IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 12, Issue 1 Ver. I (Jan. - Feb. 2016), PP 17-35
www.iosrjournals.org

The Effect of 1st Order Chemical Reaction in Convective Dusty Fluids Turbulent Flow for Three-Point Joint Distribution Functions

M. Mamun Miah [1], M. A. K. Azad[2], M. Masidur Rahman[3]
[1](Department of Applied Mathemtics, University of Rajshahi, Bangladesh)
[2](Department of Applied Mathematics, University of Rajshahi, Bangladesh)
[3](Department of Applied Mathemtics, University of Rajshahi, Bangladesh)

Abstract: *In this paper, the three-point distribution functions for simultaneous velocity, temperature and concentration fields in dusty fluids turbulent flow undergoing a first order reaction have been studied. The various properties of constructed distribution functions have been discussed. From beginning to end of the study, the transport equation for three-point distribution function undergoing a first order reaction has been obtained. The resulting equation is compared with the previous equation which related to the distribution function by many authors and the closure difficulty is to be removed as in the case of ordinary turbulence.*

Keywords: *Magnetic Temperature, Concentration, Three-point distribution functions, MHD turbulent flow, First Order Reactant.*

I. Introduction

In this study, the joint distribution functions for simultaneous velocity, temperature and concentration fields in turbulent flow undergoing a first order reaction in presence of dust particles have been studied. A distribution function may be specialized with respect to a particular set of dimensions. The various properties of the constructed joint distribution functions such as reduction property, separation property, coincidence and symmetric properties have been discussed. Particle distribution functions are often used in plasma physics to describe wave-particle interactions and velocity-space instabilities. Distribution functions are used in various fields such as fluid mechanics, statistical mechanics and nuclear physics. The mathematical analog of a distribution is a measure; the time evolution of a measure on a phase space is the topic of study in dynamical systems. In the past, several researchers discussed the distribution function in the statistical theory of turbulence. Lundgren T.S. [1967] derived the transport equation for the distribution of velocity in turbulent flow. Kumar and Patel [1974] studied the first order reactant in homogeneous turbulence before the final period of decay for the case of multi-point and single- time correlation. Kumar and Patel [1975] extended their problem for the case of multi-point and multi- time concentration correlation. Patel [1976] considered first order reactant in homogenous turbulence in numerical results. Kishore [1978] studied the distribution function in the statistical theory of MHD turbulence of an incompressible fluid. Pope [1981] derived the transport equation for the joint probability density function of velocity and scalars in turbulent flow. Dixit and Upadhyay [1989] considered the distribution function in the statistical theory of MHD turbulence of an incompressible fluid in the presence of the coriolis force. Also Kishore and Singh [1985] have been derived the transport equation for the joint distribution function of velocity, temperature and concentration in convective turbulent flow. Sarker and Kishore [1991] discussed the distribution function in the statistical theory of convective MHD turbulence of an incompressible fluid. Also Sarker and Kishore [1999] premeditated the distribution function in the statistical theory of convective MHD turbulence of mixture of a miscible incompressible fluid. Islam and Sarker [2001] deliberated the distribution function in the statistical theory of MHD turbulence for velocity and concentration undergoing a first order reaction. Azad and Sarker [2004] considered statistical theory of certain distribution function in MHD turbulence in a rotating system in presence of dust particles. Azad et *al.* [2009] studied the first order reactant in MHD turbulence before the final period of decay considering rotating system and dust particles. Sarker et al. [2009a] studied the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time in presence of dust particles. Aziz *et al.* [2010] studied the statistical theory of distribution function in Magneto-hydrodynamic turbulence in a rotating system with dust particles undergoing a first order reaction. Azad et al. [2011] premeditated the statistical theory of certain distribution function in MHD turbulent flow for velocity and concentration undergoing a first order reaction in a rotating system. In very recent, Azad et al. [2012] studied the transport equation for the joint distribution function of velocity, temperature and concentration in convective turbulent flow in presence of dust particle. Azad et al [2013] extended problem Azad et al [2012] for 1st order chemical reaction. Very recent Azad et al [2014a] derived the transport equations of three point distribution functions in MHD turbulent flow for velocity, magnetic temperature and concentration, Azad and Nazmul [2014b] considered the transport equations of three point distribution functions in MHD turbulent flow for velocity, magnetic temperature and concentration in a rotating system Azad et al [2015] derived a transport equation for the joint distribution functions of certain variables in convective dusty fluid turbulent flow in a rotating system undergoing a first order reaction. All of the above researchers had studied their problems for MHD turbulent flow.

