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Estimation of Variance of Time to Recruitment for a Two Grade Manpower System with Thresholds Having Two Components Based On Order Statistics

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Abstract: *In this paper, for a marketing organization consisting of two grades which is subject to depletion of manpower(wastages) due to policy decisions with high or low attrition rate, an important system characteristic namely the variance of time to recruitment is obtained for three mathematical models using a suitable policy of recruitment when(i) wastages form an order statistics and (ii) threshold for each grade has two components, and (iii) the inter-policy decisions form an ordinary renewal process, order statistics and geometric process. The system characteristics are analyzed by numerical illustration and the findings are reported.*

Keywords: *Order statistics, Ordinary renewal process, Policy of recruitment, Threshold with two components, Two types of policy decisions with high or low attrition rate, Variance of time to recruitment.*

I. Introduction

An exit of personnel is a common phenomenon in any marketing organization. In [1] and [2] several stochastic models for a manpower system with grades are discussed using Markovian and renewal theoretic approach. In [3] the authors have initiated the study on problem of time to recruitment for a single grade manpower system when the inter-decision times are independent and identically distributed random variables using shock model approach. In [4] the authors have studied the work in [3] when the breakdown threshold has a normal component and a component due to frequent breaks. In [5] the author has obtained the system characteristics for a single grade manpower system when the inter-decision times form an order statistics. In [6] the authors have obtained variance of time to recruitment for a two grade manpower system when (i) the loss of manpower form an order statistics and (ii) the threshold for each grades has only the normal component, and the inter-decision times form an order statistics. The present paper studies the work in [6] when the threshold has the two components cited above and the inter-decision times have low or high intensity of attrition. In this paper three mathematical models are constructed which differ from each other in the context of permitting or not permitting transfer of personnel between two grades and providing a better allowable loss of manpower in the organization. More specifically, in Model-I, the breakdown threshold is minimum of the thresholds for the loss of manpower in the two grades. In Model-II, the breakdown threshold is the maximum of the thresholds for the grades. In Model-III, the breakdown threshold is the sum of the thresholds for the grades. This paper is organized as follows: In sections II, III and IV Models I, II and III are described and analytical expressions for mean and variance of the time to recruitment are derived. The analytical results are numerically illustrated by assuming specific distributions and the influence of nodal parameters on the system characteristics is reported.

II. Model Description And Analysis For Model-I

Consider an organization having two grades in which decisions are taken at random epochs in $[0,\infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of man-hours(wastages) to the organization, if a person quits and it is linear and cumulative. Let X_i , i=1,2,3… be an independent and identically distributed exponential random variable with density function g(.) and mean 1/c,(c>0). Let $X_{(1)}, X_{(2)}\ldots X_{(k)}$ be the order statistics selected from the samples $X_1, X_2\ldots X_k$ with respective density functions $g_{X(1)}(.), g_{X(2)}(.)\ldots. g_{X(k)}(.)$. Let S_n be the total loss of man-hours in the first 'n' decisions. Let $U_i, i = 1,2,3\ldots$ be the time between i-1th and ith decisions. The best distribution when the inter-decision times have high or low intensity of attrition is the hyper exponential distribution with distribution (density) function F(.)(f(.)), and high(low) attrition rate $\lambda_h(\lambda_l)$ and p(q) be the proportion of decisions having high (low) attrition rate. Let $F_k(t)$ ($f_k(t)$) be the distribution(probability density) function of $\sum_{i=1}^{k} U_i$. Let T be a continuous random variable denoting the time for recruitment in the organization with probability distribution function (density function) $L(.)(\ell(.))$. Let $\varphi^*(.)$ be the Laplace transform of $\varphi(.)$. Let Y be the breakdown threshold for the cumulative loss of manpower in the organization. For grade A(B), let Y_{A1} (Y_{B1}) be the normal exponential threshold for depletion of manpower with mean α_{A1}, $\alpha_{A1}>0$(α_{B1},$\alpha_{B1}>0$) and Y_{A2} (Y_{B2}) be the exponential threshold of frequent breaks of existing workers with mean α_{A2}, $\alpha_{A2}>0$(α_{B2},$\alpha_{B2}>0$). In this model, the breakdown threshold for the organization $Y=\min(Y_A,Y_B)$. The loss of man-hours process and the inter-decision time process are statistically independent. The univariate policy of recruitment is ***Recruitment is done as and when the total loss of man-hours in the organization exceeds the breakdown threshold Y.*** Let $V_k(t)$ be the probability that there are exactly k-decision epochs in (0,t]. Since the number of decisions made in (0,t] form a renewal process, we note that $V_k(t)= F_k(t)-F_{k+1}(t)$, where $F_0(t)=1$. Let E(T) and V(T) be the mean and variance of time for recruitment respectively.

