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Some Common Fixed Point Theorems for Fuzzy Maps under Non-expansive Type Condition

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Abstract*: The aim of this paper is to prove some common fixed point theorems for fuzzy mapping involving non-expansive type condition. Our results extend and generalized several known results existing in the literature.*
Key Words*: Fixed point ,fuzzy mapping, and non expansive mapping.*

I. Introduction & preliminaries

The fuzzy set theory was introduced by L. Zadeh[16] in 1965 and fixed point theorems for fuzzy mappings were obtained by Chang, Heilpern and others [2-4, 5, 7, 9-10, 12-15] .In [9] & [10] Lee et al. obtained a common fixed point theorem for sequence of fuzzy mapping which generalized the result of [1].Som and Mukherjee[14] extended the result of Heilpern for nonexpansive fuzzy mappings on a metric space and gave a fixed point theorem for generalized nonexpansive fuzzy mappings to extend a result of Bose & Sahani [1]. Lee et al [9,10] define g-nonexpansive and g-nonexpansive type fuzzy mapping satisfying certain conditions on a Banach Space. In [12] Rhoades prove a common fixed point theorem involving a very general contractive condition for fuzzy maps on complete linear metric space. Later, in [13] Rhoades generalized his own result for sequence of fuzzy mappings on complete linear metric space. Recently, Saluja et al. [15] obtained a common fixed point theorem for sequence of fuzzy mappings using a more general condition involving nonexpansive self-maps in complete metric space. In this paper we establish some common fixed point theorems for fuzzy mapping involving generalized nonexpansive type condition. Throughout this paper we will be using the terminology and notations of Heilpern [7].

Definition 1.1 .A fuzzy set A in complete metric space X is a function from X into [0,1]. If $x \epsilon X$, the function value $A(x)$ is called the grade of member of X in A.The α-level set of A,denoted by $A_\alpha = \{x: A(x) \geq \alpha\}$,if $\alpha \epsilon [0,1], A_0 = \{x: A(x) > 0\}$.

Definition 1.2 A fuzzy set A is said to be an approximate quantity iff A_α is compact and convex for each $\alpha \epsilon [0,1]$ and $\sup_{x \epsilon X} A(X) = 1$
When A is an approximate quantity and $A(x_0) = 1$ for some $x_0 \epsilon X, A$ is identified with an approximation of x_0.
The collection of all fuzzy set in X is denoted by $F(X)$ and $W(X)$ is the subcollection of all approximate quantities.

Definition 1.3 Let $A, B \in W(X), \alpha \in [0,1]$.Then

$$D_\alpha(A, B) = \inf_{x \epsilon A_\alpha, y \epsilon B_\alpha} d(x, y)$$

$$D(A, B) = \sup_\alpha D_\alpha(A, B)$$

$$H_\alpha(A, B) = dist(A_\alpha, B_\alpha)$$

where " dist'' is Hausdorff distance.

Definition 1.4 Let $A, B \epsilon W(X)$, then A is said to be more accurate than B,denoted by $A \subset B$ iff $A(X) \leq B(X)$ for each $x \in X$.
The relation "$\subset$" induces a partial ordering on the family $W(X)$.

Definition 1.5 Let X and Y be two complete linear metric space. F is called a fuzzy mapping if F is a mapping from the set X into $W(X)$.
A fuzzy mapping F is a fuzzy subset of $X \times Y$ with membership function $F(x, y)$. The function value $F(x, y)$is the grade of membership of Y in $F(X)$. Each fuzzy mapping is a set valued mapping.

