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Mathematical Model on the Effect of Temperature on Dissolved Oxygen and Biochemical Oxygen Demand

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Abstract: *In this paper we develop a dissolve oxygen temperature decay mathematical model for water pollution in Damoder River at Dhanbad region. Equations for dissolved oxygen (Do) and biological oxygen demand (BOD) have been derived and solved analytically. The solution can be used to determine BOD and DO for suitable value of the parameters*

Keywords: *Dissolved Oxygen, Biological Oxygen Demand, Water pollution, Damoder River*

I. Introduction

Many of the concentration level are interrelated and change for a period of time after initial mixing. For example, the discharge of organic material into a river will create an additional demand on the existing supply of oxygen in the river (called Biological oxygen demand) (BOD), the reaction using the oxygen takes place over a period of time during which oxygen enters at the surface of the river, making available new oxygen. At any time after the effluent has been discharged. The oxygen concentration level is a function of the initial concentration of oxygen and B (=BOD) and the rate of supply (re aeration) and demand (deoxygenation), both of which are functions of the heat concentration or temperature. Given enough time the DOD level should go to zero and the temperature to equilibrium temperature determined by metrological & other conditions.

Heat and organic materials are considered degradable materials because these effects eliminated with time. Other materials do not converge to a natural equilibrium at least not a relevant time horizon.

The dissolved oxygen level is affected in two way by the temperature. First, the higher the temperature is the lower the saturation level of dissolved oxygen. This means at high temperature less oxygen will be available to oxidize the organic waste material and second the temperature level affects the rate of the oxidation re aeration of water. The simulation model of a river that in used to determine the water quality of given effluent parameters.

We consider the specific solution of water flowing in a river from factories and cities sewage sptens is being added to the river at n points denoted by j=1,2,……n. water is being taken fro the river for drinking and other purpose at m points denoted by i=1,2,………m. the quality of the water is measured by biological oxygen demand (BOD) or dissolved oxygen defect (DOD) deteriorater and it is desired to decrease the effluxes or waste water at various points so as to bring about certain minimum improvements in quality of water at the m intake points. This decrease in effluxes can be brought about by treating the waste water with chemicals in plants. In order to preserve aquatic life it is also important that temperature of water is kept at reasonable levels for aquatic life.

A first attempt to describe the relation between dissolved oxygen (DO) at atmospheric reaeration and bacterial respiration was formulated by streeter and Phelps in 1925[1]. Dobins [2,3,8] established relationship between BOD & Oxygen in streams. Graves, Hatfield and whinstone [4] used mathematical programming for water pollution. Graves, pingry and whinstone [5,6] discussed NLPP to water pollution control. Lof and Wand [7] discussed thermal pollution control. Loucks Revelle and Lynn [8] developed L.P. models for control of water pollution. Dysart and Heines [9], observed that a three equation differentiated equations model should be solved to predict the level of Do. However they did not solve the new system of equations but used the temperature at a point and assume that temperature is constant for a small river sation

II. Nomenclature:-

A	:	cross – sectional area of the stream
λ_b, μ_b	:	Value of λ & μ at b^0C
ϕ, α	:	estimated parameters which are determined of b^0C
θ_1,θ_2	:	estimated parameter
T	:	temperature
T_E	:	equilibrium basin temperature
A	:	$\frac{\lambda}{\mu-\lambda}$

A_o : $\frac{A_1}{\nu}$
D_o : value of D at t = 0
$\overline{T}$: $T_0 - T_E$

III. Mathematical Model Formulation:

Water quality is generally measured by the deviation of the concentration level of certain chemicals from desired levels. Dissolved oxygen (D) levels, dissolved oxygen deficit (DOD), heat concentration (temperature level) And concentration of various chemicals are same of the typical concentrations that are considered to be important. All of these river concentrations can be altered by mixing. The river flow with influent of various concentrations being a weighted average of the influent and mainstream concentrations.