But in this paper we considered the three-point Distribution function through this study we have derived the transport equations for three-point joint distribution functions in convective turbulent flow due to first order chemical reaction in presence of dust particles. The various properties of the constructed joint distribution functions such as reduction property, separation property, coincidence and symmetric properties have been discussed.

II. Formulation Of The Problem

The equation of motion and field equations of temperature and concentration in presence of dust particles undergoing a first order chemical reaction are given by

$$\frac{\partial u_\alpha}{\partial t}+u_\alpha\frac{\partial u_\alpha}{\partial x_\beta}=-\frac{\partial}{\partial x_\beta}\int_0^\infty\frac{1}{4\pi}\frac{\partial}{\partial x'_\beta}\left\{u_\alpha(x',t)\frac{\partial}{\partial x'_\beta}\cdot u_\alpha(x',t)\right\}\frac{dx'_\beta}{\left|x_\beta-x'_\beta\right|}+\nu\frac{\partial}{\partial x_\beta}\frac{\partial}{\partial x_\beta}u_\alpha+f(u_\alpha-v_\alpha) \tag{1}$$

$$\frac{\partial\theta}{\partial t}+u_\alpha\frac{\partial\theta}{\partial x_\beta}=\gamma\frac{\partial}{\partial x_\beta}\frac{\partial}{\partial x_\beta}\theta \tag{2}$$

$$and\ \frac{\partial c}{\partial t}+u_\alpha\frac{\partial c}{\partial x_\beta}=D\frac{\partial}{\partial x_\beta}\frac{\partial}{\partial x_\beta}c-Rc \tag{3}$$

With $\frac{\partial u_\alpha}{\partial x_\alpha}=\frac{\partial v_\alpha}{\partial x_\alpha}=0$

Where,

$u_\alpha(x,t)$ = Component of turbulent velocity,

$\theta(x,t)$ = Temperature fluctuation,

R = Constant reaction rate,

ν = Kinematics viscosity,

c = Concentration of contaminants,

N = Constant number of density of the dust particle,

ρ = Fluid density,

D = Diffusive coefficient for contaminants,

c_p = Specific heat at constant pressure,

v_α = Dust particle velocity,

k_T = Thermal conductivity.

f = $\frac{KN}{\rho}$ = Dimension of frequency,

$\gamma=\frac{k_T}{\rho c_p}$ =Thermal diffusivity,

Where, u and x are vector quantities in the whole process.

To drive the transport equation for three-point joint distribution function we have to consider the turbulence and the concentration fields are homogeneous, the chemical reaction and the local mass transfer have no effect on the velocity field and the reaction rate and the diffusivity are constant. Also consider a large ensemble of identical fluids in which each member is an infinite incompressible reacting and heat conducting fluid in turbulent state. The fluid velocity u, temperature θ and concentration c are randomly distributed functions of position and time and satisfy their field equations.

The total energy, total pressure, stress tensor which are nothing but ensemble averages at a particular time can be determined with the help of the bivariate distribution functions (defined as the averaged distribution functions with the help of Dirac delta-functions). The present aim is to construct the distribution functions, study its properties and derive an equation for its evolution of the convective dusty fluids turbulent flow for three-point joint distribution functions.

III. Distribution Function In Turbulence And Their Properties

We may consider the fluid velocity u, temperature θ and concentration c at each point of the flow field inTurbulence. Then corresponding to each point of the flow field, we have three measurable characteristics. We represent the three variables by v, ϕ and ψ and denote the pairs of the several variables at the points $x^{(1)}, x^{(2)}, -----, x^{(n)}$ as $\left(v^{(1)}, \phi^{(1)}, \psi^{(1)}\right), \left(v^{(2)}, \phi^{(2)}, \psi^{(2)}\right), ..., \left(v^{(n)}, \phi^{(n)}, \psi^{(n)}\right)$ at a fixed instant of time.

It is possible that the same pair may be occurring more than once; therefore, we simplify the problem by an assumption that the distribution is discrete (in the sense that no pairs occur more than once). Symbolically we can express the bivariate distribution as

$$\left\{ \left(v^{(1)}, \phi^{(1)}, \psi^{(1)}\right), \left(v^{(2)}, \phi^{(2)}, \psi^{(2)}\right), ------ \left(v^{(n)}, \phi^{(n)}, \psi^{(n)}\right) \right\}$$

Instead of considering discrete points in the flow field, if we consider the continuous distribution of the variables v, ϕ and ψ over the entire flow field, statistically behavior of the fluid may be described by the distribution function $F(v, \phi, \psi)$ which is normalized so that

$$\int F(v, \phi, \psi) dv d\phi d\psi = 1$$

Where, the integration ranges over all the possible values of v, ϕ and ψ. We shall make use of the same normalization condition for the discrete distributions also.

