IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 12, Issue 2 Ver. I (Mar. - Apr. 2016), PP 147-158
www.iosrjournals.org

A Retrial Queueing System With Two Type Of Arrivals And With A Control Admissible Policy On Type I Arrivals

Kalyanaraman, R.
Department Of Mathematics, Annamalai University, Annamalainagar-608002, India

Abstract : *A Retrial Queueing System With Two Types Of Batch Arrivals Has Been Considered. The Arrivals Are Called Type I And Type II Customers. The Type I Customers Arrive In Batches Of Size k With Probability c_k And Type II Customers Arrive Singly According To Two Poisson Processes With Rates $\lambda_1 \bar{c} = \lambda_1 \sum_{k=1}^{\infty} kc_k$ And λ_2 Respectively. In Addition, Let P Be The Probability Of Admission For Each Individual Type I Customer. Service Time Distributions Are Identical Independent Distributions And Are Different For Both Types Of Customers. If The Arriving Customers Are Blocked Due To Server Being Busy, Type I Customers Are Queued In A Priority Queue Of Infinity Capacity Where As Type II Customers Entered Into Retrial Group In Order To Seek Service Again After A Random Amount Of Time. For This Model The Joint Distribution Of The Number Of Customers In The Priority Queue And In The Retrial Group In Closed Form Is Obtained. Some Particular Models And Operating Characteristics Are Obtained. A Numerical Study Is Also Carried Out.*
Keywords: *Batch Arrival, Joint-Distribution, Operating Characteristics, Queueing, Retrial.*

I. Introduction

The Behaviour Of A Retrial Queue Is That A Customers Upon Arrival Who Finds The Server Busy Is Obliged To Leave The Service Area And Form A Retrial Group. After Some Random Period Of Time, The Customers In Retrial Group, Called Blocked Customers Will Try Their Luck Again. Retrial Queueing System Are Powerful Tool For Modelling, Computer And Communication Networks, Transportation Problems, Production And Manufacturing Systems, Web Access, Telecommunication Networks Etc. For A Systematic Account Of The Techniques And Results On This Topic, One Can Refer The Monograph By Falin And Templeton [1], And The Bibliographical Information Given By Artalejo [2,3]. In Addition, A Comprehensive Discussion Of Various Retrial Queueing Models Is Given In Artalejo And Falin [4].

Artalejo Et Al. [5] Considered A Retrial Queue With Two Types Of Customers In Which The Customers At The Retrial Group Have Preemptive Priority Over Customer At The Waiting Line.
Choi And Chang [6] Present A Survey Of Retrial Queues With Two Types Of Calls. Falin [7,8] And Falin And Templeton [1] Investigated Sufficient Conditions For The Existence Of The Stationary Distribution Of The Queue Lengths For The $M_1, M_2 / M / c$ Retrial Queue. Choi And Park [9] Investigated A Retrial Queue With Two Types Of Calls With No Limit On How Many Such Calls Can Be In The System. Type 2 Customers Are Placed In A Retrial Buffer Should There Be No Idle Server At The Time Of Arrivals. They Obtained The Joint Distribution Of Queue Length Using Supplementary Variable Method. Kalyanaraman And Srinivasan [10], Studied A Single Server Retrial Priority Queueing System With Primary Type I Calls, Transit Type II Calls And K Recurrent Calls. Type I Calls Have A Priority Over The Other Calls And Have Their Own Buffer. An Arriving Type II Calls Finds The Server Busy Enter Into A Retrial Buffer, And The Recurrent Call In The Retrial Buffer. Assuming General Independent Service Times, The Authors Derived The Joint Distribution Of The Number Of Calls In The Priority Queue And In The Retrial Queue Using Supplementary Variable Method. Wang [11] Discussed The $(M_1, M_2)/G_1, G_2/1$ Retrial Queues With Priority Subscribers And The Server Subject To Break Down And Repairs.

Now The Call Centers Have Been Playing A Vital Role In Many Industries And Business Organizations. In Call Centers The Customers Have Been Contacting The Call Centers By Talking To A Customer Service Representative (CSR) Or An Agent Over The Telephone. Now, In Addition To Contacting Over The Phone, The Customers Can Contact The Center Over The Internet Either Via Email Or Live Chat Sessions. In The Past, The Center Has Different Components Such As An Automatic Call Distributor (ACD), An Interactive Voice Response (IVR) Unit, Computers And Telephones. The ACD Is A Telephone Switch Located At Conveniently To Properly Distributing The Customer Calls. As The Calls Arrive The ACD Routes Them Either To The IVR Unit Where The Customer Transactions Are Handled Automatically Or To An Idle CSR, Who Provides The Necessary Service. If No CSR Is Available The Calls Are Placed In A Queue. The

CSR Responds To The Routed Calls Either Using The Telephone And Or The Computer. If The Agent Is Answering A Telephonically That Agent Can Access The Customer Information Databases Through The Computer. Arriving Calls All Terminated At The ACD Switch And Are Routed To Group Of Agents CSR's Concurrently, The Mathematical Models Applied In Practice, Are Based On Some Queueing Models. In This Paper We Study A Call Center Problem As A Retrial Queueing Model With Two Types Of Calls. The Servers Are CSR And Customers Are The Arriving Calls. The Type I Customers Are Voice Calls And Type II Customers Are Emails. In Most Of The Papers, Call Centers Are Model As The Retrial System, Where The Impact Of Preemptive Priority Of The Customers Is Not Considered. Recently, Pustova [12] Considered The Effect Of The Retrials In Call Centers.

Artalejo And Atencia [13] Considered A Single Server Retrial Queue With Batch Arrivals Which Operates Under The Linear Retrial Discipline. In Addition, Each Individual Customer Is Subject To A Control Admission Policy Upon Arrival. They Assume That Each Individual Blocked Customer Is Admitted To Join The Retrial Group With A Probability P Independently Of The Admission Of The Rest Of The Customers Arriving At The Same Batch And The Actual Size Of The Retrial Group. For This Model They Carry Out The Steady State Analysis.

Kalyanaraman And Srinivasan [14], Studied An $M/G/1$ Retrial Queue With Geometric Loss And With Type I Batch Arrivals And Type II Single Arrivals. In [15] The Author With Thillaigovindan Analyzed A Feedback Retrial Queueing System With Two Types Of Arrivals And The Type I Arrival Being Batch Arrival Of Fixed Size K. Kalyanaraman [16] Considered A Feedback Retrial Queueing System With Two Type Of Customers With Constant Retrial Rate. For This System The Author Obtained The Joint Distribution Of Number Of Customers In The Priority Queue And In The Orbit In Closed Form.