Assume

$$\frac{dB}{dt} = -\lambda B \quad (1)$$

and $$\frac{dD}{dt} = \lambda B - \mu D \quad (2)$$

From equation (1), we have,

$$B = ke^{-\lambda t} \quad (3)$$

where k is constant of integration.

or, $$B = k\ a_1 \quad (4)$$

where

$$a_1 = e^{-\lambda t} \quad (5)$$

From equation (2), we have,

$$D = \frac{\lambda k}{\mu - \lambda} e^{-\lambda t} + k_1 e^{-\mu t} \quad (6)$$

where k_1 is constant of integration and substituting the value of B from equation (4)

When

$T = 0, D = Do$

We have from (6)

$$D = a\ k\ (a_1 - a_2) + D_0\ a_2 \quad (7)$$

Where

$$a = \frac{\lambda}{\mu - \lambda} \quad (8)$$

$$a_1 = e^{-\lambda t} \quad (9)$$

$$a_2 = e^{-\mu t} \quad (10)$$

In the context of river basin if the volumetric flow and velocity V of the flow are assumed to be constant over the same river segment of length x , then time taken can be interpreted as

$$t = \frac{x}{V} \quad (11)$$

Generally the velocity of flow can be written as

$$V = \frac{F}{A} \quad (12)$$

Where A is the cross sectional area of the stream. However as volumetric flow varies in the fixed river bed, the cross sectional area also varies. So (12) can be written as

$$V = \frac{F}{A(F)} \quad (13)$$

We assume $V = e_1 F^{e_2}$

Where e_1, e_2 and F are Parameters estimated for each river segment. If we assume that the rate of re aeration depends on the velocity of flow and on the volume trice flow rate then

$$\mu_b = \phi F^{\alpha} \quad (14)$$

Where ϕ and α are estimated parameter which are determined at b^0 C.

The behavior of temperature in river has been described using an exponential decay model

$$\frac{d(T - T_E)}{dt} = -\nu\ (T - T_E) \quad (15)$$

$$\frac{dB}{dt} = -\lambda B \quad (16)$$

$$\frac{dD}{dt} = \lambda B - \mu D \quad (17)$$

$$\lambda = \lambda_b {\theta_1}^{T-b} \quad (18)$$

$$\mu = \phi F^{\alpha} \theta_2^{\,T-b} \quad (19)$$

$$\mu = \mu_b\, \theta_2^{\,T-b} \quad (20)$$

where λ_b and μ_b are the experimentally determined value of λ & μ at b^0 C .
θ_1 & θ_2 are estimated parameters from equation (15) we have,

$$T - T_E = k_2\, e^{-vt}$$

where k_2 is constant of integration.
When $t = 0, T = T_0,$
So,

$$T - T_E = (T_0 - T_E)e^{-vt}$$

$$\text{or, } T = T_E + (T_0 - T_E)a_3 \quad (21)$$

$$\text{where } \quad a_3 = e^{-vt} \quad (22)$$

From equation (16), on solving, we get,

$$\frac{dB}{dt} = -\lambda_b\, \theta_1^{T_E - b}\, \theta_1^{(T_o - T_E)a_3}\ B$$

where λ & T are being substituted from (18) and (21) respectively

$$\text{or} \quad \frac{dB}{dt} = -A_1 \theta_1^{\bar{T}a_3}\ .B \quad (23)$$

where

$$A_1 = \lambda_b \theta_1^{\bar{T}a_3 - b} \quad (24)$$

$$T = (T_0 - T_E) \quad (25)$$

on solving (23), we have,

$$B = k_5 e^{-\frac{A_1 vt}{v} + \frac{A_1}{v}\sum_{n=1}^{\bar{n}} \frac{(\bar{T}\log\theta_1)^n\, a_3^n}{n\,!\,n}}\ .\ n=1,2,\dots n$$

$$\text{or,} \quad B = k_5 e^{-A_0\, vt + \sum_{n=1}^{\bar{T}} A_n a_3^n} \quad (26)$$

where

$$A_0 = \frac{A_1}{v}$$

$$\& \quad A_n = \frac{A_0(\bar{T}\log\ \theta_1)^n}{n\,!\,n} \quad ; n = 1,2 \dots\dots..\bar{n}$$