Main Results

By definition, $S_{N(t)}$ is the total loss of man-hours in the N(t) decisions taken in (0,t]

Therefore

$$P(T>t)=P(S_{N(t)}<Y) \tag{1}$$

By using laws of probability and on simplification we get

$$P(T>t)=\gamma_1\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left(g^*(\alpha_{A1}+\alpha_{B1})\right)^k+\gamma_2\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{A2}+\alpha_{B2})\right]^k$$
$$-\gamma_3\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{A1}+\alpha_{B2})\right]^k-\gamma_4\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{A2}+\alpha_{B1})\right]^k \tag{2}$$

where

$\gamma_1=\frac{\alpha_{A2}\alpha_{B2}}{(\alpha_{A2}-\alpha_{A1})(\alpha_{B2}-\alpha_{B1})}, \gamma_2=\frac{\alpha_{A1}\alpha_{B1}}{(\alpha_{A2}-\alpha_{A1})(\alpha_{B2}-\alpha_{B1})}, \gamma_3=\frac{\alpha_{A2}\alpha_{B1}}{(\alpha_{A2}-\alpha_{A1})(\alpha_{B2}-\alpha_{B1})}$ and

$\gamma_4=\frac{\alpha_{A1}\alpha_{B2}}{(\alpha_{A2}-\alpha_{A1})(\alpha_{B2}-\alpha_{B1})}$

Case(i):

Suppose the inter-decision times U_i, i=1,2,3…. are independent and identically distributed random variables.
From (2) it can be shown that

$$P(T>t)=\gamma_1\left[1-g^*(\alpha_{A1}+\alpha_{B1})\right]\sum_{k=1}^{\infty}F_k(t)\left(g^*(\alpha_{A1}+\alpha_{B1})\right)^{k-1}+\gamma_2\left[1-g^*(\alpha_{A2}+\alpha_{B2})\right]\sum_{n=1}^{\infty}F_k(t)\left[g^*(\alpha_{A2}+\alpha_{B2})\right]^{k-1}$$

$$-\gamma_3\left[1-g^*(\alpha_{A1}+\alpha_{B2})\right]\sum_{k=1}^{\infty}F_k(t)\left[g^*(\alpha_{A1}+\alpha_{B2})\right]^{k-1}-\gamma_4\left[1-g^*(\alpha_{A2}+\alpha_{B1})\right]\sum_{k=1}^{\infty}F_k(t)\left[g^*(\alpha_{A2}+\alpha_{B1})\right]^{k-1} \tag{3}$$

Since $l(t)=\frac{d}{dt}(1-P(T>t))$, taking Laplace transform and of $l(t)$ and on simplification we get

$$\ell^*(s)=\gamma_1\frac{\left[1-g^*(\alpha_{A1}+\alpha_{B1})\right]f^*(s)}{1-f^*(s)g^*(\alpha_{A1}+\alpha_{B1})}+\gamma_2\frac{\left[1-g^*(\alpha_{A2}+\alpha_{B2})\right]f^*(s)}{1-f^*(s)g^*(\alpha_{A2}+\alpha_{B2})}-\gamma_3\frac{\left[1-g^*(\alpha_{A1}+\alpha_{B2})\right]f^*(s)}{1-f^*(s)g^*(\alpha_{A1}+\alpha_{B2})}-$$
$$\gamma_4\frac{\left[1-g^*(\alpha_{A2}+\alpha_{B1})\right]f^*(s)}{1-f^*(s)g^*(\alpha_{A2}+\alpha_{B1})} \tag{4}$$