Definition 1.6 A mapping $T: X \to W(X)$ is said to be nonexpansive, if for all $x, y \in X, d(Tx, Ty) \leq d(x, y)$

Lee[9,10] proved the following Lemma

Lemma 1.1 Let (X, d) be a complete linear metric space,F is a fuzzy map from X into $W(X)$and $x_0 \in X$ then there exist an $x_1 \in X$ such that $\{x_1\} \subset F(x_0)$.
The following two lemmas are given by Heilpern [7]
Lemma 1.2 Let$A, B \in W(X), \alpha \in [0,1]$ and $D_\alpha(A, B) = \inf_{x \epsilon A_\alpha, y \epsilon B_\alpha} d(x, y)$where
$A_\alpha = \{x: A(X) \geq \alpha\}$ Then $D_\alpha(x, A) \leq d(x, y) + D_\alpha(y, A)$ for each $x, y \in X$.
Lemma 1.3 Let $H_\alpha(A, B) = dist(A_\alpha, B_\alpha)$ where "dist" is the Hausdorff distance. If $\{x_0\} \subset A$ then $D_\alpha(x_o, B) \leq H_\alpha(A, B)$ for each $B \in W(X)$.
Rhoades [12] proved the following common fixed point theorem involving a very general contractive condition for fuzzy maps on complete linear metric space.

Theorem 1.1 Let (X, d) be a complete linear metric space F, G are fuzzy mappings from X into $W(X)$satisfying
$H(Fx, Gy) \leq Q\big(m(x, y)\big)$ For all $x, y \in X$.
Where $m(x, y) = \max\left\{d(x, y), D_\alpha(x, Fx), D_\alpha(y, Gy), \frac{1}{2}[D_\alpha(x, Gy) + D_\alpha(y, Fx)]\right\}$ (1.1)
Q is real valued function defined on D, the closure of range of d satifying the following three conditions:
(a) $0 < Q(s) < s$ for each $s \in D/\{0\}$and $Q(0) = 0$,
(b) Q in non decreasing on D,and
(c) $g(s) = s/(s - Q(s))$ is non decreasing on $D/\{0\}$,

Then there exist a point $z \in X$ such that $\{z\} \subset Fz \cap Gz$.
In [13] Rhoades generalized the result of Theorem 1.1 for sequence of fuzzy maps on complete linear metric space .He proved the following theorem:

Theorem 1.2 Let g be a non expansive self mapping of a complete linear metric space(X, d).Let $\{F_i\}$ be a sequence of fuzzy mapping from X into
$W(X)$ for each pair of fuzzy mappings F_i, F_j and for any $x \in X$,$\{u_x\} \subset F_i(x)$,there exists a$\{v_y \subset F_j(y)\}$ for all $y \in X$ such that
$$D(\{u_x\}, \{v_y\}) \leq Q(m(x, y))$$
Where $m(x, y) = \max\{d(g(x), g(u_x)), d\left(g(y), g(v_y)\right), d(g(x), g(y)), \frac{1}{2}[d\left(g(x), g(v_y)\right) + d(g(y), g(u_x)]\}$ (1.2)
Where Q satisfying the conditions (a) - (c) of theorem 1.1 then there exist $\{p\} \subset \bigcap_{i \in N} F_i(p)$.

Main Result: Now we able to prove our main result

Theorem 2.1 Let (X, d) be a complete linear metric space F, G are fuzzy mappings from X into $W(X)$satisfying.
$H(Fx, Gy) \leq a \max\{d(x, y), D_\alpha(y, Gy)\} + b \max\{D_\alpha(x, Fx), D_\alpha(y, Gy), D_\alpha(y, Fx)\} + c[\ (x, Gy), D_\alpha(y, Fx)]\}$ (2.1)
Where a, b, c are non negative real numbers such that $a + b + 2c = 1$,then there exist a point $z$in X.which is a common fixed point of maps F and G i.e. $\{z\} \subset Fz \cap Gz$.

