x(\pm a, y) = 0, 0 \leq |y| \leq \delta \quad (1.2)$$

And at y = δ

$$N_{xy}(x, \pm\delta) = 0, N_y(x, \pm\delta) = Q(x)_1, 0 \leq |x| \leq a \quad (1.3)$$

$$0 \leq |x| \leq a$$

$$M_y(x, \pm\delta) = 0, M_{y,y}(x, \pm\delta) + 2M_{xy,x}(x, \pm\delta) = 0, \quad (1.4)$$

At y = 0

$$N_{xy}(x, 0) = 0, M_{xy}(x, 0) = 0, 0 \leq |x| \leq a \quad (1.5)$$

and the mixed boundary condition

$$v^0(x, 0) = 0, b \leq |x| \leq a \quad (1.6)$$

$$N_y(x, 0) = 0, 0 \leq |x| < b \quad (1.7)$$

$$w(x, 0) = 0, b \leq |x| \leq a \quad (1.8)$$

$$M_y(x, 0) = 0, 0 \leq |x| < b \quad (1.9)$$

The boundary conditions (1.1) - (1.5) are those given by (1.1)-(1.6) respectively. The symmetry of problem reduces the mixed boundary conditions (1.6)-(1.9) for

$$V^0(x,0) = 0, b \le x \le a, \quad (1.10)$$

$$N_y(x,0) = 0, 0 \le x < b \quad (1.11)$$

And

$$w(x,0) = 0, b \le x \le a \quad (1.12)$$

$$M_y(x,0) = 0, 0 \le x < b \quad (1.13)$$

The In-plane displacement v^0 (x,0)will open the crack while out of plane i.e. transverse displacement will make the axis of crack as curved axis. we checked throughout that, see Burniston .

$$V^0(x,0) > 0, \quad 0 \le |x| < b, \quad (1.14)$$

which means that the crack really opens out and crack faces do not meet each other than at the crack-tips. In next section we shall reduce the physical problem to two decoupled series equations. The physical quantities will be given in-terms of above series equations .

II. Reduction To Series Equations

We take v^0 (x , y) with $f_1(x) = 0$ and then putting
Y = 0,

$$V^0(x,0) = -\sum_{n=1}^{\infty}\left[\beta_1 B_n + \beta_2 D_n\right]\cos(\alpha_n x) + f_2(0)$$

$$= -\sum_{n=1}^{\infty}\left[-\frac{\beta_1 t_2^1}{t_1^1} + \beta_2\right] D_n \cos\alpha_n x + f_2(0)$$

$$= \frac{\beta_1 t_2^1 - t_1^1\beta_2}{t_1^1}\sum_{n=1}^{\infty} D_n \cos(\alpha_n x) + f_2(0)$$

Thus the boundary conditions given

$$f_2(0) + \frac{\beta_1 t_2^1 - \beta_2 t_1^1}{t_1^1}\sum_{n=1}^{\infty} D_n \cos\alpha_n x = 0, \; b \le x \le a$$

$$\frac{D_0}{2} + \sum_{n=1}^{\infty} D_n \cos(\alpha_n x) = 0, \; b \le x \le a, \quad (2.1)$$

$$D_0 = f_2(0)/\beta_3, \beta_3 = \frac{t_1'}{\beta_1 t_2' - \beta_2 t_1'}, \quad (2.2)$$

$$N_y(x,0) = 0, \quad 0 \le x < b$$ Gives

$$\sum_{n=1}^{\infty}\alpha_n \cos\alpha_n x\left[t_1^0 A_n + t_2^0 C_n\right] = 0, \; 0 \le x < b,$$

Now using (2.1) in above equation we get,

$$\sum_{n=1}^{\infty} \alpha_n \cos(\alpha_n x)\left[t_1^0 \left\{ -t_6 D_n - C_2 \frac{Q_1}{C_7} \right\} + t_2^0 \left\{ t_7 D_n - \frac{C_1 Q_1}{C_7} \right\} \right] = 0,\ 0 \le x < b \tag{2.3}$$

$$\sum_{n=1}^{\infty} \alpha_n D_n \cos(\alpha_n x)\left\{ t_1 t_2^0 - t_6 t_1^0 \right\} = \sum_{n=1}^{\infty} \alpha_n \cos(\alpha_n x)\left\{ \frac{C_2 - C_1}{C_7} Q_1 \right\} \tag{2.4}$$

$$\sum \alpha_n T(n) D_n \cos(\alpha_n x) = P_1(x), 0 \le x < b,$$

$P_1(x)$ is a known function.

$$\sum_{n=1}^{\infty} \alpha_n D_n \cos(\alpha_n x) = P_1(x) - \sum_{n=1}^{\infty} \alpha_n D_n \left\langle T_{(n)} - 1 \right\rangle \cos \alpha_n x \tag{2.5}$$

$$\sum_{n=1}^{\infty} \alpha_n D_n \cos(\alpha_n x) = P_1(x) - P_{11}(x) = P_2(x), 0 \le x < b, \tag{2.6}$$

Where $P_{11}(x)$ involves unknown. This will lead to Fredholm integral equation of second kind.

The condition

$$w(x, 0) = 0, \quad b \le x \le a$$

Gives

$$\sum_{n=1}^{\infty} \left[E_n + G_n \right] \sin(\alpha_n x) = 0, b \le x \le a \tag{2.7}$$

We get

$$\sum_{n=1}^{\infty} \alpha_n^2 \sin(\alpha_n x)\left[t_3^1 E_n + t_2^3 G_n \right] + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{\beta_m^2 q_{sc}(\alpha_n, \beta_m) \sin \alpha_n x}{W} = 0 \tag{2.8}$$

The above series relations are called dual series relations involving tri geometric series. The solutions of these series equations will be given in next article.

III. RESULT

It is also observed that there is no singularity in bending moment and transverse displacement is also smooth (for crack shape and stress-intensity factors for single crack) .

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