It is known that

$$E[T]=-\left.\frac{d(\ell^*(s))}{ds}\right|_{s=0}, E\left[T^2\right]=\left.\frac{d^2(\ell^*(s))}{ds^2}\right|_{s=0} \text{ and } V[T]=E\left[T^2\right]-\left(E[T]\right)^2 \tag{5}$$

From (4) and (5) it can be shown that

$$E[T]=E[U]\left[\frac{\gamma_1}{1-g^*(\alpha_{A1}+\alpha_{B1})}+\frac{\gamma_2}{1-g^*(\alpha_{A2}+\alpha_{B2})}-\frac{\gamma_1}{1-g^*(\alpha_{A1}+\alpha_{B2})}-\frac{\gamma_1}{1-g^*(\alpha_{A2}+\alpha_{B1})}\right] \quad (6)$$

and

$$E[T^2]=E[U^2]\left[\frac{\gamma_1}{1-g^*(\alpha_{A1}+\alpha_{B1})}+\frac{\gamma_2}{1-g^*(\alpha_{A2}+\alpha_{B2})}-\frac{\gamma_3}{1-g^*(\alpha_{A1}+\alpha_{B2})}-\frac{\gamma_4}{1-g^*(\alpha_{A2}+\alpha_{B1})}\right]+$$

$$2(E[U])^2\left[\frac{\gamma_1 g^*(\alpha_{A1}+\alpha_{B1})}{(1-g^*(\alpha_{A1}+\alpha_{B1}))^2}+\frac{\gamma_2 g^*(\alpha_{A2}+\alpha_{B2})}{(1-g^*(\alpha_{A2}+\alpha_{B2}))^2}-\frac{\gamma_3 g^*(\alpha_{A1}+\alpha_{B2})}{(1-g^*(\alpha_{A1}+\alpha_{B2}))^2}-\frac{\gamma_4 g^*(\alpha_{A2}+\alpha_{B1})}{(1-g^*(\alpha_{A2}+\alpha_{B1}))^2}\right] \quad (7)$$

In (6) and (7), by hypothesis , $U=U_i$,i=1,2,3…

$$E[U]=\frac{p\lambda_l+q\lambda_h}{\lambda_h\lambda_l}\ ,\quad E[U^2]=\left(\frac{p\lambda_l^{\,2}+q\lambda_h^{\,2}}{\lambda_h^{\,2}\lambda_l^{\,2}}\right)\ and \quad (8)$$

$$g^*(\tau)=\begin{cases} kc\big/kc+\tau\ , & if\ \ g(x)=g_{x(1)}(x) \\ k!c^k\big/c+\tau)(2c+\tau)(3c+\tau).......(kc+\tau)\ , & if\ \ g(x)=g_{x(k)}(x)\end{cases} \quad (9)$$

Case (ii) :

If $U_{(1)},U_{(2)},..U_{(k)}$ be the order statistics selected from the sample $U_1,U_2,..U_k$ with respective density function $f_{u(1)}, f_{u(2)},...f_{u(k)}$ the mean and variance of time to recruitment are given by (6) and (7) where, using the theory of order statistics it can be shown that

$$E[U]=\begin{cases}\sum_{r_1=0}^{k}\dfrac{kc_{r_1}p^{r_1}q^{k-r_1}}{(\lambda_h-\lambda_l)r_1+\lambda_l k}, & if\ \ f(t)=f_{u(1)}(t)\\ \sum_{r_1=0}^{k}\sum_{r_2=0}^{k-r_1}\dfrac{(-1)^{k-r_1}kc_{r_1}(k-r_1)c_{r2}p^{r2}q^{k-r_1-r2}}{(\lambda_h+\lambda_l)r_2+\lambda_l r_1-k\lambda_l}, & if\ \ f(t)=f_{u(k)}(t)\end{cases} \quad (10)$$