& k_5 *is* constant of integration.
When $t = 0, B = B_0$, we get

$$k_5 = B_0 e^{-\sum_{n=1}^{\bar{n}} An}$$

Substituting k_5 in (26), we have

$$B = B_0 e^{-\sum_{n=1}^{\bar{n}} An} \quad e^{-A_0\, vt + \sum_{n=1}^{\bar{n}} A_n a_3^n}$$

$$or, \ B = B_0\, e^{-A_0 vt + \sum_{n=1}^{\bar{n}} A_n a_3^n - \sum_{n=1}^{\bar{n}} A_n} \quad (27)$$

$$\text{or,} \quad B = B_0\, a_4 \quad (28)$$

where $a_4 = e^{-A_0 vt + \sum_{n=1}^{n} A_n a_3^n - \sum_{n=1}^{n} A_n}$
From equation (17),

$$\frac{dD}{dt} = \lambda\, B - \mu D$$

$$\text{or,} \quad \frac{dD}{dt} = (\lambda_b\, \theta_1^{T-b})B_0\, e^{-A_0 vt + \sum_{n=1}^{\bar{n}} A_n a_3^n - \sum_{n=1}^{\bar{n}} A_n} - \phi F^{\alpha} \theta_2^{T-b}\, D \quad (29)$$

$$\text{or,} \quad F^{\alpha}\theta_2^{T-b} = A_2 \theta_2^{\overline{T\,a_3}} \quad (30)$$

where

$$A_2 = \phi F^{\alpha} \theta_2^{T_E - b} \quad (31)$$

$$\& \quad \bar{T} = T_0 - T_E \quad (32)$$

So equation (29) can be written as

$$\frac{dD}{dt} A_1 \theta_1^{\bar{T}a_3}\, B_0\, e^{-A_0 vt + \sum_{n=1}^{\bar{n}} A_n a_3^n - \sum_{n=1}^{\bar{n}} A_n} - A_2 \theta_2^{\bar{T}a_3} D \text{ [by using (24) and (31)]}$$

$$\text{or,} \quad \frac{dD}{dt} + p(t)D = q\,(t) \quad (33)$$

where $p(t) = A_2 \theta_1^{\bar{T}a_3}$ (34)

$$\& \quad q(t) = B_0\, A_1 \theta_1^{\bar{T}a_3}\, e^{-A_0 vt + \sum_{n=1}^{\bar{n}} A_n a_3^n - \sum_{n=1}^{\bar{n}} A_n} \quad (35)$$

Solving equation (33) we have,

$$D \quad e^{C_0(\bar{T}\log\theta_2 - v\,t)+\sum_{n=1}^{\bar{n}} C_n\, a_3^n} = q(0)\,t + \sum_{m=1}^{m}\frac{q^m(0)t^{m+1}}{m+1} + C \qquad (36)$$

where C is constant of integration

Initially, $t = 0$, $D = D_0$ & thus

$$C = D_0\, e^{C_0\bar{T}\log\theta_2 + \sum_1^{\bar{n}} C_n} \qquad (37)$$

Using these equations in (36) we have,

$$D = q(0)t + \sum_{m=1}^{\bar{m}}\frac{q^{m+1}(0)t^{m+1}}{(m+1)!} + D_0\, a_6 \qquad (38)$$

Where $a_6 = e^{C_0\bar{T}\log\theta_2 + \sum_1^{\bar{n}} C_n}$ (39)

Equation (38) yields the new Dissolve Oxygen equation.

IV. Conclusion

Expression (38) and (28) for dissolved oxygen level (*D*) and the biological oxygen demand (B) have been obtained. From these two expressions *t* is obvious that the temperature effect both biological oxygen demand (*B*) and dissolved oxygen (*D*)

By choosing suitable value of parameters *B* and *D* can be numerically calculated. If the temperature is assumed to be fixed at the equilibrium value ($T_E = T_0$) the expression (28) reduces to

$$B = B_o\, e^{-At} \qquad (40)$$

Which is the solution of streeter and phelps equations with temperature as a parameter in the system. This shows that the present approach is correct and streeter and phelps result is a particular case of our analysis.

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