In This Paper We Deal With A Retrial Queue With Two Types Of Customers, In Which Both Types Of Customers Arrives In Batches Of Variable Size. In Section 2, We Describe The System With Stability Condition And Notation. In Section 3, We Obtain The Joint Probability Generating Function For The Number Of Customers In The Priority Queue And In The Retrial Group When Server Is Busy As Well As Idle. The Expressions For Some Particular Models Are Deduced In Section 4. Some Operating Characteristics Are Derived In Section 5 And A Numerical Study Is Carried Out In Section 6.

The Model

A Retrial Queueing System With Two Types Of Customers Type I And Type II Respectively Has Been Considered In This Paper. The Type I Customers Arrives In Batches Of Size k With Probability $c_k, k \geq 1.$ Let $p,\ (0 \leq p \leq 1)$ Be The Probability Of Admission For Each Individual Customer And Let An Be The Probability That A Group Of Size n (≥ 0) Joins The System With $a_0 = \sum_{k=1}^{\infty} c_k (1-p)^k, a_n = \sum_{k=n}^{\infty} c_k \binom{k}{n} p^n (1-p)^{k-n}, n \geq 1$ (Artalejo And Atencia [3]). After Admitted To The System The Customers Follows The Following Policy: If The Server Is Idle Then One Customer Starts Its Service And The Rest Join The Queue And If The Server Is Busy All The Admitted Customers Go To The Queue. Type II Customers Arrives Singly To Two Independent Poisson Processes With Rates $\lambda_1 \bar{c} = \lambda_1 \sum_{k=1}^{\infty} k c_k$ And λ_2 Respectively. If A Type II Customer Upon Arrival Finds The Server Busy, They Enter In To An Orbit Of Infinite Capacity In Order To Seek Service Again After Random Amount Of Time. All The Customers In The Retrial Group Behave Independent Of Each Other. The Retrial Time Is Exponentially Distributed With Mean $\frac{1}{\alpha}$. The Type I Customers Are Queued In A Priority Queue Of Infinite Capacity After Blocking, If The Server Is Busy. As Soon As The Server Is Free, The Customers In The Priority Queue Are Served Using FCFS Rule And The Customers In The Retrial Group Are Served, If There Are No Customers In The Priority Queue.

The Service Time Distribution For Both Type Of Customers Are Identically Independently Distributed Random Variables And Have Different Distributions. Supplementary Variable Technique Is Used For The Analysis And The Variable Is The Residual Service Time Of A Customer In Service. The Service Time Density Function $b_k(x); k = 1,2$ And $B_k^*(s) = \int_0^{\infty} e^{-sx} b_k(x) dx$ Be The Laplace Transformation Of The Distribution Function $B_k(x)$.

The Stochastic Process Related To The Model Is $\{(\xi(t), N_p(t), N_r(t), S_k(t)) : t \geq 0\}$ Where

$N_p(t) =$ Number Of Customers In The Priority Queue At Time t

$N_r(t) =$ Number Of Customers In The Retrial Group At Time t

$\xi(t) =$ The Server State At Time t And

$$\xi(t) = \begin{cases} 0, & \text{When The Server Is Idle} \\ 1, & \text{When The Server Is Busy With Type I Customers} \\ 2, & \text{When The Server Is Busy With Type II Customers} \end{cases}$$

$S_k(t) =$ The Residual Service Time Of Type k Customer In Service At Time t, Is A Markov Process With State Space $\{0,1\} \times \{1,2,3\} \times (0,\infty)$ And The Corresponding Stationary Process Is $\{(\xi, N_p, N_r, S_k)\}$.

The Related Probabilities Are $q_j(t) = \Pr\{\xi(t) = 0, N_r(t) = j\}$

$p(k,i,j;x) = \Pr\{\xi(t) = k, N_p(t) = i, N_r(t) = j, S_k(t) \in (x, x+dx)\},\ k = 1, 2.$

In Steady State, The Probabilities Are $q_j = \lim_{t\to\infty} q_j(t),\ p(k,i,j;x) = \lim_{t\to\infty} p(k,i,j;x,t)$ And The Laplace Transformation Of $p(k,i,j;x)$ Is $p^*(k,i,j;s) = \int_0^\infty e^{-sx} p(k,i,j;x)dx, i = 1,2, j \geq 0.$

It Is Clear That, $p(k,i,j;0) = \int_0^\infty p(k,i,j;x)dx = \Pr\{\xi = k, N_p = i, N_r = j\}$ Is The Steady State Probability That There Are i Customers In The Priority Queue, j Customers In The Retrial Group And The Server Services To k^{th}-Type Customer.

For $|Z_1| \leq 1,\ |Z_2| \leq 1,$ The Following Probability Generating Functions

$$Q(Z_2) = \sum_{j=0}^{\infty} q_j Z_2^j$$

$$C(Z_1) = \sum_{j=1}^{\infty} c_j Z_1^j$$

$$D(Z_2) = \sum_{j=1}^{\infty} d_j Z_2^j$$

$$A(Z_1) = \sum_{n=0}^{\infty} a_n Z_1^n = C(pZ_1 + 1 - p)$$

Where The First Moment Is $\bar{a} = p\bar{c}$.

$$P^*(k,i,s,Z_2) = \sum_{j=0}^{\infty} p^*(k,i,j,s) Z_2^j;\ i \geq 0, k = 1,2.$$

$$P^*(k,s,Z_1,Z_2) = \sum_{i=0}^{\infty} P^*(k,i,s,Z_2) Z_1^i;\ k = 1,2.$$

$$P^*(k,i,0,Z_2) = \sum_{j=0}^{\infty} p^*(k,i,j,0) Z_2^j;\ i \geq 0, k = 1,2.$$

$$P^*(k,0,Z_1,Z_2) = \sum_{i=0}^{\infty} P^*(k,i,0,Z_2) Z_1^i;\ k = 1,2.$$

Are Defined For The Analysis.