$$E[U^2]=\begin{cases}2\sum_{r_1=0}^{k}\dfrac{kc_{r_1}p^{r_1}q^{k-r_1}}{((\lambda_h-\lambda_l)r_1+\lambda_l k)^2}, & if\ \ f(t)=f_{u(1)}(t)\\ 2\sum_{r_1=0}^{k}\sum_{r_2=0}^{k-r_1}\dfrac{(-1)^{k-r_1}kc_{r_1}(k-r_1)c_{r2}p^{r2}q^{k-r_1-r2}}{((\lambda_h+\lambda_l)r_2+\lambda_l r_1-k\lambda_l)^2}, & if\ \ f(t)=f_{u(k)}(t)\end{cases} \quad (11)$$

Case (iii) :

Assume that the inter-decision times U_i form a geometric process with parameter ‘a’.

Since $\{U_i\}$ is a geometric process it is known that

$$f_k^*(s)=\prod_{n=1}^{k} f^*\left(s\big/a^{n-1}\right) \quad (12)$$

From (4),(5) and (12) we get

$$E[T]=a\ E[U]\left[\frac{\gamma_1}{a-g^*(\alpha_{A1}+\alpha_{B1})}+\frac{\gamma_2}{a-g^*(\alpha_{A2}+\alpha_{B2})}-\frac{\gamma_3}{a-g^*(\alpha_{A1}+\alpha_{B2})}-\frac{\gamma_4}{a-g^*(\alpha_{A2}+\alpha_{B1})}\right] \quad (13)$$

and

$$E[T^2]=2a^2\ E[U^2]\left[\begin{array}{l}\dfrac{\gamma_1}{a^2-g^*(\alpha_{A1}+\alpha_{B1})}+\dfrac{\gamma_2}{a^2-g^*(\alpha_{A2}+\alpha_{B2})}-\\ \dfrac{\gamma_3}{a^2-g^*(\alpha_{A1}+\alpha_{B2})}-\dfrac{\gamma_4}{a^2-g^*(\alpha_{A2}+\alpha_{B1})}\end{array}\right]-$$

$$(E[U])^2\left(\sum_{k=0}^{\infty}\left[\left(\sum_{i=1}^{k}\frac{1}{a^{i-1}}\right)^2-\left(\sum_{i=1}^{k+1}\frac{1}{a^{i-1}}\right)^2\right]\right)\times\begin{pmatrix}\gamma_1\left(g^*(\alpha_{A1}+\alpha_{B1})\right)^k+\gamma_2\left(g^*(\alpha_{A2}+\alpha_{B2})\right)^k\\-\gamma_3\left(g^*(\alpha_{A1}+\alpha_{B2})\right)^k-\gamma_4\left(g^*(\alpha_{A2}+\alpha_{B1})\right)^k\end{pmatrix} \qquad (14)$$

In (13) and (14) E[U], E[U^2] and $g^*(\tau)$ are given by (8) and (9).

III. Model Description And Analysis For Model-II

For this model, $Y=\max(Y_A,Y_B)$. All the other assumptions and notations are as in Model-I. In this model it can be shown that

$$P(T>t)=\gamma_3\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{A1}+\alpha_{B2})\right]^k+\gamma_4\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{A2}+\alpha_{B1})\right]^k+$$

$$\gamma_5\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{A1})\right]^k+\gamma_7\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{B1})\right]^k-$$

$$\gamma_6\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{A2})\right]^k-\gamma_8\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{B2})\right]^k-$$

$$\gamma_1\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left(g^*(\alpha_{A1}+\alpha_{B1})\right)^k-\gamma_2\sum_{n=1}^{k}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{A2}+\alpha_{B2})\right]^k \qquad (15)$$

where $\gamma_1=\frac{\alpha_{A2}\alpha_{B2}}{(\alpha_{A2}-\alpha_{A1})(\alpha_{B2}-\alpha_{B1})},\gamma_2=\frac{\alpha_{A1}\alpha_{B1}}{(\alpha_{A2}-\alpha_{A1})(\alpha_{B2}-\alpha_{B1})},\gamma_3=\frac{\alpha_{A2}\alpha_{B1}}{(\alpha_{A2}-\alpha_{A1})(\alpha_{B2}-\alpha_{B1})}$

