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Mathematical Modeling of Concentration of Biochemical Oxygen Demand with Different Zones in Damoadar River

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Abstract*: In this paper various mathematical models developed to date accounts for only that portion of BOD which is in dissolved form and not the least in settleable form. These models also do not account for the storage zone in a scattered way in rivers and hence do not represents the actual situation caused by the discharge of partially treated/ untreated waste water, which contains a significant portion of BOD in settleable form into the water body with a large width where the water becomes stagnant anywhere, due to rag/garbage in rivers. The present paper represents a model to predict the concentration of total BOD when partially treated/untreated waste water is discharged into the Damodar river having staggered storage zones and thus address the above stated situation.*
Keywords: *Damodar river, Mathematical modeling, Biochemical oxygen demand, Settleable BOD,Main zone, Scattered storage zone.*

I. Introduction

Damodar is the main river and a large number of tributaries join it in the course of its flow. One of its main source streams,locally known as Sonasathi originates from Khamar pat of Palamu near Chutanpani and later falls nearly 30 m. down and form the Sitabi Nala. Damodar river drops nearly 450m. within a distance of nearly 1.5 K.ms and then flow between Ranchi and Hazaribagh Plateau. The main damodar river flows from West to East and its different tributaries join from North and South.The Barakar and important tributary of the damodar ,runs almost parallel to it before it joins the main stream. Damodar river is considerably wide and many tributaries carry the effluentsof different industries such as coal mines,coal washeries, coke plants and townships with various types of pollutants, which have different characteristics in nature to degrate the river ecosystem. Dissolved Oxygen is the surrogate variable for the general health of an aquatic eco- system. Low dissolved oxygen in river adversely affects the aquatic system and consequently life of human being. The waste dumped into the river consumes the oxygen dissolved in river water for the process of stabilization. The amount of oxygen consumed by bacteria to stabilize the organic waste aerobically at a stated temperature and in specified period of time, called BOD, is used as an adjunct to DO determination. An increase in use of coal as a source of energy has resulted into serious environmental problems. The problem of water pollution is one among them .In Jharia area Damodar river is the main source of water and the river also acts as receiver of waste water . Even the ground water in these area has become contaminated due to seepage to waste water specially leachets of over burden dumps .Water pollution due to coal mining may be classified into-
(a)Mine water pumped out of coal mines both – open cast as well as underground.(b) Leachets and wash off from overburden dumps and solid waste deposits (c)Waste water from coal based and related industries.Row coal possess various types of inorganic impurities like pyrites , marc rite, hematite and siderite which generally produce sulphuric acid and other soluble salts when it come in contact with air moisture. This result to the problem of acid mine discharge (A.M.D.).The acid generation is pH dependent process which is completed in three steps.The AMD contains high concentration of different types of inorganic constituents. The acid dissolution of the silicates and bedrocks produces high concentration of Ai, Si, Ca, Mg, Na, and K. The second major source of inorganic pollutants is the oxidation of metallic sulphides. This results into a higher concentration of Fe, Mn, Zn, Cu, Pb, and As. These pollutants eventually contaminate the surface water making it unsuitable for use. The discharge of mine water should be diverted and collected in a settling pond. A large settling pond centrally located in three or four mines may be better choice .The mine water collected in the pond may be used for irrigating nearby fields also. A suitable treatment plant may make the mine water even potable The model presented by Streeter and Phelps 1925 [15] and subsequent mathematical formulation by Fair 1939[14] are the first published mathematical models which were used to determine the DO condition in a stream below single point source under steady state conditions. The interesting characteristic of Streeter and Phelps model is the idea that the river may be represented by a single onedimensional system. The model was well suited with the computational capabilities of that time, but it did not include that part of BOD which is in settleable form. Modeling of river flow studied by Vlatschii (2010) [1]. This situation arises when partially treated/untreated waste enters the river. Bhargava 1983, 1986(b) [11,9] incorporated the settleable part of BOD

