IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 12, Issue 1 Ver. II (Jan. - Feb. 2016), PP 01-10
www.iosrjournals.org

Pgrw-Continuous and Pgrw-Irresolute Maps in Topological Spaces

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Abstract: *This paper introduces pre generalized regular weakly continuous maps , pgrω–irresolute maps, strongly pgrω–continuous maps , perfectly pgrω–continuous maps and studies some of their properties.*
Keywords: *pgrω–closed sets, pgrω–open sets, pgrω–continuous maps, pgrω-irresolute maps, strongly pgrω-continuous maps, perfectly pgrω–continuous maps.*

I. Introduction

N .Levine[1] introduced Semi-open sets and semi-continuity in topological spaces. The concept of regular continuous and Completely–continuous functions was first introduced by Arya. S. P. and Gupta.R [2]. Later Y. Gnanambal [3] studied the concept of generalized pre regular continuous functions. Also, the concept of ωα-continuous functions was introduced by S S Benchalli et al [4]. R S Wali et al[5] introduced and studied the properties of αrω–Continuous and αrω–Irresolute Maps.Recently R S Wali et al[6] introduced and studied the properties of pgrω-closed sets. The purpose of this paper is to introduce a new class of functions, namely, pgrω-continuous functions and pgrω-irresolute functions, strongly pgrω-continuous maps , perfectly pgrω-continuous maps. Also we study some of the characterization and basic properties of pgrω-continuous functions.

II. Preliminaries

Definition2.1: A subset A of a topological space (X, T) is called

- a pre-open set[7] if A ⊆ int(cl(A)) and pre-closed set if cl(int(A)) ⊆ A.
- an α-open set [8]if A ⊆ int(cl(int(A))) and α -closed set if cl(int(cl(A)))⊆ A.
- a semi-preopen set (=β-open)[9] if A⊆cl(int(cl(A)))) and a semi-pre closed set (=β-closed) if int(cl(int(A)))⊆A.
- a regular open set [10] if A = int(clA)) and a regular closed set if A = cl(int(A)).
- a generalized closed set (briefly g-closed)[11] if cl(A) ⊆ U whenever A ⊆ U and U is open in X.
- a regular generalized closed set(briefly rg-closed)[11] if cl(A)⊆U whenever A⊆U and U is regular open in X.
- a α –generalized closed set(briefly αg -closed)[12] if αcl(A)⊆U whenever A⊆U and U is open in X.
- a generalized pre regular closed set(briefly gpr-closed)[3] if pcl(A) ⊆ U whenever A ⊆ U and U is regular open in X.
- a generalized semi-pre closed set(briefly gsp-closed)[13] if spcl(A) ⊆ U whenever A⊆U and U is open in X.
- a regular generalized α-closed set[14] (briefly, rgα-closed)if αcl (A)⊆ U whenever A⊆ U and U is regular α-open in X.
- an α-generalized regular closed[15] (briefly αgr-closed) set if αcl(A)⊆ U whenever A⊆U and U is regular-open in X.
- a ωα- closed set[16] if αcl(A) ⊆ U whenever A⊆ U and U is ω-open in X.
- a generalized pre closed (briefly gp-closed) set[17] if pcl(A)⊆U whenever A⊆U and U is open in X.
- a α-regular w- closed set[5] if αcl(A) ⊆U whenever A⊆U and U is rw -open in X.
- a generalized pre regular weakly closed (briefly gprw-closed) set [18] if pcl(A)⊆U whenever A⊆ U and U is regular semi- open in X.
- a #rg-closed[19] if cl(A)⊆U whenever A⊆U and U is rw-open.
- a regular generalized weak (briefly rgw-closed) set[20] if cl(int(A)) ⊆ U whenever A ⊆U and U is regular semi open in X.
- ageneralized semi pre regular closed (briefly gspr-closed) set [21] if spcl(A)⊆ U whenever A⊆U and U is regular open in X.

The complements of the above mentioned closed sets in (5) - (18), are called the respective open sets.

Definition 2.2: Let (X, T) be a topological space and A⊆X. The intersection of all closed (resp pre-closed, α-closed and semi-pre-closed) subsets of the space X containing A is called the closure (resp pre-closure, α-closure and Semi-pre-closure) of A and is denoted by cl(A) (resp pcl(A), αcl(A), spcl(A)).

2.3 Pre Generalised Regular Weakly Closed Set:
Definition: A subset A of a topological space (X, T) is called a pre generalised regular weakly closed set [6]if pcl(A)⊆U whenever A ⊆ U and U is a rw-open set.

- **Theorem:** Every pgrw-closed set is gp-closed
- **Theorem:** Every pre-closed set is pgrw-closed.
- **Corollary:** Every α- closed set is pgrw- closed.
- **Corollary:** Every closed set is pgrw-closed.
- **Corollary:** Every regular closed set is pgrw-closed.
- **Theorem :** Every #rg- closed set is pgrw- closed**.**
- **Theorem:** Every αrw-closed set is pgrw-closed.
- **Theorem :** Every pgrw- closed set is gsp-closed.
- **Corollary:** Every pgrw- closed set is gspr- closed.
- **Corollary :** Every pgrw- closed set is gpr- closed.
- **Theorem:** If A is open and gp-closed, then A is pgrw-closed.
- **Theorem:** If A is both w- open and wα- closed, then A is pgrw- closed.
- **Theorem:** If A is both regular-open and rg-closed, then A is pgrw-closed.
- **Theorem:** If A is both open and g-closed, then A is pgrw -closed.
- **Theorem:** If A is regular-open and gpr-closed, then it is pgrw-closed.
- **Theorem**: If A is regular-open and αgr -closed, then it is pgrw -closed.
- **Theorem:** If A is open and αg-closed, then it is pgrw -closed**.**
- **Theorem**: If A is regular open and pgrw-closed, then A is pre-closed.

2.4: Definition: A subset A of a topological space X is called a pre generalised regular-weakly open (briefly pgrw-open) set in X if the complement A^c of A is pgrw-closed in X.
Theorem: (X ,T) is a topological space.
i) Every open (α-open, regular-open, αrω-open, #rg-open, pgpr-open) set is pgrw-open.
ii) Every pgrw-open set is gspr–open (gsp-open, gp-open and gpr-open).