$\gamma_4=\frac{\alpha_{A1}\alpha_{B2}}{(\alpha_{A2}-\alpha_{A1})(\alpha_{B2}-\alpha_{B1})}$, $\gamma_5=\frac{\alpha_{A2}}{(\alpha_{A2}-\alpha_{A1})},\gamma_6=\frac{\alpha_{A1}}{(\alpha_{A2}-\alpha_{A1})},\gamma_7=\frac{\alpha_{B2}}{(\alpha_{B2}-\alpha_{B1})}$ and $\gamma_5=\frac{\alpha_{B1}}{(\alpha_{B2}-\alpha_{B1})}$

Case (i) :

Proceeding as in Model-I we get

$$\ell^*(s)=\gamma_3\frac{\left[1-g^*(\alpha_{A1}+\alpha_{B2})\right]f^*(s)}{1-f^*(s)\,g^*(\alpha_{A1}+\alpha_{B2})}+\gamma_4\frac{\left[1-g^*(\alpha_{A2}+\alpha_{B1})\right]f^*(s)}{1-f^*(s)\,g^*(\alpha_{A2}+\alpha_{B1})}+\gamma_5\frac{\left[1-g^*(\alpha_{A1})\right]f^*(s)}{1-f^*(s)\,g^*(\alpha_{A1})}+$$

$$\gamma_7\frac{\left[1-g^*(\alpha_{B1})\right]f^*(s)}{1-f^*(s)\,g^*(\alpha_{B1})}-\gamma_6\frac{\left[1-g^*(\alpha_{A2})\right]f^*(s)}{1-f^*(s)\,g^*(\alpha_{A2})}-\gamma_8\frac{\left[1-g^*(\alpha_{B2})\right]f^*(s)}{1-f^*(s)\,g^*(\alpha_{B2})}-$$

$$\gamma_1\frac{\left[1-g^*(\alpha_{A1}+\alpha_{B1})\right]f^*(s)}{1-f^*(s)\,g^*(\alpha_{A1}+\alpha_{B1})}-\gamma_2\frac{\left[1-g^*(\alpha_{A2}+\alpha_{B2})\right]f^*(s)}{1-f^*(s)\,g^*(\alpha_{A2}+\alpha_{B2})} \qquad (16)$$

From (5) and (16) it can be shown that

$$E[T]=E[U]\begin{bmatrix}\frac{\gamma_3}{1-g^*(\alpha_{A1}+\alpha_{B2})}+\frac{\gamma_4}{1-g^*(\alpha_{A2}+\alpha_{B1})}+\frac{\gamma_5}{1-g^*(\alpha_{A1})}+\frac{\gamma_7}{1-g^*(\alpha_{B1})}-\\\frac{\gamma_6}{1-g^*(\alpha_{A2})}-\frac{\gamma_8}{1-g^*(\alpha_{B2})}-\frac{\gamma_1}{1-g^*(\alpha_{A1}+\alpha_{B1})}-\frac{\gamma_2}{1-g^*(\alpha_{A2}+\alpha_{B2})}\end{bmatrix} \qquad (17)$$

and

$$E[T^2]=E[U^2]\begin{bmatrix}\frac{\gamma_3}{1-g^*(\alpha_{A1}+\alpha_{B2})}+\frac{\gamma_4}{1-g^*(\alpha_{A2}+\alpha_{B1})}+\frac{\gamma_5}{1-g^*(\alpha_{A1})}+\frac{\gamma_7}{1-g^*(\alpha_{B1})}-\\\frac{\gamma_6}{1-g^*(\alpha_{A2})}-\frac{\gamma_8}{1-g^*(\alpha_{B2})}-\frac{\gamma_1}{1-g^*(\alpha_{A1}+\alpha_{B1})}-\frac{\gamma_2}{1-g^*(\alpha_{A2}+\alpha_{B2})}\end{bmatrix}+$$