The Analysis

Using The Mean Drift Argument Of Falin [8], It Can Be Show That The System Is Stable If $\rho_1+\rho_2 \le 1$ Where $\rho_1 = -\lambda_1 p\bar{c}B_1^{*'}(0), \rho_2 = -\lambda_2 B_2^{*'}(0).$

Usual Arguments Lead To The Following Differential-Difference Equations:

For $j \ge 0, x \ge 0, i \ge 0$

$$(\lambda + j\alpha)q_j = p(1,0,j;0) + p(2,0,j;0) \tag{1}$$

$$-p^{'}(1,0j;x) = -\lambda p(1,0,j;x) + \lambda_1 b_1(x)q_j + b_1(x)p(1,i,j;0) + p(1,0,j-1;x) \tag{2}$$

$$-p^{'}(1,i,j;x) = -\lambda p(1,i,j;x) + b_1(x)p(1,i+1,j:0) + \lambda_1\sum_{k=1}^{i} a_k p(1,i-k,j;x) + \lambda_2 p(1,i,j-1;x) \tag{3}$$

$$-p^{'}(2,0,j;x) = -\lambda p(2,0,j;x) + \lambda_2 b_2(x)q_j + (j+1)\alpha b_2(x)q_{j+1} + \lambda_2 p(2,0,j-1;x) \tag{4}$$

$$-p^{'}(2,i,j;x) = \lambda p(2,i,j;x) + \lambda_1\sum_{k=1}^{n} a_k p(2,i-k,j;x) + \lambda_2 p(2,i,j-1;x) \tag{5}$$

And The Normalization Condition Is

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\int_0^{\infty} p(1,i,j;x)dx + \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\int_0^{\infty} p(2,i,j;x)dx + \sum_{j=0}^{\infty} q_j = 1 \tag{6}$$

Where $\lambda = \lambda_1 + \lambda_2$

By Taking Laplace Transformation To Equations (1) To (5) And Multiplying By Z_2^j And Then Summing Over j, The Following Equations Can Be Obtained

$$\lambda Q(Z_2) + \alpha Z_2 Q^{'}(Z_2) = P(1,0;0,Z_2) + P(2,0;0,Z_2) \tag{7}$$

$$(s-\lambda+\lambda_2 Z_2)P^*(1,0;s,Z_2) = P(1,0;0,Z_2) - \lambda_1 B_1^*(s)Q(Z_2) - B_1^*(s)P(1,1;0,Z_2) \tag{8}$$

$$(s-\lambda+\lambda_2 Z_2)P^*(1,i;s,Z_2) = P(1,i;0,Z_2) - B_1^*(s)P(1,i+1;0,Z_2) - \lambda_1\sum_{k=1}^{i} a_k P^*(1,i-k;s,Z_2) \tag{9}$$

$$(s-\lambda+\lambda_2 Z_2)P^*(2,0;s,z_2) = P(2,0;0,Z_2) - \lambda_2 B_2^*(s)Q(Z_2) - \alpha B_2^*(s)Q^{'}(Z_2) \tag{10}$$

$$(s-\lambda+\lambda_2 Z_2)P^*(2,i;,s,Z_2) = -\lambda_2\sum_{k=1}^{i} a_k P^*(2,i-k;s,Z_2) \tag{11}$$

$$p(2,i,j;0) = 0$$

Multiplying Equations (9) And (11) By Z_1^i And Summing Over i And Using Equations (8) And (10) Leads To

$$(s-\lambda+\lambda_1 A(Z_1)+\lambda_2 Z_2)P^*(1,s;Z_1,Z_2) = \frac{[Z_1 - B_1^*(s)]}{Z_1}P(1,0;Z_1,Z_2) + \frac{B_1^*(s)}{Z_1} \times [P(1,0;0,Z_2) - \lambda_1 Z_1 Q(Z_2)] \tag{12}$$

$$(s-\lambda+\lambda_1 A(Z_1)+\lambda_2 Z_2)P^*(2,s;Z_1,Z_2) = P(2,0;0,Z_2) - B_2^*(s)[\alpha Q^{'}(Z_2) + \lambda_2 Q(Z_2)] \tag{13}$$

By Substituting $s = \lambda - \lambda_1 A(Z_1) - \lambda_2 Z_2$ In (12) And (13), We Get

$$P(1,0;0,Z_2) = \lambda_1 Z_1 Q(Z_2) - \frac{[Z_1 - B_1^*(l)]}{B_1^*(l)}P(1,0;Z_1,Z_2) \tag{14}$$

$$P(2,0;0,Z_2) = B_2^*(l)[\alpha Q^{'}(Z_2) + \lambda_2 Q(Z_2)] \tag{15}$$

Where $l = \lambda - \lambda_1 A(Z_1) - \lambda_2 Z_2$

Using Equations (14) And (15) In (7) And On Simplification One Can Get The Following Equation

$$\alpha[Z_2 - B_2^*(l)]Q^{'}(Z_2) + [\lambda - \lambda_1 Z_1 - \lambda_2 B_2^*(l)]Q(Z_2) = \frac{[B_1^*(l) - Z_1]}{B_1^*(l)}P(1,0;Z_1,Z_2) \tag{16}$$

Define $f(Z_1, Z_2) = \dfrac{B_1^*(l) - Z_1}{B_1^*(l)}$ For Each Fixed $Z_2, |Z_2| \leq 1$, By Rouche's Theorem, There Is A Unique Solution $Z_1 = h(Z_2)$ Of The Equation $f(Z_1, Z_2) = 0$, Now (16) Becomes

$$Q^{'}(Z_2) = \frac{1}{\alpha} \frac{\lambda - \lambda_1 h(Z_2) - \lambda_2 U(Z_2)}{U(Z_2) - Z_2} Q(Z_2) \tag{17}$$

Where $h(Z_2)$ Is The Root Of The Equation $Z_1 = B_1^*(\lambda - \lambda_1 A(Z_1) - \lambda_2 Z_2)$ And $U(Z_2) = B_2^*(\lambda - \lambda_1 A(Z_1) - \lambda_2 Z_2)$

Using Equation (17) In Equation (16), It Can Be Seen That

$$P(1,0;Z_1,Z_2) = \frac{\{L[Z_2 - B_2^*(l)] + R[U(Z_2) - Z_2]\} B_1^*(l) Q(Z_2)}{[B_1^*(l) - Z_1][U(Z_2) - Z_2]} \tag{18}$$

Where $L = \lambda - \lambda_1 h(Z_2) - \lambda_2 U(Z_2), R = \lambda - \lambda_1 Z_1 - \lambda_2 B_2^*(l)$

Using Equation (18) In (16), Leads To

$$P(1,0;0,Z_2) = \frac{\{L[Z_2 - B_2^*(l)] + [\lambda_1 Z_1 + R][U(Z_2) - Z_2]\} Q(Z_2)}{[U(Z_2) - Z_2]} \tag{19}$$

Using Equation (17) In Equation (15), Leads To

$$P(2,0;Z_1,Z_2) = \frac{\{L + \lambda_2 [U(Z_2) - Z_2]\} B_2^*(l) Q(Z_2)}{[U(Z_2) - Z_2]} \tag{20}$$

The General Solution Of The Differential Equation (17) Is

$$Q(Z_2) = Q(1) \exp\left\{ -\frac{1}{\alpha} \int_{Z_2}^{1} \frac{\lambda - \lambda_1 h(x) - \lambda_2 U(x)}{U(x) - x} dx \right\} \tag{21}$$

Where $Q(1)$ Is A Constant, Which Is The Probability That The Server Is Idle.