Definition 2.5: A map f: (X,τ)→(Y,σ) is said to be

- Completely–continuous[22] if f^{-1} (V) is regular closed in X for every closed subset V of Y
- Strongly–continuous[23] if f^{-1} (V) is Clopen (both open and closed) in X for every subset V of Y.
- α–continuous[8] if f^{-1} (V) is α–closed in X for every closed subset V of Y.
- rwg–continuous[24] if f^{-1} (V) is rwg–closed in X for every closed subset V of Y.
- gp–continuous[25] if f^{-1} (V) is gp–closed in X for every closed subset V of Y.
- gpr–continuous[3] if f^{-1} (V) is gpr–closed in X for every closed subset V of Y.
- αgr–continuous[15] if f^{-1} (V) is αgr–closed in X for every closed subset V of Y.
- ωα–continuous[4] if f^{-1} (V) is ωα–closed in X for every closed subset V of Y.
- gspr–continuous[21] if f^{-1} (V) is gspr–closed in X for every closed subset V of Y.
- g–continuous[25] if f^{-1} (V) is g–closed in X for every closed subset V of Y
- ω–continuous[26] if f^{-1} (V) is ω–closed in X for every closed subset V of Y
- rgα–continuous[14] if f^{-1} (V) is rgα–closed in X for every closed subset V of Y
- gsp–continuous[13] if f^{-1} (V) is gsp–closed in X for every closed subset V of Y.
- gprw–continuous[18] if f^{-1} (V) is gprw–closed in X for every closed subset V of Y
- wgrα–continuous[27] if f^{-1} (V) is wgrα–closed in X for every closed subset V of Y
- #rg-continuous [28] if f^{-1} (V) is #rg-closed in (X,τ) for every closed set V of Y.
- pre-continuous [7] then f^{-1} (V) is preopen in X for every open set V in Y.
- rg continuous [29] if the inverse image of every closed set inY is rg-closed in X
- semi-pre continuous (β- continuous)[30] if the inverse image of each open set in Y is a semi-preopen set in X.
- semi-generalized continuous (sg-continuous)[31] if for every closed set F of Y the inverse image f^{-1} (F) is sg-closed in X.
- rω–continuous[32] if f^{-1} (V) is rw–closed in X for every closed subset V of Y
- α regular ω continuous (αrω–Continuous)[5] if f^{-1}(V) is αrω–Closed set in X for every closed set V in Y.

- contra continuous [16] if $f^{-1}(V)$ is open in X for every closed subset V of Y .

Definition 2.6: A map f: (X,τ)→(Y,σ) is said to be

- α–irresolute [8] if $f^{-1}(V)$ is α-closed in X for every α-closed subset V of Y.
- irresolute [33] if $f^{-1}(V)$ is semi- closed in X for every semi-closed subset V of Y.
- contra ω–irresolute [26] if $f^{-1}(V)$ is ω-open in X for every ω-closed subset V of Y
- contra irresolute [17] if $f^{-1}(V)$ is semi-open in X for every semi-closed subset V of Y
- contra r–irresolute [34] if $f^{-1}(V)$ is regular-open in X for every regular-closed subset V of Y

III. Pgrw-Continuous Map:

Definition 3.1 :A map f: (X, T_1)→(Y, T_2) is called a pre generalised regular weakly- continuous map (pgrw-continuous map) if the inverse image $f^{-1}(V)$ of every closed set V in Y is pgrw-closed in X .

Example3.2: Let X ={a,b,c,d},T_1 = {X, ϕ,{a},{a,b}, {a,b,c}} and Y={a,b,c} T_2={Y, ϕ,{a}}.
Define a map f: X→Y by f(a)=b, f(b)=c , f(c)=a, f(d)=c. The closed sets in T_2 are Y, ϕ,{b,c}.
The pgrw-closed sets in T_1 are X , ϕ, {c},{d},{b,c},{c,d},{a,d},{b,d}, {b,c,d}, {a,c,d}, {a,b,d}
Inverse images of Y, ϕ, {b,c} are X , ϕ, {a,b,d} which are pgrw closed sets in X.
∴ f is pgrw-continuous map.

Theorem 3.3 : A map f: (X, T_1)→(Y, T_2) is pgrw-continuous if and only if the inverse image of every open set in Y is a pgrw-open set in X.

Proof : Suppose a map f: (X, T_1)→(Y, T_2) is pgrw-continuous.
Let U be an open set in Y. Then U^c is closed in Y. Therefore $f^{-1}(U^c)$ is pgrw-closed in X.
$f^{-1}(U^c) = X-f^{-1}(U)$. Therefore $f^{-1}(U)$ is pgrw-open in X.
Conversely
Suppose f: (X, T_1)→(Y, T_2) is such that the inverse image of every open set in Y is pgrw-open in X. Let F be a closed set in Y. Then F^c is open in Y. ∴ $f^{-1}(F^c)$ is pgrw-open.
$F^{-1}(F^c)=X-f^{-1}(F)$ ∴ $f^{-1}(F)$ is pgrw-closed in X.
∴ f is a pgrw-continuous map.

Theorem 3.4: If a map f: (X, T_1)→(Y, T_2) is continuous, then it is pgrw-continuous.

Proof : Let F be a closed subset in Y.
f is continuous. So $f^{-1}(F)$ is a closed set in X.As every closed set is pgrw-closed, $f^{-1}(F)$ is pgrw-closed. ∴ f is pgrw-continuous map.
The converse is not true.

Example 3.5 : Consider example 3.2. {b,c} is closed in Y and its inverse image {a,b,d} is not closed in X.
∴ f is pgrw-continuous.But not continuous.

Theorem 3.6 : If a map f: (X, T_1)→(Y, T_2) is completely continuous, then f is pgrw-continuous.

Proof : Suppose amap f: (X, T_1)→(Y, T_2) is completely continuous.
Let F be a closed set in Y. Then $f^{-1}(F)$ is regular-closed in X.
∴ $f^{-1}(F)$ is pgrw-closed in X as every regular-closed set is pgrw-closed .
∴ f is pgrw-continuous.
Converse is not true.

Example 3.7: In the above example 3.2 f is pgrw-continuous. But not completely continuous.

Theorem 3.8 : If a map f: (X, T_1)→(Y, T_2) is pre-continuous, then f is pgrw-continuous.