$$2(E[U])^2\begin{bmatrix}\frac{\gamma_3g^*(\alpha_{A1}+\alpha_{B2})}{\left(1-g^*(\alpha_{A1}+\alpha_{B2})\right)^2}+\frac{\gamma_4g^*(\alpha_{A2}+\alpha_{B1})}{1\left(-g^*(\alpha_{A2}+\alpha_{B1})\right)^2}+\frac{\gamma_5g^*(\alpha_{A1})}{\left(1-g^*(\alpha_{A1})\right)^2}+\frac{\gamma_7g^*(\alpha_{B1})}{\left(1-g^*(\alpha_{B1})\right)^2}-\\\frac{\gamma_6g^*(\alpha_{A2})}{\left(1-g^*(\alpha_{A2})\right)^2}-\frac{\gamma_8g^*(\alpha_{B2})}{\left(1-g^*(\alpha_{B2})\right)^2}-\frac{\gamma_1g^*(\alpha_{A1}+\alpha_{B1})}{\left(1-g^*(\alpha_{A1}+\alpha_{B1})\right)^2}-\frac{\gamma_2g^*(\alpha_{A2}+\alpha_{B2})}{\left(1-g^*(\alpha_{A2}+\alpha_{B2})\right)^2}\end{bmatrix} \qquad (18)$$

In (17) and (18) E[U], E[U^2] and $g^*(\tau)$ are given by (8) and (9)

Case (ii) :

Proceeding as in Model-I , the mean and variance of time to recruitment in this case are given by (17) and (18) where $g^*(\tau)$, E[U] and E[U^2] are given by (9), (10) and (11)

Case (iii) :

Proceeding as in Model-I it is found that

$$E[T] = a\ E[U]\left[\begin{array}{l}\dfrac{\gamma_3}{a-g^*(\alpha_{A1}+\alpha_{B2})}+\dfrac{\gamma_4}{a-g^*(\alpha_{A2}+\alpha_{B1})}+\dfrac{\gamma_5}{a-g^*(\alpha_{A1})}+\dfrac{\gamma_7}{a-g^*(\alpha_{B1})}-\\ \dfrac{\gamma_6}{a-g^*(\alpha_{A2})}-\dfrac{\gamma_8}{a-g^*(\alpha_{B2})}-\dfrac{\gamma_1}{a-g^*(\alpha_{A1}+\alpha_{B1})}-\dfrac{\gamma_2}{a-g^*(\alpha_{A2}+\alpha_{B2})}\end{array}\right] \tag{19}$$

and

$$E[T^2] = 2a^2E\left[U^2\right]\left[\begin{array}{l}\dfrac{\gamma_3}{a^2-g^*(\alpha_{A1}+\alpha_{B2})}+\dfrac{\gamma_4}{a^2-g^*(\alpha_{A2}+\alpha_{B1})}+\dfrac{\gamma_5}{a^2-g^*(\alpha_{A1})}+\dfrac{\gamma_7}{a^2-g^*(\alpha_{B1})}-\\ \dfrac{\gamma_6}{a^2-g^*(\alpha_{A2})}-\dfrac{\gamma_8}{a^2-g^*(\alpha_{B2})}-\dfrac{\gamma_1}{a^2-g^*(\alpha_{A1}+\alpha_{B1})}-\dfrac{\gamma_2}{a^2-g^*(\alpha_{A2}+\alpha_{B2})}\end{array}\right]$$

$$+\left(E[U]\right)^2\left(\sum_{k=0}^{\infty}\left[\left(\sum_{i=1}^{k}\frac{1}{a^{i-1}}\right)^2-\left(\sum_{i=1}^{k+1}\frac{1}{a^{i-1}}\right)^2\right]\right)\times\left(\begin{array}{l}\gamma_3\left(g^*(\alpha_{A1}+\alpha_{B2})\right)^k+\gamma_4\left(g^*(\alpha_{A2}+\alpha_{B1})\right)^k+\\ \gamma_5\left(g^*(\alpha_{A1})\right)^k+\gamma_7\left(g^*(\alpha_{B1})\right)^k-\gamma_6\left(g^*(\alpha_{A2})\right)^k-\\ \gamma_8\left(g^*(\alpha_{B2})\right)^k-\gamma_1\left(g^*(\alpha_{A1}+\alpha_{B1})\right)^k-\\ \gamma_2\left(g^*(\alpha_{A2}+\alpha_{B2})\right)^k\end{array}\right) \tag{20}$$