Putting $s = 0$ In Equation (8) And In Equation (9) And Summing Over $i = 0$ To ∞, Which Leads To

$$\lambda_2 (Z_2 - 1) \sum_{i=0}^{\infty} P^*(1,i;0,Z_2) = P(1,0;0,Z_2) - \lambda_1 Q(Z_2) \tag{22}$$

Putting $s = 0$ In Equation (10) And In Equation (11) And Summing Over $i = 0$ To ∞, Which Leads To

$$\lambda_2 (Z_2 - 1) \sum_{i=0}^{\infty} P^*(2,i;0,Z_2) = P(2,0;0,Z_2) - \lambda_2 Q(Z_2) - \alpha Q^{'}(Z_2) \tag{23}$$

Adding Equations (22) And (23) And Using Equation (7), Which Leads To

$$\lambda_2 (Z_2 - 1) \sum_{i=0}^{\infty} \sum_{k=1}^{2} P^*(k,i;0,Z_2) = \alpha (Z_2 - 1) Q^{'}(Z_2) \tag{24}$$

Evaluating At $Z_2 = 1$ And Using Normalization Condition Leads To

$$Q^{'}(1) = \frac{\lambda_2}{\alpha} [1 - Q(1)] \tag{25}$$

Putting At $Z_2 = 1$ In Equation (17), We Get

$$Q^{'}(1) = \frac{\lambda_2 (\rho_1 + p\bar{c}\rho_2) Q(1)}{\alpha p\bar{c}(1 - \rho_1 - \rho_2)} \tag{26}$$

From Equation (25) And (26), Leads To

$$P_I = Q(1) = \frac{p\bar{c}(1 - \rho_1 - \rho_2)}{(p\bar{c} + \rho_1 - p\bar{c}\rho_1)} \tag{27}$$

Is The Probability That The Server Is Idle. In Steady State, The Probability Generating Function Of Number Of Customers In The Orbit When The Server Is Idle Is Obtained From Equation (26) And Equation (21).

Substituting $s = 0$ In Equation (12), We Get

$$P^*(1,0;Z_1,Z_2) = \frac{P(1;0,Z_1,Z_2)(1-Z_1)+\lambda_1 Z_1 Q(Z_2) - P(1,0,0;Z_2)}{lZ_1} \tag{28}$$

(28) Together With Equation (18) And (19) Yields The Joint Probability Generating Function Of The Number Of Customers In The Priority Queue And In The Orbit When The Server Is Busy With Type I Customer And Is

$$P^*(1,0;Z_1,Z_2) = \frac{[1-B_1^*(l)]\{L[Z_2 - B_2^*(l)] + R[U(Z_2)-Z_2]\}}{l[U(Z_2)-Z_2][B_1^*(l)-Z_1]} Q(Z_2) \tag{29}$$

Putting $s = 0$ In Equation (13), We Get

$$P^*(2,0;Z_1,Z_2) = \frac{\alpha Q'(Z_2) + \lambda_2 Q(Z_2 - P(2,0;0,Z_2)}{l} \tag{30}$$

(30) Together With Equations (17) And (20) Yields The Joint Probability Generating Function Of The Number Of Customers In The Priority Queue And In The Orbit When The Server Is Busy With Type II Customer And Is

$$P^*(2,0;Z_1,Z_2) = \frac{[1-B_2^*(l)][\lambda - \lambda_1 h(Z_2) - \lambda_2 Z_2]}{l[U(Z_2)-Z_2]} Q(Z_2) \tag{31}$$

Thus We Have The Following Theorem.

Theorem

The Stationary Distribution Of $\{(\xi, N_p, N_r, S_k)\}$ Has The Following Generating Functions

$$Q(Z_2) = \frac{p\bar{c}(1-\rho_1-\rho_2)}{(p\bar{c}+\rho_1 - p\bar{c}\rho_1)} \exp\left\{\frac{1}{\alpha}\int_1^{Z_2} \frac{\lambda - \lambda_1 h(x) - \lambda_2 U(x)}{U(x)-x} dx\right\}$$

$$P^*(1,0;Z_1,Z_2) = \frac{[1-B_1^*(l)]\{L[Z_2 - B_2^*(l)] + R[U(Z_2)-Z_2]\}}{l[U(Z_2)-Z_2][B_1^*(l)-Z_1]} Q(Z_2)$$

$$P^*(2,0;Z_1,Z_2) = \frac{[1-B_2^*(l)][\lambda - \lambda_1 h(Z_2) - \lambda_2 Z_2]}{l[U(Z_2)-Z_2]} Q(Z_2)$$

Corollary

The Probability That The Server Busy Is

$$P_B = P^*(1,0;1,1) + P^*(2,0;1,1) = \frac{\rho_1 + \rho_2 p\bar{c}}{p\bar{c} + \rho_1 - \rho_1 p\bar{c}} \tag{32}$$

Particular Models

By Taking Particular Values To Some Parameters Of The Above Model, The Following Models Can Be Obtained:

(I) When $c_k = 0, k \neq 1, p = 1$ And $B_1(x) = B_2(x) = B(x)$ The System Coincides With That Of Choi And Park [9].

(Ii) When $c_k = 0, k \neq 1, p = 1$ And The Above Results Coincides With The Results Of Falin Et Al. [17].