Proof: A map f : X→Y is pre-continuous.
Let F be a closed set in Y. Then $f^{-1}(F)$ is pre-closed in X.
Then $f^{-1}(F)$ is pgrw-closed in X as every pre-closed set is pgrw-closed.
∴ f is pgrw-continuous.
The converse is not true.

Example3.9: X ={a,b,c,d},T_1 = {X, φ, {a}, {b}, {a,b}, {a,b,c}}
Y={a,b,c},T_2 = {Y, φ, {a}, {b}, {a,b }}
Closed sets in T_2 are Y, φ, {b,c }, {a,c }, {c }.
Pgrw-closed sets in T_1 are X, φ, {c}, {d}, {b,c}, {c,d}, {a,d}, {b,d}, {b,c,d}, {a,c,d}, {a,b,d}.
Define f(a)=c, f(b)=a, f(c)=b, f(d)=c. Inverse images of closed sets in Y are X, φ, {a,c,d},{a,b,d}, {a,d}.
Then f is pgrw-continuous. But f is not pre-continuous since $f^{-1}(\{a,c\})$= {a,b,d} is not preclosed.

Theorem 3.10 : If a map f: (X, T_1)→(Y, T_2) is α-continuous, then f is pgrw-continuous.
Proof : A map f : X→Y is α-continuous.
Let F be closed in Y. Then $f^{-1}(F)$ is α-closed in X.
Then $f^{-1}(F)$ is pgrw-closed in X because every α-closed is pgrw-closed.
∴ f is pgrw-continuous map.
The converse is not true.
Example 3.11 : X=Y={a,b,c,d},
T_1 = {X, φ, {a}, {b}, {a,b}, {a,b,c}}
T_2 = {Y, φ, {a}, {b}, {a,b}, {a,b,c}}
Closed sets in T_2 are Y, φ, {b,c,d}, {a,c,d}, {c,d}, {d}
Pgrw-closed sets in T_1 are X, φ, {c}, {d}, {b,c}, {c,d}, {a,d}, {b,d}, {b,c,d}, {a,c,d}, {a,b,d}.
Define f(a)=c, f(b)=a, f(c)=b, f(d)=d. Inverse images of closed sets in Y are X, φ, {a,c,d},{a,b,d}, {a,d}, {d}.
Then f is pgrw-continuous. But f is not α-continuous.

Theorem 3.12: If a map f: (X, T_1)→(Y, T_2) is #rg –continuous, then f is pgrw-continuous.
The converse is not true.
Example 3.13:X ={a,b,c,d},T_1= {X, φ, {a}, {b}, {a,b}, {a,b,c}}
Y={a,b,c}, T_2={Y, ϕ,{a}}. Closed sets in (Y, T_2) are Y, ϕ,{b,c}.
Pgrw-closed sets in T_1 are X, φ, {c}, {d}, {b,c}, {c,d}, {a,d}, {b,d}, {b,c,d}, {a,c,d}, {a,b,d}.
#rg-closed sets are X, φ, {d},{c,d},{a,d}, {b,d},{a,c},{b,c,d}, {a,c,d}.
Define f(a)=b, f(b)=c, f(c)=a, f(d)=c.
Inverse images of closed sets in Y are X, φ, {a,b,d}. f is pgrw-continuous but not #rg –continuous.

Theorem 3.14 : If a map f: (X, T_1)→(Y, T_2) is αrw-continuous,then f is pgrw-continuous.
The converse is not true.
Example 3.15:X ={a,b,c,d},T_1 = {X, φ, {a}, {b}, {a,b}, {a,b,c}}
Y={a,b,c}, T_2={Y, ϕ,{a}}. Closed sets in (Y, T_2) are Y, ϕ,{b,c}.
Pgrw-closed sets in T_1 are X, φ, {c}, {d}, {b,c}, {c,d}, {a,d}, {b,d}, {b,c,d}, {a,c,d}, {a,b,d}.
αrw -closed sets are X, φ, {c}, {d},{c,d},{a,d}, {b,d},{b,c,d}, {a,c,d},{a,b,d}.
Define f(a)=d, f(b)=c, f(c)=b, f(d)=a
Inverse images of closed sets in Y are X, φ, {b,c}. f is pgrw-continuous but not αrw -continuous.
Theorem 3.16: If a map f: (X, T_1)→(Y, T_2) is α–irresolute, then it is pgrω– continuous.
Proof : Suppose that a map f: (X, T_1)→(Y, T_2) is α–irresolute. Let V be an open set in Y. Then V is α-open in Y. Since f is α–irresolute , $f^{-1}(V)$ is α–open and hence pgrω–open in X. Thus f is pgrω–continuous.

Theorem 3.17: If a map f: (X, T_1)→(Y, T_2) is pgrw-continuous, then f is gsp- continuous.
Proof: f : X→Y is pgrw-continuous. Let F be a closed set in Y. Then $f^{-1}(F)$ is pgrw-closed.
⇒ $f^{-1}(F)$ is gsp-closed . '.' Every pgrw-closed set is gsp-closed.⇒f is gsp-continuous.
Converse is not true.
Example3.18: X ={a,b,c},T_1 = {X, φ, {a}, {b}, {a,b}}
Y={a,b,c},T_2 = {Y, φ, {a }}Closed sets in T_2 are Y, φ, {b,c }.
Pgrw-closed sets in T_1 are X, φ, {c}, {b,c}, {a,c}.
Define f(a)=b, f(b)=c, f(c)=a. Inverse images of closed sets in Y are X, φ, {a,b}.f^{-1}({b,c})={a,b} which is not pgrw-closed. So f is not pgrw-continuous.gsp-closed sets are X, φ,{a},{b} {c},,{a,b},{b,c},{a,c}. f is gsp-continuous.