along with the soluble part and evaluated the model for accurate prediction of the DO-sag related parameters. Bhargava 1983, 1986(a) and 1986(b) [11, 8 & 9], however, did not include the dispersion term in his model. Tyagi. et. al. 1999 [3] accounted for both the parts of BOD in their model along with dispersion. Various authors (Chapra and Runkel 1999 [5], Thackston and Schnelle 1970 [14], Gooseff 2005 [7]) explored and analyzed the impact of dead zone(also referred to as storage zone) in the last three decades. Several approaches have been developed to determine the impact of these storage zones on solute transport (Rutherford 1994 [10]. The model presented by Chapra and Runkel (1999)[4] is extended to incorporate settleable part of BOD along with the dissolved part which represents the situation in which the partially treated/untreated waste enters the river having stagnant zone on the bank of river. The present work, therefore, addresses such a situation and develops a model to study the cumulative effect of storage zone and two types of BOD below a point source, under steady state condition 2.

Nomenclature

λ : scattered storage zone coefficient
α : storage zone exchange coefficient
x' : distance at which all the setteable BOD removed
u : mean cross- sectional flow velocity
B_d : dissolved part of BOD in the main zone
B_s : dissolved part of BOD in the storage zone
L : settleable part of BOD in the main zone
L_o : initial setteable BOD in the main zone
D : DO deficit in the main zone
D_s : DO deficit in the storage zone
K_d : decomposition rate of the dissolved BOD in the main zone
K'_d : decomposition rate of the dissolved BOD in the storage zone
K_s : decomposition rate of settleable BOD in the main zone
v : settling velocity of the particles
d : depth of the river

II. Formulation Of Mathematical Model

A mathematical model is developed for the physical system as mentioned below:

2.1 Physical System

Consider a river in which there is an immobile storage zone on both the banks of a river. Let the partially treated/untreated oxygen demanding waste be released continuously at a constant rate into the river through a point source.

2.2 Mathematical Representation

The cross-section of the river is divided into two zone's namely the main zone in the centre of the river and the scattered storage zone where the velocity is assumed to be zero. A mathematical model is developed for the above stated system based on the following assumptions.

(a)The entire BOD in the waste is in two forms namely dissolved and settleable. The dissolved part of BOD is decaying according to first order kinetics, while the settleable part of BOD is being removed by a linear law The ratio of settleable part to total BOD is assumed to be fixed at outfall.

(b) The size of storage zone is As and it consists of two parts located near the two banks of the river while the size of main zone is A which is located in the centre of river.

(c) No transverse gradient exists within any of the two zones. However, there is exchange of mass between the two zones which is linearly related to the difference in the respective concentrations.

(d) In the main zone, advection, reaction and exchange of mass are considered to be the relevant phenomena.

(e) In the storage zone only exchange of mass with the main zone and reaction within the storage zone are considered.

(f) In the storage zone the settleable BOD is settled at the outfall itself while in the main zone it is carried forward with the flow and is settled only after a particular distance downstream. Hence the effects of advective forces are considered and included in the transition time Ts, in which all
the settleable part get removed from the waste. The transition time Ts = d/v would be longer for deeper rivers and for smaller flocculated particle size.

(g) Exchange of mass between two zones is considered only for dissolved part of BOD.

(h) The temperature effect on decomposition rates is same in each zone.

(i) There is no other source and sink of BOD in the river.

Using the above stated assumptions, the steady-state mass balance equations for BOD in the main zone and the storage zone respectively are given as follows:

2.2.1 *Main-Zone*

$$0 = u\frac{dB_d}{dx} - k_d B_d + \alpha(\lambda B_s - B_d) \quad (1)$$

$$L(x) = L_0\left\{1 - \frac{\frac{x}{u}}{\frac{d}{v}}\right\}, x \le x' \quad (2)$$

$$= 0, x > x'$$

2.2.2 Scattered Storage Zone

$$0 = -\alpha\frac{A}{A_s}(\lambda B_s - B_d) - K'_d B_s \quad (3)$$

2.3 Boundary Conditions

The associated boundary conditions reflecting the release of BOD causing material are $B_d = B_0$ at $x=0$.