Operating Characteristics

Using Straight Forward Calculations, The Operating Characteristics Like The Mean Number Of Customers In The Priority Queue And The Mean Number Of Customers In The Orbit Have Been Calculated. After Putting $Z_2 = 1$ In Equation (29) And In Equation (31) The Differential Coefficient With Respect To Z_1 Has Been Obtained And Then Taking $Z_1 = 1$.

$$\lim_{Z_1 \to 1} P^{*'}(1,0;Z_1,1) = \frac{[\lambda_1^2 \bar{c}^3 p^2 \beta_1 + pc_2\rho_1][1-\rho_2(1-p\bar{c})] - pc_2\rho_1(1-\rho_1-\rho_2)}{2\bar{c}(1-\rho_1)(p\bar{c}+\rho_1 - p\bar{c}\rho_1)} + \frac{\lambda_1\lambda_2 p\bar{c}\beta_2\rho_1}{2(1-\rho_1)} \tag{33}$$

$$\lim_{Z_1 \to 1} P^{*'}(2,0;Z_1,1) = \frac{\lambda_1 \lambda_2 p\bar{c}\beta_2}{2} \tag{34}$$

After Putting $Z_1 = 1$ In Equation (29) And In Equation (31) The Differential Coefficient With Respect To Z_2 Has Been Obtained And Then Taking $Z_2 = 1$.

$$\lim_{Z_2 \to 1} P^{*'}(1,0;1,Z_2) = \frac{\lambda_2[(1-\rho_2)D_1 + \lambda_1\lambda_2\bar{c}^2\beta_2 D_2]}{2\lambda_1\bar{c}^2(1-\rho_1)(1-\rho_1-\rho_2)(p\bar{c}+\rho_1-p\bar{c}\rho_1)} - \frac{\lambda_2^2 p\bar{c}\beta_2(1-\rho_1-\rho_2)}{2(p\bar{c}+\rho_1-p\bar{c}\rho_1)}$$

$$+ \frac{\lambda_2\rho_1(\rho_1+p\bar{c}\rho_2)(1-\rho_2+p\bar{c}\rho_2)}{\alpha p\bar{c}(1-\rho_1-\rho_2)(p\bar{c}+\rho_1-p\bar{c}\rho_1)} \tag{35}$$

$$\lim_{Z_2 \to 1} P^{*'}(2,0;1,Z_2) = \frac{[\lambda_1\lambda_2\bar{c}^2 D_3 + \lambda_2\rho_2 c_2\rho_1^2(\rho_1+p\bar{c}\rho_2)]}{2\lambda_1\bar{c}^2(1-\rho_1)(1-\rho_1-\rho_2)(p\bar{c}+\rho_1-p\bar{c}\rho_1)} + \frac{\lambda_2\rho_2(\rho_1+p\bar{c}\rho_2)}{\alpha p\bar{c}(1-\rho_1-\rho_2)} \tag{36}$$

Where

$D_1 = \lambda_1^2\bar{c}^3 p\beta_1(1-\rho_2+p\bar{c}\rho_2) + \rho_1^2 c_2(\rho_1+p\bar{c}\rho_2),$

$D_2 = (1-\rho_1-\rho_2)[p\bar{c}(1-\rho_2)+\rho_1(\rho_1+p\bar{c}\rho_2)] + \rho_1(\rho_1+p\bar{c}\rho_2)$ And

$D_3 = \lambda_2\beta_2(p\bar{c}+\rho_1-p\bar{c}\rho_1)[(1-\rho_1)^2+\rho_1\rho_2] + \lambda_1\beta_1\rho_2 p\bar{c}(1-\rho_2+p\bar{c}\rho_2)$

From Equation (26) And (27), We Get

$$Q'(1) = \frac{\lambda_2(\rho_1+p\bar{c}\rho_2)}{\alpha(p\bar{c}+\rho_1-\rho_1 p\bar{c})} Q(1) \quad (3$$

(I) Mean Number Of Customers In The Priority Queue Is

$$N_p = \lim_{Z_1 \to 1} P^{*'}(1,0;Z_1,1) + \lim_{Z_1 \to 1} P^{*'}(2,0;Z_1,1) \tag{38}$$

Adding Equations (33) And (34) We Get Equation (38).

(Ii) Mean Number Of Customers In The Orbit Is

$$N_r = \lim_{Z_2 \to 1} P^{*'}(1,0;1,Z_2) + \lim_{Z_2 \to 1} P^{*'}(2,0;1,Z_2) + Q'(1) \tag{39}$$

Adding Equations (35), (36) And (37) We Get Equation (39).

(Iii) Mean Busy Period

Busy Period T_b Is The Length Of The Time Interval That Keeps The Server Busy Continuously And This Continues Till The Instant The Server Becomes Free Again And Let T_0 Be The Length Of The Idle Period. For This Model, T_b And T_0 Generates An Alternating Renewal Process And Therefore

$\dfrac{E(T_b)}{E(T_0)} = \dfrac{\Pr\{T_b\}}{1-\Pr\{T_b\}} = \dfrac{P_B}{1-P_B}$. But $E(T_0) = \dfrac{1}{\lambda}$.

Hence $$E(T_b) = \frac{P_B}{\lambda(1-P_B)} \tag{40}$$

Using Equation (32) On Equation (40), We Get

$$E(T_b) = \frac{(\rho_1+p\bar{c}\rho_2)}{\lambda p\bar{c}(1-\rho_1-\rho_2)}$$

Special Cases

By Assigning Particular Distribution On $\{c_n\}$, We Obtain The Following Results Corresponding To The Results In The Section 3.

The Distribution Of The Number Of Customers Arriving In A Batch Is Geometric That Is, $c_k = (1-q)q^{k-1},\ k \geq 1,\ q \in [0,1),\ \bar{a} = \dfrac{p}{1-q}$.