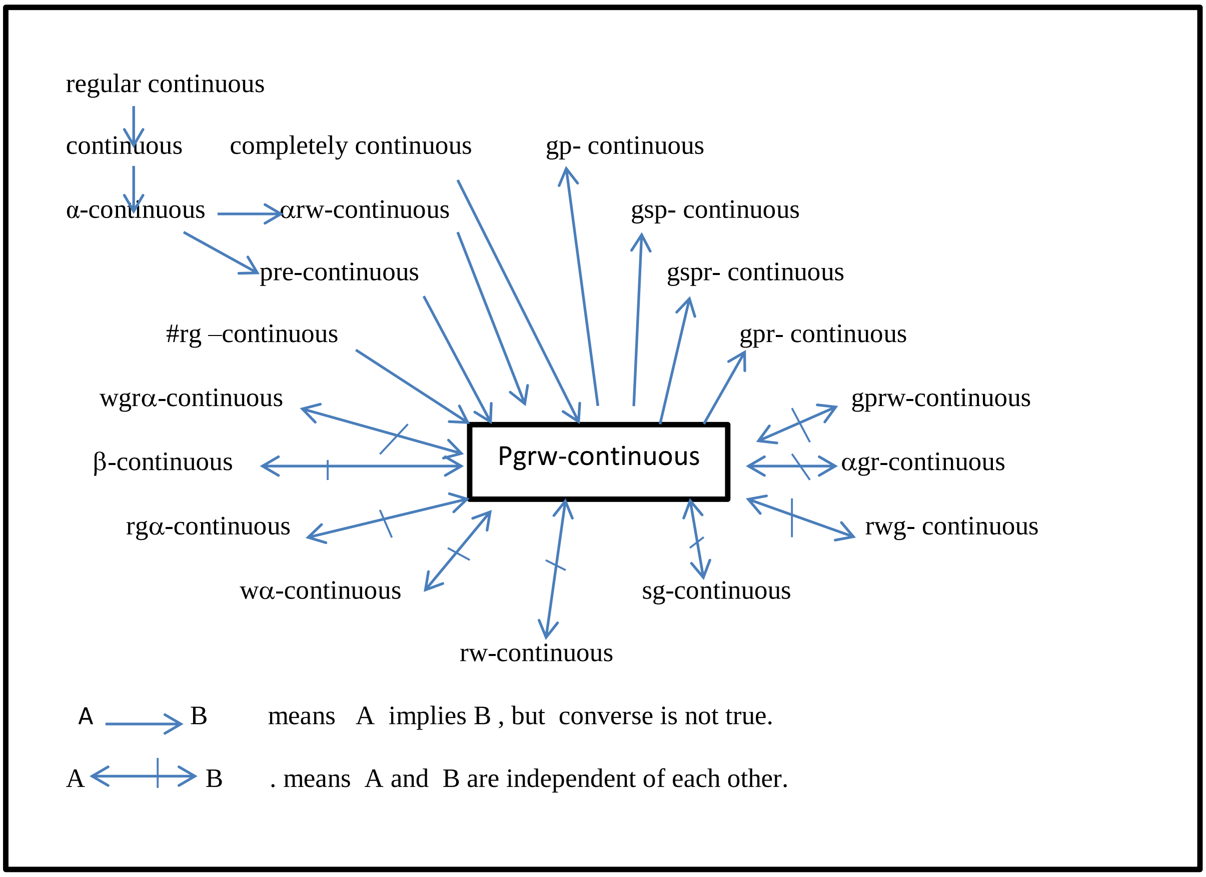
Theorem 3.19: If a map f: (X, T_1)→(Y, T_2) is pgrw-continuous, then f is gspr- continuous.
Proof: We can prove it using the fact that every pgrw-closed set is gspr closed.
Converse is not true. For example,
X ={a,b,c,d},T_1 = {X, φ, {a}, {b}, {a,b},{a,b,c}}
Y={a,b,c},T_2 = {Y, φ, {a }}
Closed sets in T_2 are Y, φ, {b,c }
Pgrw-closed sets in T_1 are X, φ, {c}, {d}, {b,c}, {c,d}, {a,d}, {b,d}, {b,c,d}, {a,c,d}, {a,b,d}.
Define f(a)=b, f(b)=c, f(c)=a f(d)=a. Inverse images of closed sets are are X, φ, {a,b}.
f^{-1}({b,c})={a,b} which is not pgrw-closed. So f is not pgrw-continuous.All subsets of X are gspr-closed.
f is gspr-continuous.

Theorem 3.20: If a map f: (X, T_1)→(Y, T_2) is pgrw-continuous, then f is gpr- continuous.
We can prove it using the fact that every pgrw-closed set is gpr closed.
Converse is not true.
Example 3.21: Consider example 3.18, f is not pgrw-continuous. gpr-closed sets are X,φ, {c} ,{d}, {a,b}, [b,c},{c,d} ,{a,c},{a,d},{b,d} ,{a,b,c},{a,b,d},{a,c,d} {b,c,d}. f is gpr-continuous.

Theorem 3.22: If a map f: (X, T_1)→(Y, T_2) is pgrw-continuous,then f is gp- continuous.
We can prove it using the fact that every pgrw-closed set is gp-closed
The coverse is not true.
Example 3.23: X={a,b,c},T_1 = {X, φ, {a }}, Y={a,b,c},T_2 = {Y, φ, {a}, {b}, {a,b}}.
Closed sets in T_2 are Y, φ, {b,c}, {a,c},{c}. Pgrw-closed sets in T_1 are X, φ,{b},{c},{b,c}.
gp-closed sets in T_1 are X, φ,{b},{c},{a,b},{b,c} ,{a,c}.
Define f(a)=b, f(b)=c, f(c)=a .
Inverse images of closed sets are X, φ,{a,b}, {b,c},{b} . Then f is gp-continuous but not pgrw-continuous.

