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An adjoint primal is a Primal Ideal which is related Krull dimension with decomposition

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Abstract: *This paper will focus mainly on the relation with commutative ring R related on primal ideals. It will represent an efficient decomposition of an ideal A of a commutative ring R of primal ideals. Here primal decomposition is denoted as $=\cap_{p\in X_A} A_{(P)}$, where $A_{(P)}$ is the isolated components of A. To prove $P \in Spec\ R$ that an ideal $A \subseteq P$ is an intersection of P - primal ideals iff the elements of the primal ideal to primal decomposition that is denoted as A.*

Keywords: *Primal Ideal, Decomposition of an ideal, Associated Prime, Set-Theoretic Union, Krull-dimension.*

I. Introduction

It has been proved that the Artinian ring which is satisfied by the ascending chain condition on ideals every ideal is the intersection of a finite number of irreducible ideals related with a commutative ring and the irreducible ideals are primary ideals. Among rings without the ascending chain condition, the rings in which such decomposition holds for all ideals. In this paper, it will be mainly focused establishing a more efficient decomposition of an ideal into the intersection of prime ideals.

1.1 Commutative ring:

A Ring is a set R equipped with two binary operations, i.e. operations combining any two elements of the ring to a third. They are called addition and multiplication and commonly denoted by "$+$" and "$.$" e.g $a + b$ and $a.b$

A commutative ring is a ring in which the multiplication operation is commutative. The study of commutative rings is called commutative algebra.

1.2 Primal Ideal:

A proper ideal Q of a commutative ring A is said to be primal if the elements that are not prime to it form an ideal.

1.3 Decomposition of an ideal:

The study of the decomposition of ideals in rings began as a remedy for the lack of unique factorization in rings like

$Z[\sqrt{-5}]$ in which

$$6 = 2.3 = \left(1 + \sqrt{-5}\right)\left(1 - \sqrt{-5}\right)$$

If a number does not factor uniquely into primes then the ideal generated by the number may still factor into the intersection of powers of prime ideals.

1.4 Associated Prime:

A nonzero R module N is called a prime module if the annihilator $A_{nn_R}(N) = A_{nn_R}(N')$ for any non zero submodule N' of N.For a prime module N. $A_{nn_R}(N)$ is a prime ideal in R.

An associated prime of an R-module M is an ideal of the form $A_{nn_R}(N)$ where N is a prime submodule of M.

1.5 Set-theoretic union:

The union of two sets A and B is the set of elements which are in A, in B or in both A and B. In symbols $A \cup B = \{x : x\epsilon A \ \ or \ \ x\epsilon B\}$.

1.6 Krull-dimension/Krull Associated Prime:

A chain of prime ideals of the form $P_0 \subset P_1 \subset .... \subset P_n$ has length n. That is , the length is the number of strict inclusions , not the number of primes , these differ by 1. It would be defined the Krull dimension of R to be the supreme of the lengths of all chains of prime ideals in R.

II. Primal Ideals with Ad joint Primes

2.1 Lemma:

There is a rare maximal member in the set $\{ P_1, P_{2,} .... P_n \}$ iff a reduced intersection

$A = A_1 \cap A_2 \cap \dots \dots \cap A_n$ of primal ideals A_i with adjoint primes P_i is again primal.

Proof:

The closed sets of the prime spectrum $SpecR$ of the ring R in the Zariski topology are the sets $V(I) = \{P \in SpecR : P \supseteq I\}$ with I raining over the set of ideals of R. $SpecR$ is called Noetherian if the closed subsets in the Zariski topology satisfy the descending chain condition or equivalently, if the radical ideals of R satisfy the ascending chain condition. The maximal Spectrum of R is set $MaxR$ of maximal ideals of $R$with the subspace topology from $SpecR$. It has been said that R has Noetherian maximal spectrum if the closed subsets of $MaxR$ satisfy the descending chain condition or equivalently, if the J −radical ideals of R satisfy the ascending chain condition, where an ideal is a J-radical ideal if it is an intersection of maximal ideals.

III. Primal Ideal which is generated by Ideal

3.1 Theorem: A prime ideal P of the ring R is contained a finitely generated ideal which is denoted by A. Then $(AP)_{(P)}$ is a P-Primal ideal of R when $A_p \neq 0$ is a P-primal ideal of R when $A_p \neq 0$.

Proof: Set $B = AP$. clearly, the elements of R not prime to $B_{(p)}$ are contained in P , so to show $B_{(p)}$ is a P-primal ideal of R, it suffices to prove that the elements of P are not prime to $B_{(p)}$.

Since A is finitely generated and $A_p \neq 0$, it implies that $B_p \neq A_p$ and it follows that $B_{(p)} \neq A_{(p)}$. Thus $B_{(p)} \subset A_{(p)} \subseteq B_{(p)}:P$, so there exists $y \in {}^{R}/_{B_{(P)}}$ such that $yP \subseteq B_{(p)}$. This proves that the elements of P are not prime to $B_{(p)}$ and $B_{(p)}$ is a P-primal ideal.

IV. Associated Primes

4.1 Lemma: Let a proper ideal which is denoted by A of the ring R. Every Weak-Bourbaki associated prime of A is a Krull associated prime of A. A prime ideal Q of R is a set-theoretic union of Weak-Bourbaki associated primes of A iff Q of R is a Krull associated prime of A.

Proof:

Let P be a Weak-Bourbaki associated prime of A. Then P is a minimal prime of $A{:}\,x$ for some $x \notin A$. It follows that the ideal $(A{:}\,x)_{(p)}$ is P-primary. Thus given $u \in P$, there is a smallest integer $k \geq 1$ such that $U^k \in (A{:}\,x)_{(P)}$.Hence $u^k v \in A{:}\,x$ for some $v \notin p$. Evidently, $u \in A{:}\,xu^{k-1}v$. If $A{:}\,xu^{k-1}v \subseteq P$ were not true.

V. Conclusion

Observing the puzzle in difficult way Lemma 2.1 portrays that there is a rare maximal member in the set $\{ P_1, P_{2,} .... P_n \}$ iff a reduced intersection

$A = A_1 \cap A_2 \cap \dots \dots \cap A_n$ of primal ideals A_i with adjoint primes P_i is again primal.

Theorem 3.1 states that a prime ideal P of the ring R is contained a finitely generated ideal which is denoted by A. Then $(AP)_{(P)}$ is a P-Primal ideal of R when $A_p \neq 0$ is a P-primal ideal of R when $A_p \neq 0$.

And in the final stage it has been depicted that a proper ideal which is denoted by A of the ring R. Every Weak-Bourbaki associated prime of A is a Krull associated prime of A. A prime ideal Q of R is a set-theoretic union of Weak-Bourbaki associated primes of A iff Q of R is a Krull associated prime of A

Then it could be found a $w \notin P$ with $w \in A{:}xu^{k-1}v$. But then $u^{k-1}vw \in A{:}\,x$ and $u^{k-1} \in (A{:}\,x)_{(p)}$ is impossible.Thus $A{:}xu^{k-1}v \subseteq P$ I indeed. It follows that a prime ideal Q of R is Krull associated prime of A if it is a set –theoretic union of Weak-Bourbaki associated primes of A. The converse is clear, since $x \in A{:}\,y \subseteq Q$ implies that x is contained in every minimal prime of $A{:}\,y$.

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