Numerical Study

In This Section, Some Numerical Examples Related To The Model Analyzed In This Article Are Given. For The Sake Of Convenience, It Has Been Assumed That The Type I And Type II Service Times Are Exponentially Distributed Random Variables With Mean $\frac{1}{\mu_1}$ And $\frac{1}{\mu_2}$. In Order To See The Effect Of The Parameters Type I Service Rate μ_1, Type II Service Rate μ_2, Type I Arrival Rate λ_1, Type II Arrival Rate λ_2 On The Mean Number Of Customers In The Priority Queue, The Mean Number Of Customers In The Orbit, The Mean Busy Period, The Probability That The Server Is Idle And The Probability That The Server Is Busy Of The Model Discussed In This Paper By Fixing The Values Of The Retrial Rates (α), Probability $(p, \bar{c})$ And c_2. Some Numerical Results Are Obtained. The Results Are Presented In Graphs And Tables. The Figures 1 To 3 (4 To 6) Represents The Surface Of Type I Arrival Rate, Type I Service Rate (Type II Service Rate) And The Mean Number Of Customers In The Priority Queue, And The Mean Number Of Customers In The Orbit, And The Mean Busy Period Respectively. The Figures 7 To 9 (10 To 12) Represents The Surface Of Type II Arrival Rate, Type I Service Rate (Type II Service Rate)And The Mean Number Of Customers In The Priority Queue, The Mean Number Of Customers In The Orbit, And The Mean Busy Period Respectively. Figures 1, 4 Shows That As Type I Arrival Rate Increases, The Mean Number Of Customers In The Priority Queue Increases Steadily For Small Value Of Type I Service Rate, That Is, 4 Whereas The Increment Is Too Small For Comparatively Big Values, That Is, Like 8. The Same Situation Has Been Encounter In The Case Of Type II Service Rate. Figure 7 Represents The Surface Of Type II Arrival Rate, Type I Service Rate And The Mean Number Of Customers In The Priority Queue Is Convex Surface For Increasing The Values Of Type I Service Rate, Type II Service Rate And The Mean Number Of Customers In The Priority Queue Is Convex With Respect To Type II Service Rate But Increases With Respect To Type II Arrival Rate. The Surface Of The Mean Number Of Customers In The Priority Queue, Type I Arrival Rate (Type II Arrival Rate) And Type I Service Rate In The Figures 2 (8) Is Almost A Convex Surface At The Comparatively Large Value Of Type I Arrival Rate And The Small Value Of Type I Service Rate (Is A Flat And Increasing Surface For Increasing Value Of Type II Arrival Rate). Figures 5 And 11 Represents The Surface Area Of The Mean Number Of Customers In The Orbit Versus Type I Arrival Rate Versus Type II Service Rate And The Mean Number Of Customers In The Orbit Versus Type II Arrival Rate Versus Type II Service Rate Respectively. The First Surface Is Convex With Respect To Type II Service Rate But Slightly Increases For The Increasing Values Of Type I Arrival Rate Whereas The Second Surface Is Again Convex With Respect To Type II Service Rate And Increases With Respect To Type II Arrival Rate For Smallest Values Of Type II Service Rate. Figures 3, 6, 9 And 12 Represents The Surface Of The Mean Busy Period Versus Type I Arrival Rate Or Type II Arrival Rate Versus Type I Service Rate Or Type II Service Rate. All The Surfaces Are Convex With Respect To Service Rate And Are Increases For Increasing Values Of Arrival Rate. Tables 1-4 Shows The Probabilities That The Server Is Idle And The Server Is Busy For The Various Values Of λ_1, λ_2, μ_1 And μ_2. From Tables It Can Be Seen That, For Increasing Values Of Type I Arrival Rate And Type II Arrival Rate, For Fixed Values Of Type I Service Rate And Type II Service Rate The Idle Probabilities Decreases Whereas The Busy Probabilities Increases As Expected.

Table 1

The Probabilities P_I And P_B

λ_1	$\alpha = 0.7, \lambda_2 = 3.0, \mu_2 = 5.0, \bar{c} = 0.4, c_2 = 0.05, p = 0.6$					
	$\mu_1 = 4.0$		$\mu_1 = 6.0$		$\mu_1 = 8.0$	
	P_I	P_B	P_I	P_B	P_I	P_B
0.1	0.3780	0.6220	0.3852	0.6148	0.3888	0.5070
0.2	0.3574	0.6426	0.3710	0.6290	0.3780	0.5142
0.3	0.3379	0.6621	0.3574	0.6426	0.3675	0.5214
0.4	0.3195	0.6805	0.3443	0.6557	0.3574	0.5289
0.5	0.3022	0.6978	0.3317	0.6683	0.3475	0.5365
0.6	0.2857	0.7143	0.3195	0.6805	0.3379	0.5443
0.7	0.2701	0.7299	0.3078	0.6922	0.3286	0.5523
0.8	0.2553	0.7447	0.2966	0.7034	0.3195	0.5604
0.9	0.2412	0.7588	0.2857	0.7143	0.3107	0.5688
1.0	0.2278	0.7722	0.2752	0.7248	0.3022	0.5774

Table 2

The Probabilities P_I And P_B

	$\alpha = 0.7, \lambda_2 = 3.0, \mu_2 = 5.0, \bar{c} = 0.4, c_2 = 0.05, p = 0.6$

λ_1	$\mu_2 = 4.0$		$\mu_2 = 6.0$		$\mu_2 = 8.0$	
	P_I	P_B	P_I	P_B	P_I	P_B
0.1	0.2360	0.7640	0.4798	0.5202	0.6018	0.3982
0.2	0.2227	0.7773	0.4607	0.5393	0.5796	0.4204
0.3	0.2100	0.7900	0.4424	0.5576	0.5586	0.4414
0.4	0.1979	0.8021	0.4249	0.5751	0.5384	0.4616
0.5	0.1864	0.8136	0.4083	0.5917	0.5192	0.4808
0.6	0.1753	0.8247	0.3924	0.6076	0.5009	0.4991
0.7	0.1648	0.8352	0.3771	0.6229	0.4833	0.5167
0.8	0.1547	0.8453	0.3625	0.6375	0.4665	0.5335
0.9	0.1450	0.8550	0.3485	0.6515	0.4503	0.5497
1.0	0.1356	0.8644	0.3351	0.6649	0.4348	0.5652