Proof : Let $x_0 \in X$,then by Lemma1.1,we can choose $x_1 \in X$ such that$\{x_1\} \subset Fx_0$.choose x_2 such that $d(x_1, x_2) \leq H(Fx_0, Gx_1)$, continuing the process, we obtain a sequence $\{x_n\}$ such that $\{x_{2n+1}\} \subset Fx_{2n}$,$\{x_{2n+2}\} \subset Gx_{2n+1}$ and

$$d(x_{2n+1}, x_{2n+2}) \leq H(Fx_{2n}, Gx_{2n+1}) \text{ where } n = 1,2,3$$
Applying(2.1) and using triangular inequality, we have
$$d(x_{2n}, x_{2n+1}) \leq H(Fx_{2n-1}, Gx_{2n})$$

$$\leq a\max\{d(x_{2n-1},x_{2n}),D_\alpha(x_{2n},Gx_{2n})\}$$
$$+\,b\max\{D_\alpha(x_{2n-1},Fx_{2n-1}),D_\alpha(x_{2n},Gx_{2n}),D_\alpha(x_{2n},Fx_{2n-1})\}$$
$$+\,c[\,D_\alpha(x_{2n-1},Gx_{2n}),D_\alpha(x_{2n},Fx_{2n-1})]\}$$

$$\leq a\max\{d(x_{2n-1},x_{2n}),d(x_{2n},x_{2n+1})\}$$
$$+\,b\max\{d(x_{2n-1},x_{2n}),d(x_{2n},x_{2n+1}),d(x_{2n},x_{2n})\}$$
$$+\,c[\,d(x_{2n-1},x_{2n+1}),d(x_{2n},x_{2n})]$$

$$\leq a\max\{d(x_{2n-1},x_{2n}),d(x_{2n},x_{2n+1})\}$$
$$+\,b\max\{d(x_{2n-1},x_{2n}),d(x_{2n},x_{2n+1})\}+\,c[\,d(x_{2n-1},x_{2n})+d(x_{2n},x_{2n+1})]$$

$$\leq (a+b)\max\{d(x_{2n-1},x_{2n}),d(x_{2n},x_{2n+1})\}+c[\,d(x_{2n-1},x_{2n})+d(x_{2n},x_{2n+1})]$$

If $d(x_{2n-1},x_{2n})<d(x_{2n},x_{2n+1})$ for some n,we have

$$d(x_{2n},x_{2n+1})<(a+b+2c)d(x_{2n},x_{2n+1})$$
$$=d(x_{2n},x_{2n+1})$$

A contradiction .Thus $d(x_{2n},x_{2n+1})<d(x_{2n-1},x_{2n})$.

Hence for all positive integer n,

$$d(x_{2n},x_{2n+1})\leq d(x_0,x_1) \qquad (2.2)$$

Again applying (2.1) and using (2.2),we get

$$\leq H(Fx_1,Gx_2)$$

$$d(x_2,x_3)\leq a\max\{d(x_1,x_2),d(x_2,x_3)\}+b\max\{d(x_1,x_2),d(x_2,x_3),d(x_2,x_2)\}+c[\,d(x_1,x_3)+d(x_2,x_2)]$$
$$\leq a\max\{d(x_0,x_1),d(x_0,x_1)\}+b\max\{d(x_0,x_1),d(x_0,x_1)\}+c[\,d(x_1,x_3)] \qquad (2.3)$$

Applying (2.1) again and using (2.2),we have

$$d(x_1,x_3)\leq H(Fx_0,Gx_2)$$

$$\leq a\max\{d(x_0,x_2),d(x_2,x_3)\}+b\max\{d(x_0,x_1),d(x_2,x_3),d(x_2,x_1)\}+c[\,d(x_0,x_3)+d(x_2,x_1)]$$

$$\leq 2a\,d(x_0,x_1)+b\,d(x_0,x_1)+4c\,d(x_0,x_1)$$
$$\leq [2(a+b+2c)-b]d(x_0,x_1)$$
$$\leq (2-b)\,d(x_0,x_1) \qquad (2.4)$$