In (19) and (20) E[U], E[U^2] and $g^*(\tau)$ are given by (8) and (9).

IV. Model Description And Analysis For Model-III

For this model, $Y = Y_A + Y_B$. All the other assumptions and notations are as in Model-I. Proceeding as in Model-I it can be shown that

$$P(T>t)=\gamma_9\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{A1})\right]^k+\gamma_{10}\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{B1})\right]^k -$$

$$\gamma_{11}\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{A2})\right]^k-\gamma_{12}\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{B2})\right]^k \tag{21}$$

where

$$\gamma_9=\frac{\alpha_{A1}\alpha_{A2}\alpha_{B1}\alpha_{B2}}{(\alpha_{A2}-\alpha_{A1})(\alpha_{B1}-\alpha_{A1})(\alpha_{B2}-\alpha_{A1})},\gamma_{10}=\frac{\alpha_{A1}\alpha_{A2}\alpha_{B1}\alpha_{B2}}{(\alpha_{B2}-\alpha_{B1})(\alpha_{B1}-\alpha_{A1})(\alpha_{B1}-\alpha_{A2})},$$

$$\gamma_{11}=\frac{\alpha_{A1}\alpha_{A2}\alpha_{B1}\alpha_{B2}}{(\alpha_{A2}-\alpha_{A1})(\alpha_{B1}-\alpha_{A2})(\alpha_{B2}-\alpha_{A2})}\ and\ \frac{\alpha_{A1}\alpha_{A2}\alpha_{B1}\alpha_{B2}}{(\alpha_{B2}-\alpha_{B1})(\alpha_{B2}-\alpha_{A1})(\alpha_{B2}-\alpha_{A2})}$$

Case (i) :

Proceeding as in Model-I we get

$$\ell^*(s)=\gamma_9\frac{\left[1-g^*(\alpha_{A1})\right]f^*(s)}{1-f^*(s)\,g^*(\alpha_{A1})}+\gamma_{10}\frac{\left[1-g^*(\alpha_{B1})\right]f^*(s)}{1-f^*(s)\,g^*(\alpha_{B1})}-\gamma_{11}\frac{\left[1-g^*(\alpha_{A2})\right]f^*(s)}{1-f^*(s)\,g^*(\alpha_{A2})}-\gamma_{12}\frac{\left[1-g^*(\alpha_{B2})\right]f^*(s)}{1-f^*(s)\,g^*(\alpha_{B2})} \tag{22}$$

From (5) and (22) it can be shown that

$$E[T]=E[U]\left[\frac{\gamma_9}{1-g^*(\alpha_{A1})}+\frac{\gamma_{10}}{1-g^*(\alpha_{B1})}-\frac{\gamma_{11}}{1-g^*(\alpha_{A2})}-\frac{\gamma_{12}}{1-g^*(\alpha_{B2})}\right] \tag{23}$$

and

$$E[T^2]=E[U^2]\left[\frac{\gamma_9}{1-g^*(\alpha_{A1})}+\frac{\gamma_{10}}{1-g^*(\alpha_{B1})}-\frac{\gamma_{11}}{1-g^*(\alpha_{A2})}-\frac{\gamma_{12}}{1-g^*(\alpha_{B2})}\right]-$$

$$2(E[U])^2\left[\frac{\gamma_9 g^*(\alpha_{A1})}{\left(1-g^*(\alpha_{A1})\right)^2}+\frac{\gamma_{10} g^*(\alpha_{B1})}{\left(1-g^*(\alpha_{B1})\right)^2}-\frac{\gamma_{11} g^*(\alpha_{A2})}{\left(1-g^*(\alpha_{A2})\right)^2}-\frac{\gamma_{12} g^*(\alpha_{B2})}{\left(1-g^*(\alpha_{B2})\right)^2}\right] \tag{24}$$

In (23) and (24) E[U], E[U^2] and $g^*(\tau)$ are given by (8) and (9)

Case (ii) :

Proceeding as in Model-I , the mean and variance of time to recruitment in this case are given by (23) and (24) where $g^*(\tau)$, E[U] and E[U^2] are given by (9), (10) and (11).