Table 3

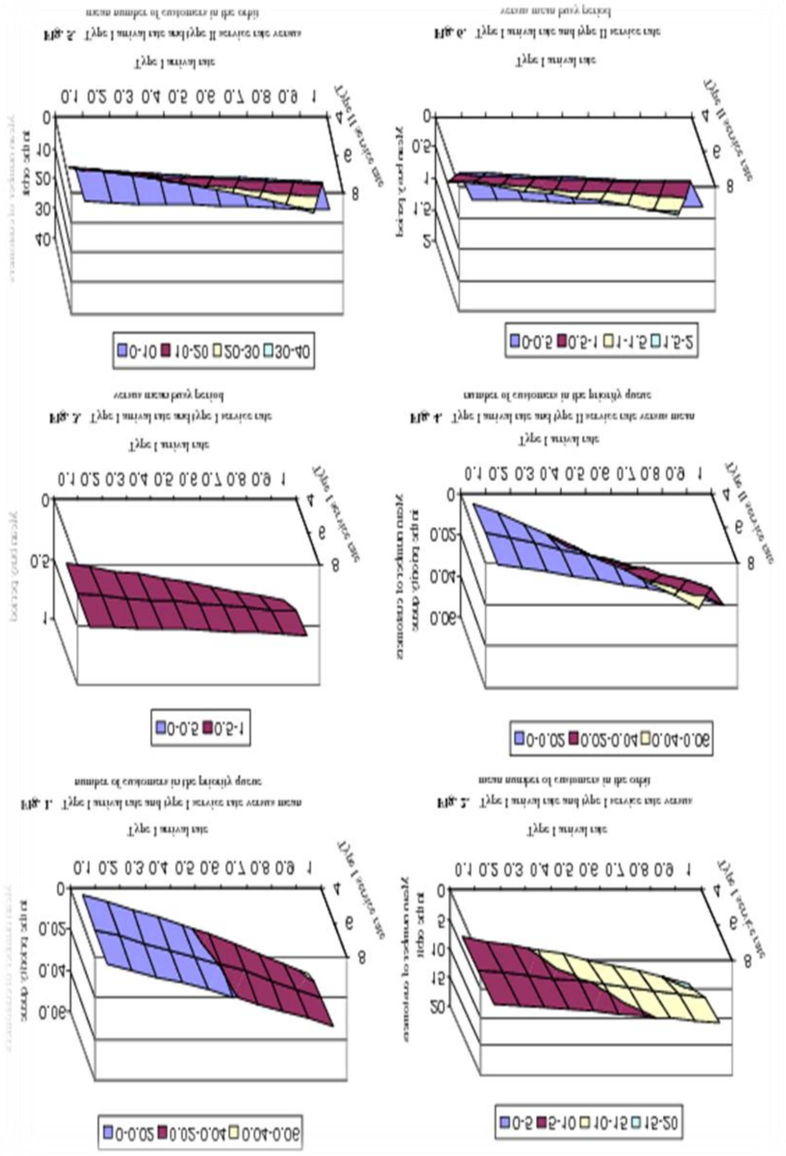
The Probabilities P_I And P_B

λ_2	$\alpha = 0.7, \lambda_2 = 3.0, \mu_2 = 5.0, \bar{c} = 0.4, c_2 = 0.05, p = 0.6$					
	$\mu_1 = 4.0$		$\mu_1 = 6.0$		$\mu_1 = 8.0$	
	P_I	P_B	P_I	P_B	P_I	P_B
0.1	0.3487	0.6513	0.4776	0.5224	0.5627	0.4373
0.2	0.3385	0.6615	0.4653	0.5347	0.5492	0.4508
0.3	0.3282	0.6718	0.4531	0.5469	0.5356	0.4644
0.4	0.3179	0.6821	0.4408	0.5592	0.5220	0.4780
0.5	0.3077	0.6923	0.4286	0.5714	0.5085	0.4915
0.6	0.2974	0.7026	0.4163	0.5837	0.4949	0.5051
0.7	0.2872	0.7128	0.4041	0.5959	0.4814	0.5186
0.8	0.2769	0.7231	0.3918	0.6082	0.4678	0.5322
0.9	0.2769	0.7231	0.3918	0.6082	0.4678	0.5458
1.0	0.2564	0.7436	0.3673	0.6327	0.4407	0.5593

Table 4

The Probabilities P_I And P_B

λ_2	$\alpha = 0.7, \lambda_2 = 3.0, \mu_2 = 5.0, \bar{c} = 0.4, c_2 = 0.05, p = 0.6$					
	$\mu_2 = 4.0$		$\mu_2 = 6.0$		$\mu_2 = 8.0$	
	P_I	P_B	P_I	P_B	P_I	P_B
0.1	0.4176	0.5824	0.4223	0.5777	0.4247	0.5753
0.2	0.4034	0.5966	0.4129	0.5871	0.4176	0.5824
0.3	0.3892	0.6108	0.4034	0.5966	0.4105	0.5895
0.4	0.3750	0.6250	0.3939	0.6061	0.4034	0.5966
0.5	0.3608	0.6392	0.3845	0.6155	0.3963	0.6037
0.6	0.3466	0.6534	0.3750	0.6250	0.3892	0.6108
0.7	0.3324	0.6674	0.3655	0.6345	0.3821	0.6179
0.8	0.3182	0.6818	0.3561	0.6439	0.3750	0.6250
0.9	0.3040	0.6960	0.6534	0.2408	0.3679	0.6321
1.0	0.2898	0.7102	0.6629	0.2477	0.3608	0.6392


0-10 10-20 20-30 30-40
0-0.5 0.5-1 1-1.5 1.5-2
0-0.5 0.5-1
0-0.02 0.02-0.04 0.04-0.06
0-0.02 0.02-0.04 0.04-0.06
0-5 5-10 10-15 15-20

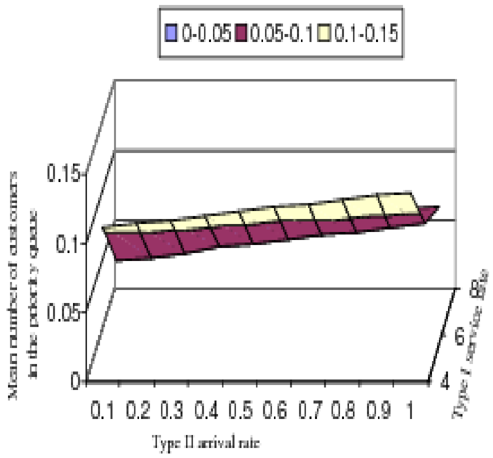


Fig. 7. Type II arrival rate and type I service rate versus mean number of customers in the priority queue

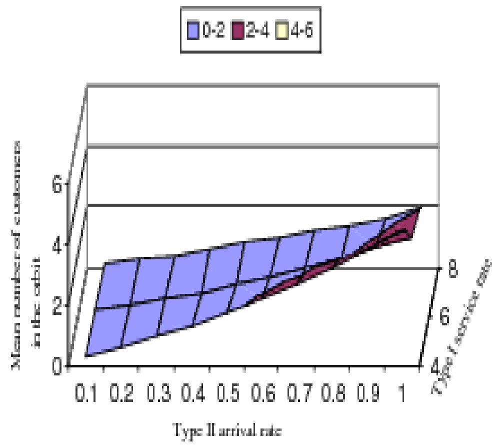


Fig. 8. Type II arrival rate and type I service rate versus mean number of customers in the orbit