Using (2.3) & (2.4) we get

$$d(x_2,x_3)\leq ad(x_0,x_1)+b\,d(x_0,x_1)+(2c-bc)d(x_0,x_1)$$
$$\leq (1-bc)d(x_0,x_1)$$

It is easy to show that

$$d(x_{n+1},x_n)\leq (1-bc)^{[n/2]}d(x_0,x_1)$$

Where $[n/2]$means the greatest integer not exceeding $n/2$

We conclude that $\{x_n\}$ is a Cauchy sequence.Since X is complete, therefore $\{x_n\}$ converges to the point z(say).
Since $\alpha\in[0,1]$ then using Lemma 1.2, 1.3 and (2.1) we have

$$D_\alpha(z,Fz)\leq d(z,x_{2n+2})+D_\alpha(x_{2n+2},Fz)$$

$$\leq d(z,x_{2n+2})+H_\alpha(Fz,Gx_{2n+1})$$
$$\leq d(z,x_{2n+2})+H(Fz,Gx_{2n+1})$$

Letting n tend to infinity,we get

$$D_\alpha(z,Fz)\leq \lim_{n\to\infty}d(z,x_{2n+2})+\lim_{n\to\infty}H(Fz,Gx_{2n+1})$$
$$\leq \lim_{n\to\infty}H(Fz,Gx_{2n+1}) \qquad (2.5)$$

Again using (2.1),we have

$$H(Fz, Gx_{n+1}) \leq \quad a\max\{d(z, x_{2n+1}), D_\alpha(x_{2n+1}, Gx_{2n+1})\}$$
$$+ b\max\{D_\alpha(z, Fz), D_\alpha(x_{2n+1}, Gx_{2n+1}), D_\alpha(x_{2n+1}, Fz)\} + \quad c\,[D_\alpha(z, Gx_{2n+1})$$
$$+ D_\alpha(x_{2n+1}, Fz)]$$

$$H(Fz, Gx_{n+1})$$
$$\leq \quad a\max\{d(z, x_{2n+1}), d(x_{2n+1}, x_{2n+2})\}$$
$$+ b\max\{d(z, Fz), d(x_{2n+1}, x_{2n+2}), d(x_{2n+1}, Fz)\} + \quad c\,[d(z, x_{2n+2}) + d(x_{2n+1}, Fz)]$$

$$\leq a\max\{d(z, x_{2n+1}), d(x_{2n+1}, x_{2n+2})\} + b\max\{d(z, Fz), d(x_{2n+1}, x_{2n+2}), d(x_{2n+1}, Fz)\} +$$
$$c[\,d(z, x_{2n+1}) + d(x_{2n+1}, x_{2n+2}) + d(x_{2n+1}, Fz)]\}$$

Letting $n \to \infty$,we have
$$\lim_{n\to\infty} H(Fz, Gx_{2n+1}) < (a + b + 2c)d(z, Fz)$$
$$= d(z, Fz) \qquad (2.6)$$
Using (2.5) & (2.6) we have
$$D_\alpha(z, Fz) < d(z, Fz).$$
a contradiction. Hence we must have $D_\alpha(z, Fz) = 0$. since α is an arbitrary number in $[0,1]$ it follows that $D(z, Fz) = 0$ which implies that $\{z\} \subset Gz$
Similarly it can be shown that $\{z\} \subset Fz$ i.e. z is common fixed point of set valued maps F and G.
Next we prove a common fixed point theorem for sequence of fuzzy mappings.