Remark: The following examples show that pgrω-continuous map is independent of gprw-continuous, αgr-continuous, β-continuous, wgrα-continuous, sg-continuous, rw-continuous, wα-continuous,rgα-continuous, rwg- continuous.
Example 3.24: Let X ={a,b,c,d},T_1 = {X, φ, {a}, {b}, {a,b},{a,b,c}}
Y={a,b,c}, T_2 = {Y, φ, {a}, {b}, {a,b}}.
Define f : X→Y as f(a)= c , f(b)=a , f(c)=b , f(d)=c.
Closed sets in T_2 are Y, φ, {b,c}, {a,c},{c}.
Pgrw-closed sets in T_1 are X, φ, {c}, {d}, {b,c}, {c,d}, {a,d}, {b,d}, {b,c,d}, {a,c,d}, {a,b,d}.
Inverse images are X, φ, {a,c,d},{a,b,d},{a,d}. Here f is pgrw-continuous but not αgr-continuous, β-continuous, wgrα-continuous, sg-continuous, rw-continuous, wα-continuous, rgα-continuous.
Example 3.25: Let X ={a,b,c,d},T_1 = {X, φ, {a}, {b}, {a,b},{a,b,c}}
Y={a,b,c}, T_2 = {Y, φ, {a }}. Define f(a)=b, f(b)=a, f(c)=a , f(d)=c.
Closed sets in T_2 are Y, φ, {b,c} .
Pgrw-closed sets in T_1 are X, φ, {c}, {d}, {b,c}, {c,d}, {a,d}, {b,d}, {b,c,d}, {a,c,d}, {a,b,d}.
Inverse images are X, φ, {a,d}. Here f is pgrw-continuous but not gprw-continuous.
Example 3.26: X ={a,b,c} T_1 = {X, φ, {a}, {b}, {a,b}} , Y={a,b,c}, T_2 = {Y, φ, {a }}.
Closed sets in T_2 are Y, φ, {b,c} . Pgrw-closed sets in T_1 are X, φ, {c}, {a,c}, {b,c}.
Define f(a)=c, f(b)=a, f(c)=b. Inverse images are X, φ, {a,c}. f is pgrw-continuous but not swg-continuous.
Example3.27: Let X ={a,b,c,d},T_1 = {X, φ, {a}, {b}, {a,b},{a,b,c}}
Y={a,b,c}, T_2 = {Y, φ, {a }}. Closed sets in T_2 are Y, φ, {b,c}.
Pgrw-closed sets in T_1 are X, φ, {c}, {d}, {b,c}, {c,d}, {a,d}, {b,d}, {b,c,d}, {a,c,d}, {a,b,d}.
Define f(a)=b, f(b)=a, f(c)=c, f(d)=b. Inverse images are X, φ, {a,c,d}.f is pgrw-continuous but not rwg-continuous.
Example 3.28: X ={a,b,c}, T_1 = {X, φ, {a}, {b}, {a,b}} , Y={a,b,c}, T_2 = {Y, φ, {a }}. Closed sets in T_2 are Y, φ, {b,c} .
Pgrw-closed sets in T_1 are X, φ, {c}, {a,c}, {b,c}. Define f(a)=b, f(b)=c, f(c)=a.
Inverse images are X, φ, {a,b}. f is not pgrw-continuous but f is β-continuous, wgrα-continuous,sg-continuous, rw-continuous, αgr- continuous, rwg- continuous.
Example 3.29: X ={a,b,c}, T_1 = {X, φ, {a}} , Y={a,b,c}, T_2 = {Y, φ, {a }, {b,c}}. Closed sets in T_2 are Y, φ, {b,c},{a}. Pgrw-closed sets in T_1 are X, φ, {c}, {b}, {b,c}. Define f(a)=b, f(b)=c, f(c)=a
Inverse images are X, φ, {b}, {a,b}. f is not pgrw-continuous but f is rgα-continuous.
Example 3.30: Let X ={a,b,c,d},T_1 = {X, φ, {a}, {b}, {a,b},{a,b,c}}
Y={a,b,c}, T_2 = {Y, φ, {a }}. Closed sets in T_2 are Y, φ, {b,c}.
Pgrw-closed sets in T_1 are X, φ, {c}, {d}, {b,c}, {c,d}, {a,d}, {b,d}, {b,c,d}, {a,c,d}, {a,b,d}.
Define f(a)=b, f(b)=c, f(c)=a , f(d)=a. Inverse images are X, φ, {a,b}. f is not pgrw-continuous but f is wα-continuous,gprw-continuous.

Remark 3.31: From the above discussion and known results we have the following implications.



Theorem 3.32:f: $(X, T_1)\to(Y, T_2)$ is a map . Then the following statements hold.
1) If f is agp–continuous and contra continuous map, then f is pgrω –continuous.
2) If f is a ωα –continuous and contra-w- irresolute map, then f is pgrω –continuous
3) If f is a rg–continuous and contra- r–irresolute map, then f is pgrω –continuous .
4) If f is a g–continuous and contra continuous map, then f is pgrω –continuous .
5) If f is a gpr–continuous and contra- r–irresolute map, then f is pgrω –continuous.
6) If f is aαgr–continuous and contra- r–irresolute map, then f is pgrω –continuous .
7) If f is aαg- continuous and contra continuous map, then f is pgrw-continuous.
8) If f is a pgrw–continuous and contra- r–irresolute map ,then f is pre–continuous.
Proof: 1) Let V be a closed set of Y. Then $f^{-1}(V)$ is open and gp-closed in X('.' f is gp-continuous and contra continuous map) . Then $f^{-1}(V)$ is pgrw-closed set in X ('.' every open and gp-closed set is pgrw-closed).
Thus f is pgrw-continuous.
Similarly , we can prove2), 3) , 4) , 5), 6), 7) ,8).

Theorem 3.33: If a map f: $(X, T_1)\to(Y, T_2)$is pgrω–continuous, then f(pgrωcl(A)) ⊆ cl(f(A)) for every subset A of X.
Proof: f(A) ⊆ cl(f(A)) implies that A ⊆ f^{-1}(cl(f(A))). Since cl(f(A)) is a closed set in Y and f is pgrω–continuous f^{-1}(cl(f(A))) is a pgrω–closed set in X containing A. Hence pgrωcl(A)⊆f^{-1}(cl(f(A))). Therefore f(pgrωcl(A)) ⊆ cl(f(A)).

IV. Perfectly Pgrω–Continuous Map:

Definition 4.1: A function f: (X, T_1)→(Y, T_2) is called a perfectly pre generalized regular ωeakly- continuous (briefly perfectly pgrω–Continuous) function, if f^{-1}(V) is a clopen (closed and open) set in X for every pgrω–open set V in Y.