Method Of Solution

The value of B_s computed from equation (3) as follows:

$$B_s = \frac{1}{\lambda}\left[\frac{\frac{\alpha}{k'_d}}{\left(\frac{\alpha}{K'_d}+\frac{A_s}{A}\right)}\right] B_d \quad (4)$$

$\lambda = 1, at\ x \le 50\ m;\ \lambda = 0.75\ at\ x > 50m$

B_s is substituted in equation (1) to give the following equation

$$0 = -u\frac{dB_d}{dx} - E_d k_d B_d \quad (5)$$

Where

$$E_d = 1 + \left[\frac{\left(\frac{\alpha}{K_d}\right)\left(\frac{A_s}{A}\right)}{\left(\frac{\alpha}{K'_d}+\frac{A_s}{A}\right)}\right]$$

E_d can be calculated as dimensionless enhancement factor representing the impact of the storage Zone on decomposition.

Let $\overline{K_d} = E_d K_d$

Where $\overline{K_d}$ *an apparent decomposition ratio that accounts for the scattered storage – zone (per* day).

Equation (5) now converts to

$$0 = -u\frac{dB_d}{dx} - \overline{K_d}\, B_d \quad (6)$$

On solving (6) with the boundary condition $B_d = B_0$ at x = 0, we get

$$B_d = B_0 e^{-\left(\frac{K_d}{u}\right)x} \quad (7)$$

So that the total BOD (B_M) in the main zone is given by the following equations

$$B_M = B_0 e^{-\left(\frac{K_d}{u}\right)x} + L_0\left(1 - \frac{\frac{x}{u}}{\frac{d}{v}}\right), x \le x' \quad (8)$$

$$B_M = B_0 e^{-\left(\frac{K_d}{u}\right)x},\ x > x' \quad (9)$$

Using equation (4), (8a) and (8b), then the total BOD in the scattered storage zone is given by

$$B_s = \left[\frac{\left(\frac{\alpha}{K'_d}\right)}{\left(\frac{\alpha}{K'_d}+\frac{A_s}{A}\right)}\right]\left[B_0\, e^{-\left(\frac{K_d}{u}\right)x} + L_0\left(1 - \frac{\frac{x}{u}}{\frac{d}{v}}\right)\right], x = 0 \quad (10)$$

$$B_s = \left[\frac{\left(\frac{\alpha}{K'_d}\right)}{\left(\frac{\alpha}{K'_d}+\frac{A_s}{A}\right)}\right]\left[B_0\, e^{-\left(\frac{K_d}{u}\right)x}\right] > 0 \quad (11)$$

III. Results And Discussion

To analyze the cumulative impact of settleable BOD and storage zone on Damodar river's DO, the model is applied to Uvas Creek for which the physical parameters with their values are outlined in Table-**1.**

To predict the concentration of BOD in the considered Damodar river system, some kinetic and chemical parameters are appropriately taken from the literature and their values are given in Table-**2**.The model is applied to simulate a 4 Km stretch of the Creek along with the present model. The result of conventional S-P model and storage zone version of S-P model (presented by Chapra) are also displayed for comparison.

Case-1: The entire BOD is in dissolved form and the Creek has no storage zone.

Case-2: The entire BOD is in dissolved form and the Creek has storage zone .
Case-3: A part of total BOD is in settleable form and the Creek has storage zone.

Parameter	Value	Unite
U	2707	m/day
α	3.67	Per day
B_0	6.0	Mg/L
L_0	4.0	Mg/L
A	0.4	m^2
A_s	0.7	m^2
(v/d)	6	Per day
x'	650	m