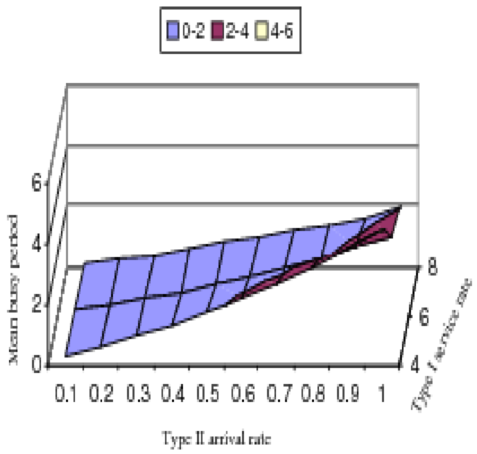


Fig. 9. Type II arrival rate and type I service rate versus mean busy period

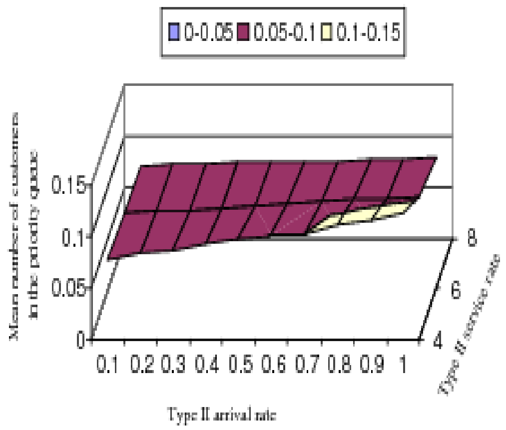


Fig. 10. Type II arrival rate and type II service rate versus mean number of customers in the priority queue

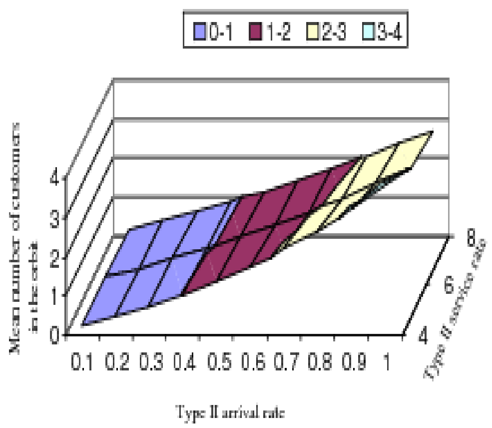


Fig. 11. Type II arrival rate and type II service rate versus mean number of customers in the orbit

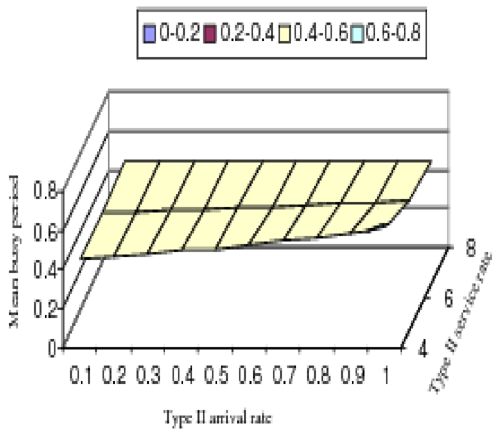


Fig. 12. Type II arrival rate and type II service rate versus mean busy period

II. Conclusion

In The Fore Going Analysis, An $M/G/1$ Retrial Queueing System With Two Type Of Arrivals And With A Control Admissible Policy On Type I Arrivals Is Considered To Obtain Queue Length Distribution And Mean Queue Length. Extensive Numerical Work Has Been Carried Out To Observe The Trends Of The Operating Characters Of The System.

References

[1]. G.I. Falin, And J. G. C. Templeton, Retrial Queues (Chapman And Hall, London, 1997).
[2]. J.R. Artalejo, Accessible Bibliography On Retrial Queues, Mathematical And Computer Modelling, 30, 1999, 1-6.
[3]. J.R. Artalejo, A Classical Bibliography Of Research On Retrial Queues: Progress In 1990 -1997, Top, 7, 1999, 187-211.
[4]. J..R. Artalejo, And G.I. Falin, Standard And Retrial Queueing Systems: A Comparative Analysis, Rev. Mat. Complut., 15, 2002, 101-129.
[5]. J.R. Artalejo, A.N. Dudin, And V.I. Klimenok, Stationery Analysis Of A Retrial Queue With Preemptive Repeated Attempts, Operations Research Letters, 28, 2001,173-190.
[6]. B.D. Choi, And Y. Chang, Single Server Retrial Queues With Priority Calls, Mathematical And Computer Modelling, 30, 1999, 7-32.
[7]. G.I. Falin, On Sufficient Condition For Ergodicity Of Multichannel Queueing Systems With Repeat Calls, Advances In Applied Probability, 16, 1984, 447-448.
[8]. G.I. Falin, Multichannel Queueing Systems With Repeat Calls Under High Intensity Of Repetition, Journal Of Information
[9]. B.D. Choi, And K. K. Park, The M|G|1 Retrial Queueing Systems With Bernoulli Schedule, Queueing Systems, 7, 1990, 219-227.
[10]. R. Kalyanaraman, And B. Srinivasan, A Single Server Retrial Queue With Two Types Of Calls And Recurrent Repeated Calls, International Journal Of Information And Management Sciences, 14, 2003, 49-62.
[11]. J. Wang, On The Single Server Retrial Queue With Priority Subscribers And Server Breakdowns, Journal Of Systems Science And Complexity, 21, 2009, 304-315.
[12]. S.V. Pustova, Investigation Of Call Centers As Retrial Queueing Systems, Cybernetics And System Analysis, 46, 2010, 494-499.
[13]. J.R. Artalejo, And I.Atencia, On The Single Server Retrial Queue With Batch Arrivals, Sankhya : The Indian Journal Of Statistics, 66 (1), 2004, 140-158.
[14]. R. Kalyanaraman, And B. Srinivasan, A Retrial Queueing System With Two Types Of Calls And Geometric Loss, International Journal Of Information And Management Sciences, 15 (4), 2004, 75-88.
[15]. N. Thillaigovindan And R. Kalyanaraman, A Feedback Retrial Queueing System With Two Types Of Arrivals, Proceeding Of QTNA 2011, Korea, 2011, 177-181.
[16]. R. Kalyanaraman, A Feedback Queueing System With Two Types Of Arrivals And With Constant Retrial Rate, Proceeding Of International Conference On Mathematical Modelling And Applications In Industrial Problems, Mac Millan, 2012, 357-367.
[17]. G.I. Falin, J. R. Artalejo And M. Martin, On The Single Server Retrial Queue With Priority Customers, Queueing Systems, 14, 1993, 439-455.