Theorem 4.2: If a map f: (X, T_1)→(Y, T_2) is perfectly pgrω–continuous, then
(i) f is pgrω–continuous.
(ii) f is gsp–continuous.
(iii) f is gspr–continuous.
(iv) f is gpr–continuous.
(v) f is gp–continuous.
Proof: (i) Let F be an open set in Y. Then F is pgrω–open in Y. Since F is perfectly pgrω–continuous, f^{-1}(F) is clopen in X, so open f^{-1}(F) is pgrω–open in X. Hence f is pgrω–continuous.
(ii) Let F be an open set in Y. As every open set is pgrω–open in Y andf is perfectly pgrω-continuous and so f^{-1}(F) is both closed and open in X, as every open set is pgrω-open that implies gsp–open. Then f^{-1}(F) is gsp-open in X. Hence f is gsp–continuous.
Similarly, we can prove (iii), (iv) and (v).

Theorem 4.3: (X,τ) is a discrete topological space and (Y,σ) is any topological space. Then every function f: (X, τ)→(Y, σ) is perfectly pgrw-continuous.
Proof: Let U be a pgrω -open set in (Y,σ). Since(X,τ) is a discrete space F^{-1}(U) is both open and closed in (X,τ). Hence f is perfectly pgrω-continuous.

Theorem 4.4: If f: (X, T_1)→(Y, T_2) is a strongly continuous map, then it is perfectly pgrw-continuous.
Proof: Let V be a pgrw-open set in Y. As f is strongly continuous and V is a subset of Y, f^{-1}(V) is clopen in X. So f is perfectly pgrw-continuous.

V. Pgrω*–Continuos Map

Definition 5.1: A function f: (X, T_1)→(Y, T_2) is called a pre generalized regular ωeakly*- continuous function (pgrω*–continuous function) if f^{-1}(V) is a pgrω–closed set in X for every pre-closed set V in Y.

Theorem 5.2: If a map f: (X, T_1)→(Y, T_2) is is pgrω*–continuous, then it is pgrω–continuous.
Proof: f: (X, T_1)→(Y, T_2) is pgrω*–continuous. Let F be any closed set in Y. Then F is pre-closed in Y. Since f is pgrω*-continuous., the inverse image f^{-1}(F) is pgrω-closed in X. Therefore f is pgrω-continuous.

VI. Pgrω–Irresolute map

Definition6.1: A map f: (X,T_1)→(Y, T_2) is called a pre generalized regular ωeakly irresolute (pgrω–irresolute) map if f^{-1}(V) is a pgrω–closed set in X for every pgrω–closed set V in Y.

Theorem 6.2: A map f: (X,T_1)→(Y, T_2) is pgrω–irresolute if and only if the inverse image f^{-1}(V) is pgrω–open in X for every pgrω–open set V in Y.
Proof: Assume that f: (X,T_1)→(Y, T_2) is pgrω–irresolute. Let G be a pgrω–open set in Y. Then G^c is pgrω–closed in Y. Since f is pgrω– irresolute, $f^{-1}(G^c)$ is pgrω–closed in X. But $f^{-1}(G^c) = X – f^{-1}(G)$. ∴ f^{-1}(G) is pgrω–open in X.
Conversely
Assume that the inverse image f^{-1}(V) of every pgrw- open set V in Y is pgrω–open in X. Let F be any pgrω–closed set in Y. Then F^c is pgrw-open in Y.By assumption $f^{-1}(F^c)$ is pgrω–open in X. But $f^{-1}(F^c) = X – f^{-1}(F)$.
∴ X– f^{-1}(F) is pgrω–open in X and so f^{-1}(F) is pgrω–closed in X. Therefore f is pgrω– irresolute.

Theorem 6.3: Every perfectly pgrw-continuous map is pgrw-irresolute.
Proof: Let f: (X, T_1)→(Y, T_2) be a perfectly pgrw-continuous map. Let V be a pgrw-open set in Y. Then f^{-1}(V) is clopen in X.and so f^{-1}(V) is open . As every open set is pgrw-open, f^{-1}(V) is pgrw-open.
∴ f is pgrw-irresolute.

Theorem 6.4: If a map f: (X, T_1)→(Y, T_2) is pgrω–irresolute, then it is pgrω*–continuous.
Proof: f: (X, T_1)→(Y, T_2) is pgrω–irresolute. Let F be any pre-closed set in Y. Then F is pgrω–closed in Y. Since f is pgrω–irresolute , f^{-1}(F) is pgrω–closed in X. Therefore f is pgrω*–continuous.

Theorem6.5: If a map f: (X, T_1)→(Y, T_2) is pgrω–irresolute, then it is pgrω–continuous.

Proof: f: $(X, T_1)\rightarrow(Y, T_2)$ is a pgrω–irresolute map. Let F be any closed set in Y. Then F is pgrω–closed in Y. Since f is pgrω–irresolute , the inverse image $f^{-1}(F)$ is pgrω–closed set in X. Therefore f is pgrω–continuous.

Theorem6.6: If a map f: $(X, T_1)\rightarrow(Y, T_2)$ is pgrω–irresolute, then for every subset A of X , f(pgrωcl(A) ⊆ pcl(f(A)).
Proof : A⊆ X. Then pcl(f(A)) is pgrω-closed in Y. Since f is pgrω–irresolute ,f^{-1}(pcl(f(A))) is pgrω-closed in X. Further A⊆ f^{-1}(f(A))⊆ f^{-1}(pcl(f(A))). Therefore by definition of pgrω-closure pgrωcl(A)⊆f^{-1}(pcl(f(A))), consequently f(pgrωcl(A) ⊆ pclf((A)).