Theorem 2.2 Let g be a non expansive self mapping of a complete linear metric space(X, d).Let $\{F_i\}$be a sequence of fuzzy mappings from X into $W(X)$.For each pair of fuzzy mapping F_i, F_j and for any $x \in X, \{u_x\} \subset F_i(x)$,there exist $\{v_y\} \subset F_j(y)$for all $y \in X$such that
$$D(\{u_x\}, \{v_y\}) \leq a\ \max\left\{d(g(x), g(y)), d\left(g(y), g(v_y)\right)\right\} +$$
$$b\max\left\{d(g(x), g(u_x)), d\left(g(y), g(v_y)\right), d(g(y), g(u_x))\right\} +$$
$$c[\ d\left(g(x), g(v_y)\right) + d(g(y), g(u_x))]\} \qquad (2.7)$$

Where a, b, c are non negative real numbers such that $a + b + 2c = 1$,then there exists $\{p\} \subset \bigcap_{i\in N} F_i(p)$. i.e. p is a common fixed point of sequence of fuzzy mappings.

Proof : Let $x_0 \in X$,then by Lemma1.1,we can choose $x_1 \in X$such that $\{x_1\} \subset F(x_0)$.similarly for $x_1 \in X$ we can choose $x_2 \in X$ such that $\{x_2\} \subset F(x_1)$. In general $\{x_{n+1}\} \subset F_{n+1}(x_n)$.
Applying (2.7) and using triangular inequality, we have
$$d(x_n, x_{n+1}) = D(\{x_n\}, \{x_{n+1}\})$$
$$\leq a\max\{d(g(x_{n-1}), g(x_n)), d(g(x_n), g(x_{n+1}))\} +$$
$$b\max\{d(g(x_{n-1}), g(x_n)), d(g(x_n), g(x_{n+1})), d(g(x_n), g(x_n))\} +$$
$$c[\ d(g(x_{n-1}), g(x_{n+1})) + d(g(x_n), g(x_n))\ ]$$
Since g is non expansive and $D(\{x_n\}, \{x_{n+1}\}) = d(x_n, x_{n+1})$,we get

$$d(x_n, x_{n+1}) \leq a\max\{d(x_{n-1}, x_n), d(x_n, x_{n+1})\}$$
$$+ b\max\{d(x_{n-1}, x_n), d(x_n, x_{n+1}), d(x_n, x_n)\} \ + c[\ d(x_{n-1}, x_{n+1}) + d(x_n, x_n)]$$
$$\leq (a + b)\max\{d(x_{n-1}, x_n), d(x_n, x_{n+1})\} + c[\ d(x_{n-1}, x_n) + d(x_n, x_{n+1})]$$
If $\quad d(x_{n-1}, x_n) < d(x_n, x_{n+1})$ for some n,then we have

$$d(x_n, x_{n+1}) \leq (a + b)\max\{d(x_n, x_{n+1}), d(x_n, x_{n+1})\} + c[\ d(x_n, x_{n+1}) + d(x_n, x_{n+1})]$$
$$= (a + b + 2c)\ d(x_n, x_{n+1})$$
$$= d(x_n, x_{n+1})$$
A contradiction. Thus $d(x_n, x_{n+1}) \leq d(x_{n-1}, x_n)$,
Hence for all positive intgers n,

$d(x_n, x_{n+1}) \leq d(x_0, x_1)$ ... (2.8)

Again applying (2.7) and (2.8),we get

$$d(x_2, x_3) = D(x_2, x_3)$$
$$\leq a \max\{d(g(x_1), g(x_2)), d(g(x_2), g(x_3))\}$$
$$+ b \max\{d(g(x_1), g(x_2)), d(g(x_2), g(x_3)), d(g(x_2), g(x_2))\} + c[\ d(g(x_1), g(x_3))$$
$$+ d(g(x_2), g(x_2))]$$

Since g is nonexpansive ,then

$$d(x_2, x_3) \leq a \max\{d(x_1, x_2), d(x_2, x_3)\} + b \max\{d(x_1, x_2), d(x_2, x_3), d(x_2, x_2)\} + c[\ d(x_1, x_3) + d(x_2, x_2)]$$
$$\leq a \max\{d(x_0, x_1), d(x_0, x_1)\} + b \max\{d(x_0, x_1), d(x_0, x_1)\} + c[\ d(x_1, x_3)]$$
$$\leq (a + b)d(x_0, x_1) + cd(x_1, x_3) \quad \ldots (2.9)$$