Table:1 Hydraulic Parameters

Parameter	value	Units
K_d	4	Per day
K'_d	4	Per day
K_s	8	Per day

. **Table: 2 Chemical and kinetic parameters**

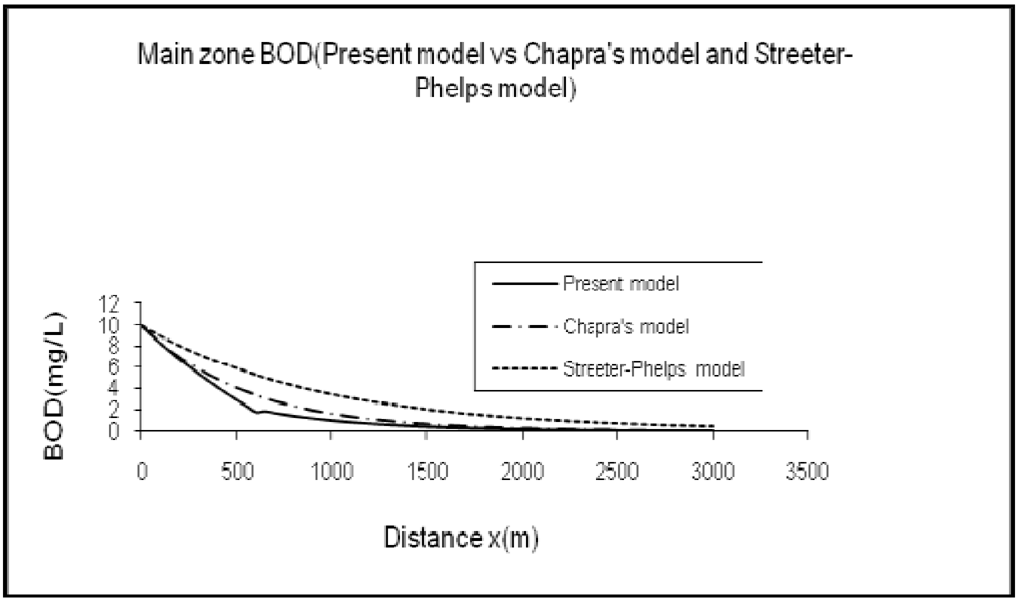

Figure1 Depicts the comparative BOD distribution in the main zone as predicted by the conventional Streeter-Phelps model, Streeter-Phelps storage zone model (i.e.Chapra's model) and the present model. It is observed from the figure that the concentration of BOD by the present model is less than that predicted by S-P model as well as the Chapra's model. Since the settle able part is removed faster, it assimilates more BOD and consequently the remaining BOD would be less with distance downstream.

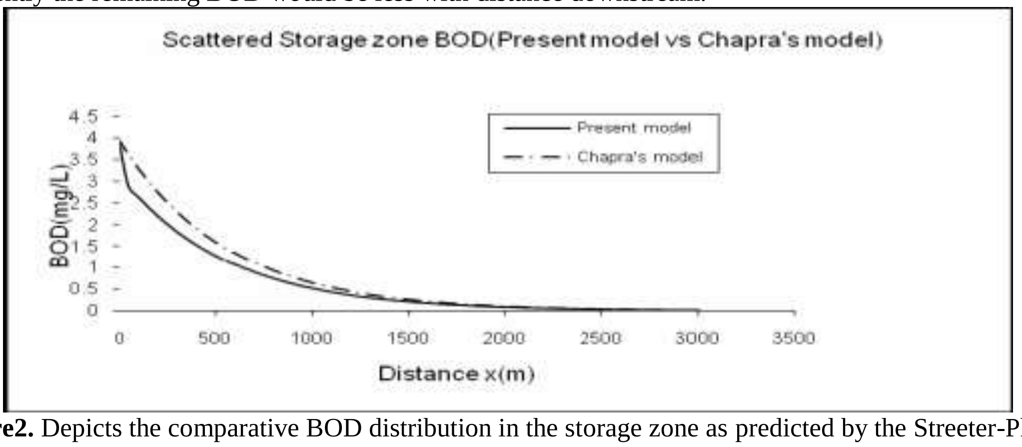

Figure2. Depicts the comparative BOD distribution in the storage zone as predicted by the Streeter-Phelps storage zone model (i.e., Chapra's model) and the present model(with scattered storage zone).

It is observed from the figure that for the beginning stretch (0-50 meters) of the downstream the concentration of BOD by the present model is very less than the concentration of BOD by Chapra's model. Because the settle able part of BOD in the present model settled at the starting point of the downstream, so the remaining BOD of present model is very less than the remaining BOD of Chapra's model. And the difference of concentration of BOD of two models becomes less gradually with the distance downstream. The cumulative impact of settle able part of BOD in the waste water and presence of the scattered storage zone in the river is to increase the demand of oxygen from river's DO for stabilization of waste discharged into the river.

IV. Conclusion

It is concluded that predicting the distribution of BOD (which ultimately affect the DO in the river) with the scattered storage zone depends on the treatment of waste. If untreated or partially treated waste enters such river then the model presented by Chapra shall be of little use. The mathematical model would be able to predict the concentration of BOD more accurately and consequently the decision based on such a prediction would be more rational for such a real life situation.

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