Theorem6.7:f: $(X, \tau)\rightarrow(Y,\sigma)$ and g : $(Y, \sigma) \rightarrow(Z,\eta)$ are two functions.
(i) If f is pgrω– irresolute and g is r-continuous ,then g o f : $(X, \tau)\rightarrow(Z,\eta)$ is pgrω–continuous.
(ii) If fand g are pgrω–irresolute , then g o f : $(X, \tau)\rightarrow(Z,\eta)$ is pgrω–irresolute.
(iii) If f is pgrω–irresolute and g is continuous ,then g o f : $(X, \tau)\rightarrow(Z,\eta)$ is pgrω–continuous.
(iv) If f: $(X, T_1)\rightarrow(Y, T_2)$ is a pgrω–continuous function and g:Y→Z is a continuous function, then g∘f: X→Z is pgrω–continuous.
Proof:(i) Let U be an open set in (Z, η) . Since g is r-continuous, $g^{-1}(U)$ is r-open in (Y, σ). As every r-open set is pgrω–open, so $g^{-1}(U)$ is pgrω–open in Y. As f is pgrω–irresolute$f^{-1}(g^{-1}(U))$ is a pgrω–open set in (X, τ). Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is a pgrω–open set in (X, τ) and hence gof is pgrω–continuous.
(ii) Let U be a pgrω–open set in (Z, η) . Since g is pgrω–irresolute, $g^{-1}(U)$ is pgrω–open in (Y, σ). Since f is pgrω–irresolute, $f^{-1}(g^{-1}(U))$ is a pgrω–open set in (X, τ).
Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is a pgrω–open set in (X, τ) and hence gof is pgrω– irresolute.
(iii) Let U be an open set in (Z, η) . Since g is continuous, $g^{-1}(U)$ is open in (Y, σ). As every open set is pgrω-open, $g^{-1}(U)$ is pgrω-open set in (Y, σ). Since f is pgrω- irresolute$f^{-1}(g^{-1}(U))$ is a pgrω-open set in (X, τ). Thus for every open set U in Z, $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is a pgrω-open set in (X, τ) and hence gof is pgrω–continuous.
(iv) Let V be an open set in Z. As g is continuous $g^{-1}(V)$ is open in Y. Since f is pgrω–continuous, $F^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ is pgrω–open in X. Hence g∘f is pgrω–continuous.

VII. Strongly Pgrω–Continuous map:

Definition 7.1: A map f: $(X, T_1)\rightarrow(Y, T_2)$ is called a strongly pre generalized regular weakly -continuous (strongly pgrω–continuous) map if $f^{-1}(V)$ is a closed set in X for every pgrω–closed set V in Y.

Theorem 7.2: A map f: $(X, T_1)\rightarrow(Y, T_2)$ is strongly pgrω–continuous if and only if $f^{-1}(G)$ is an open set in X for every pgrω–open set G in Y.
Proof : f: $(X, T_1)\rightarrow(Y, T_2)$ is strongly pgrω–continuous. Let G be pgrω–open in Y. The G^c is pgrω–closed in Y. Since f is strongly pgrω-continuous, $f^{-1}(G^c)$ is closed in X. But $f^{-1}(G^c) = X–f^{-1}(G)$. ∴ $f^{-1}(G)$ is open in X.
Conversely
Assume that the inverse image of every pgrw- open set in Y is open in X. Let F be any pgrω-closed set in Y. Then F^c is pgrω -open in Y. ∴ $f^{-1}(F^c)$ is open in X . But $f^{-1}(F^c) = X–f^{-1}(F)$. ∴ $X– f^{-1}(F)$ is open in X and so $f^{-1}(F)$ is closed in X. Therefore f is strongly pgrω-continuous.

Theorem 7.3: If a map f: $(X, \tau)\rightarrow (Y,\sigma)$ is strongly pgrω–continuous and A is an open subset of X, then the restriction f/A: $(A, \tau_A) \rightarrow (Y,\sigma)$ is strongly pgrω–continuous.
Proof: Let V be any pgrω-open set of Y.Since f is strongly pgrω–continuous $f^{-1}(V)$ is open in X. Since A is open in X
$(f/A)^{-1}(V)=A\cap f^{-1}(V)$ is open in A. Hence f/A is strongly pgrω–continuous.

Theorem7.4: If a map f: $(X, T_1)\rightarrow(Y, T_2)$ is strongly pgrω–continuous, then it is continuous.
Proof: Assume that f: $(X, T_1)\rightarrow(Y, T_2)$ is strongly pgrω-continuous, Let F be a closed set in Y. As every closed set is pgrω–closed, F is pgrω-closed in Y. Since f is strongly pgrω–continuous so $f^{-1}(F)$ is closed set in X. Therefore f is continuous.

Theorem 7.5 : If a map f: $(X, T_1)\rightarrow(Y, T_2)$ is strongly pgrω–continuous, then it is pgrω– irresolute.
Proof :f is astrongly pgrω–continuous map. Let V be a pgrw-closed set in Y. Then $f^{-1}(V)$ is closed in X. Every closed set is pgrw-closed. ∴ $f^{-1}(V)$ is pgrw-closed in X. ∴ f is pgrω– irresolute.

Theorem 7.6: Every perfectly pgrw-continuous map is strongly pgrω–continuous.

Proof: Let f: (X, T_1)→(Y, T_2) be a perfectly pgrw-continuous map. Let U be a pgrω-open set in Y. As f is perfectly pgrw-continuous $f^{-1}(U)$ is both open and closed in (X,τ). $f^{-1}(U)$ is open in (X,τ). Hence f is strongly pgrω-continuous.

Theorem 7.7: If a map f: (X, T_1)→(Y, T_2) is strongly continuous , then it is strongly pgrω–continuous.
Proof: f: (X, T_1)→(Y, T_2)is strongly continuous. Let G be pgrω–open in Y.As f is strongly continuous and G is a subset of Y , $f^{-1}(G)$ is clopen in X and so open in X. Therefore f is strongly pgrω–continuous

Theorem7.8: For all discrete spaces X and Y **,** if a map f: (X, T_1)→(Y, T_2) is strongly pgrω–continuous, then it is strongly continuous.
Proof: Let F be a subset of Y. As Y is adiscrete space F isclopen.

$$\Rightarrow\begin{Bmatrix} \text{F is open} \Rightarrow \text{F is pgrw} - \text{open} \Rightarrow f^{-1}(\text{F})\text{is open.} \\ \text{F is closed} \Rightarrow \text{F is pgrw} - \text{close} \Rightarrow f^{-1}(\text{F})\text{is closed.} \end{Bmatrix}$$

⇒$f^{-1}(F)$ is clopen. Hence f is strongly continuous.