Again applying (2.7)& (2.8),we have

$$d(x_1, x_3) = D\big(\{x_1\}, \{x_{3\}}\big)$$
$$\leq a \max\{d(g(x_0), g(x_2)), d(g(x_2), g(x_3))\}$$
$$+ b \max\big\{d\big(g(x_0), g(x_1)\big), d\big(g(x_2), g(x_3)\big), d\big(g(x_2), g(x_1)\big)\big\} + c[\ d(g(x_0), g(x_3))$$
$$+ d(g(x_2), g(x_1))]$$

$$d(x_2, x_3) \leq a \max\{d(x_0, x_2), d(x_2, x_3)\} + b \max\{d(x_0, x_1), d(x_2, x_3), d(x_2, x_1)\} + c[\ d(x_0, x_3) + d(x_2, x_1)]$$
$$\leq a \max\{d(x_0, x_1) + d(x_1, x_2), d(x_2, x_3)\}$$
$$+ b \max\{d(x_0, x_1), d(x_2, x_3), d(x_2, x_1)\} + c[\ d(x_0, x_1) + d(x_1, x_2) + d(x_2, x_3) + d(x_2, x_1)]$$

$$\leq a \max\{d(x_0, x_1) + d(x_0, x_1), d(x_0, x_1)\} + b \max\{d(x_0, x_1), d(x_0, x_1), d(x_0, x_1)\ \} + c[\ d(x_0, x_1) + d(x_0, x_1) + d(x_0, x_1) + d(x_0, x_1)]$$
$$\leq 2a\ d(x_0, x_1) + bd(x_0, x_1) + 4c\ d(x_0, x_1)$$
$$\leq (2a + b + 4c)d(x_0, x_1)$$
$$= [2(a + b + 2c) - b]d(x_0, x_1)$$
$$= (2 - b)d(x_0, x_1) \quad \ldots (2.10)$$

Using (2.9) & (2.10),we get

$$d(x_2, x_3) \leq (a + b)d(x_0, x_1) + (2c - bc)d(x_0, x_1)$$
$$\leq (a + b + 2c - bc)d(x_0, x_1)$$
$$= [1 - bc]d(x_0, x_1)$$

It is easy to show that

$$d(x_{n+1}, x_n) \leq (1 - bc)^{[n/2]} d(x_0, x_1)$$

Where $[n/2]$means the greatest integer not exceeding $n/2$

Since $bc < 1$,therefore $\{x_n\}$ is a Cauchy sequence and hence converges to the limit p(say).

Let F_m be an arbitrary member of $\{F_i\}$.Since $\{x_n\} \subset F_m(x_{n-1})$ by Lemma 1.1,there exists a $v_n \in X$.such that $\{v_n\} \subset F_m(p)$ for all n .

Applying (2.7) again and using (2.8),we have

$$d(x_n, v_n) = D(\{x_n\}, \{v_n\})$$
$$\leq\ a \max\{d(x_{n-1}, p), d(p, v_n)\} + b \max\{d(x_{n-1}, x_n), d(p, v_n), d(p, x_n)\} + c[\ d(x_{n-1}\ , v_n) + d(p, x_n)]$$

If $\lim_{n\to\infty} v_n \neq p$ letting the limit $n \to \infty$,we have

$$d(p, v_n) \leq\ a \max\{d(p, p), d(p, v_n)\} + b \max\{d(p, p), d(p, v_n), d(p, p)\} +\ c[\ d(p, v_n) + d(p, p)]$$
$$\leq (a + b + c)d(p, v_n)$$
$$< d(p, v_n)$$

A contradiction .Hence $\lim_{n\to\infty} v_n = p$.

Since F_m is arbitrary, therefore $\{p\} \subset \bigcap_{i=1}^{n} F_i(p)$.

This completes the poof.

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