Case (iii) :

Proceeding as in Model-I it is found that

$$E[T]=a\ E[U]\left[\frac{\gamma_9}{a-g^*(\alpha_{A1})}+\frac{\gamma_{10}}{a-g^*(\alpha_{B1})}+\frac{\gamma_{11}}{a-g^*(\alpha_{A2})}+\frac{\gamma_{12}}{a-g^*(\alpha_{B2})}\right] \tag{25}$$

and

$$E[T^2]=2a^2E[U^2]\left[\frac{\gamma_9}{a-g^*(\alpha_{A1})}+\frac{\gamma_{10}}{a-g^*(\alpha_{B1})}+\frac{\gamma_{11}}{a-g^*(\alpha_{A2})}+\frac{\gamma_{12}}{a-g^*(\alpha_{B2})}\right]-$$

$$(E[U])^2\left(\sum_{k=0}^{\infty}\left[\left(\sum_{i=1}^{k}\frac{1}{a^{i-1}}\right)^2-\left(\sum_{i=1}^{k+1}\frac{1}{a^{i-1}}\right)^2\right]\right)\times\begin{pmatrix}\gamma_9\left(g^*(\alpha_{A1})\right)^k+\gamma_{10}\left(g^*(\alpha_{B1})\right)^k-\\ \gamma_{11}\left(g^*(\alpha_{A2})\right)^k-\gamma_{12}\left(g^*(\alpha_{B2})\right)^{k^k}\end{pmatrix} \tag{26}$$

In (25) and (26) E[U], E[U^2] and $g^*(\tau)$ are given by (8) and (9).

V. Numerical Illustration

The influence of parameters on the performance measures namely the mean time for recruitment derived in case(i) is studied numerically. In the following table these performance measures are calculated by varying the parameter ‘c’ and taking α_{A1}=0.1, α_{A2}=0.2, α_{B1}=0.3 , α_{B2}=0.4 , p=0.4, λ_h=0.3and λ_l=0.2

Table : Effect of ‘c’ and ‘k’ on the performance measure E[T] for case (i)

c	k	Model-I		Model-II		Model-III	
		$E(T)$		$E(T)$		$E(T)$	
		g(x)=g$_{x(1)}$(x)	**g(x)=g$_{x(k)}$(x)**	**g(x)=g$_{x(1)}$(x)**	**g(x)=g$_{x(k)}$(x)**	**g(x)=g$_{x(1)}$(x)**	**g(x)=g$_{x(k)}$(x)**
3	3	11.96	2.3480	69.03	10.4165	72.93	13.6677
4	3	15.86	3.0194	91.78	13.8853	96.98	18.0408
5	3	19.76	3.7291	114.53	17.3353	121.03	22.4481
3	**6**	23.66	1.7524	137.28	9.7749	145.08	10.3064
3	**7**	27.56	1.6642	160.03	9.2593	169.13	9.7664
3	**8**	31.46	1.4432	182.78	8.8427	193.18	9.3207

Findings

From the above table it is found that

1. E(T) increases in all the three models as ‘c’ increases when both g(x)=$g_{X(1)}$(x) and g(x)=$g_{X(k)}$(x) .

2. E(T) increases in all the three models as ‘k’ increases when g(x)=$g_{X(1)}$(x) but it decreases when g(x)=$g_{X(k)}$(x) .

VI. Conclusion

Since the time to recruitment is more elongated in Model-III than the first two models, Model-III is preferable from the organization point of view.

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