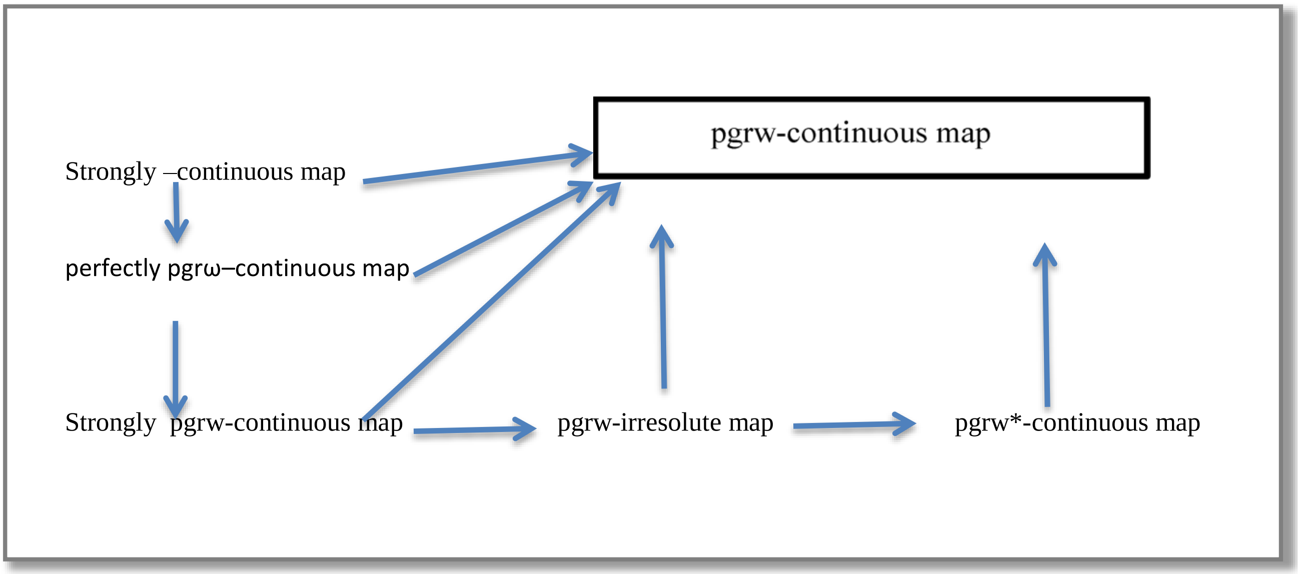
Theorem 7.9: If a map f: (X, T_1)→(Y, T_2) is strongly pgrω–continuous, then it is pgrω–continuous.
Proof: Let G be an open set in Y . As every open set is pgrω–open , G is pgrω–open in Y. Since f is strongly pgrω–continuous, $f^{-1}(G)$ is open in X. As every open is pgrw-open , $f^{-1}(G)$ is pgrω–open in X . Hence f is pgrω–continuous.
Theorem7.10: f: (X, τ)→(Y,σ) and g: (Y, σ) →(Z,η) aretwo functions.
(i) If f and gare strongly pgrω–continuous, then g o f : (X, τ)→(Z,η) is strongly pgrω–continuous.
(ii) If f is continuous and g is strongly pgrω–continuous , then g o f : (X, τ)→(Z,η) is strongly pgrω-continuous.
(iii) If f is pgrω–continuous and g is strongly pgrω–continuous , then g o f : (X, τ)→(Z,η) is pgrω–irresolute.
(iv) If f is strongly pgrω–continuous and g is pgrω–continuous ,then g o f : (X, τ)→(Z,η) is continuous.
Proof: (i) Let U be a pgrω–open set in (Z, η) . Since g is strongly pgrω–continuous, $g^{-1}(U)$ is open in (Y, σ). As every open set is pgrω–open, $g^{-1}(U)$ is pgrω–open set in (Y, σ). Since f is strongly pgrω–continuous $f^{-1}(g^{-1}(U))$ is an open set in (X, τ). Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an open set in (X, τ) and hence gof is strongly pgrω–continuous.
(ii) Let U be a pgrω–open set in (Z, η) . Since g is strongly pgrω–continuous, $g^{-1}(U)$ is open in (Y, σ). Since f is continuous $f^{-1}(g^{-1}(U))$ is an open set in (X, τ).
Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an open set in (X, τ) and hence gof is strongly pgrω–continuous.
(iii) Let U be a pgrω–open set in (Z, η) . Since g is strongly pgrω–continuous, $g^{-1}(U)$ is open in (Y, σ). Since f is pgrω–continuous, $f^{-1}(g^{-1}(U))$ is a pgrω–open set in (X, τ).
Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is a pgrω–open set in (X, τ) and hence gof is pgrω–irresolute.
(iv) Let U be an open set in (Z, η) . Since g is pgrω–continuous $g^{-1}(U)$ is a pgrω–open set in (Y, σ). Since f is strongly pgrω–continuous $f^{-1}(g^{-1}(U))$ is an open set in (X, τ).
Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an open set in (X, τ) and hence gof is continuous.

Theorem7.11: f: (X, τ)→(Y,σ) and g: (Y, σ) →(Z,η) are two functions.

1. If f is continuous and g is perfectly pgrω–continuous, then gof : (X, τ)→(Z,η) is strongly pgrω–continuous.
2. If f is perfectly pgrω–continuous and g is strongly pgrω–continuous ,then g o f : (X, τ)→(Z,η) is perfectly pgrω–continuous.
3.If f and g are perfectly pgrω–continuous functions, then g o f: (X, τ)→(Z,η) is perfectly pgrω–continuous.

Proof: 1. Let U be a pgrω–open set in (Z, η) . Since g is perfectly pgrω–continuous, $g^{-1}(U)$ is clopen in (Y, σ). ∴ $g^{-1}(U)$ is open in (Y, σ). Since f is continuous, $f^{-1}(g^{-1}(U))$ is an open set in (X, τ).
Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an open set in (X, τ) and hence gof is strongly pgrω–continuous .
2. Let U be a pgrω–open set in (Z, η) . Since g is strongly pgrω–continuous, $g^{-1}(U)$ is an open set in (Y, σ) and so pgrw-open. Since f is perfectly pgrω-continuous , $f^{-1}(g^{-1}(U))$ is a clopen set in (X, τ).
Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an clopen set in (X, τ) and hence gof is perfectly pgrω–continuous.
3). Let U be a pgrw-open set in Z. As g is perfectly pgrω–continuous, $g^{-1}(U)$ is clopen in Y and so open. As every open set is pgrw-open $g^{-1}(U)$ is pgrw-open in Y. As f is perfectly pgrω-continuous $f^{-1}(g^{-1}(U))$ is clopen in X. Hence$(gof)^{-1}(U)$ is clopen in X. Hence gof is perfectly pgrω–continuous.

The following diagram shows the relation between above discussed maps.



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