The distribution functions of the above quantities can be defined in terms of Dirac delta function. The one-point distribution function $F_1^{(1)}\left(v^{(1)}, \phi^{(1)}, \psi^{(1)}\right)$, defined so that $F_1^{(1)}\left(v^{(1)}, \phi^{(1)}, \psi^{(1)}\right) dv^{(1)} d\phi^{(1)} d\psi^{(1)}$ is the probability that the fluid velocity, temperature and concentration at a time t are in the element $dv^{(1)}$ about $v^{(1)}$, $d\phi^{(1)}$ about $\phi^{(1)}$ and $d\psi^{(1)}$ about $\psi^{(1)}$ respectively and is given by

$$F_1^{(1)}\left(v^{(1)}, \phi^{(1)}, \psi^{(1)}\right) = \left\langle \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right) \right\rangle \tag{4}$$

Where, δ is the Dirac delta-function defined as

$$\int \delta(u - v) dv = \begin{cases} 1 & \text{at the point } u=v \\ 0 & \text{elsewhere} \end{cases}$$

Two-point distribution function is given by

$$F_2^{(1,2)} = \left\langle \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right) \delta\left(u^{(2)} - v^{(2)}\right) \delta\left(\theta^{(2)} - \phi^{(2)}\right) \delta\left(c^{(2)} - \psi^{(2)}\right) \right\rangle \tag{5}$$

Three-point distribution function is given by

$$F_3^{(1,2,3)} = \left\langle \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right) \delta\left(u^{(2)} - v^{(2)}\right) \delta\left(\theta^{(2)} - \phi^{(2)}\right) \delta\left(c^{(2)} - \psi^{(2)}\right) \delta\left(u^{(3)} - v^{(3)}\right) \right.$$
$$\left. \delta\left(\theta^{(3)} - \phi^{(3)}\right) \delta\left(c^{(3)} - \psi^{(3)}\right) \right\rangle \tag{6}$$

And four-point distribution function is given by

$$F_4^{(1,2,3,4)} = \left\langle \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right) \delta\left(u^{(2)} - v^{(2)}\right) \delta\left(\theta^{(2)} - \phi^{(2)}\right) \delta\left(c^{(2)} - \psi^{(2)}\right) \right.$$
$$\left. \delta\left(u^{(3)} - v^{(3)}\right) \delta\left(\theta^{(3)} - \phi^{(3)}\right) \delta\left(c^{(3)} - \psi^{(3)}\right) \delta\left(u^{(4)} - v^{(4)}\right) \delta\left(\theta^{(4)} - \phi^{(4)}\right) \delta\left(c^{(4)} - \psi^{(4)}\right) \right\rangle \tag{7}$$

Similarly, we can define an infinite numbers of multi-point distribution functions $F_4^{(1,2,3,4,5)}$, $F_5^{(1,2,3,4,5,6)}$ and so on. The following properties of the constructed distribution functions can be deduced from the above definitions:

3.1. Reduction Properties

Integration with respect to pair of variables at one-point lowers the order of distribution function by one. For example,

$$\int F_1^{(1)} dv^{(1)} d\phi^{(1)} d\psi^{(1)} = 1,$$

$$\int F_2^{(1,2)} dv^{(2)} d\phi^{(2)} d\psi^{(2)} = F_1^{(1)},$$

$$\int F_3^{(1,2,3)} dv^{(3)} d\phi^{(3)} d\psi^{(3)} = F_2^{(1,2)}$$

$$\int F_4^{(1,2,3,4)} dv^{(4)} d\phi^{(4)} d\psi^{(4)} = F_3^{(1,2,3)}$$

and so on. Also the integration with respect to any one of the variables, reduces the number of Delta-functions from the distribution function by one as

$$\int F_1^{(1)} dv^{(1)} = \left\langle \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right) \right\rangle ,$$

$$\int F_1^{(1)} d\theta^{(1)} = \left\langle \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right) \right\rangle ,$$

$$\int F_1^{(1)} d\phi^{(1)} = \left\langle \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right) \right\rangle ,$$

And

$$\int F_2^{(1,2)} dv^{(2)} = \left\langle \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right) \delta\left(\theta^{(2)} - \phi^{(2)}\right) \delta\left(c^{(2)} - \psi^{(2)}\right) \right\rangle$$

3.2. Separation Properties

If two points are far apart from each other in the flow field, the pairs of variables at these points are statistically independent of each other i.e.,

$$\lim_{\left|x^{(2)} - x^{(1)}\right| \to \infty} F_2^{(1,2)} = F_1^{(1)} F_1^{(2)}$$

Similarly,

$$\lim_{\left|x^{(3)} - x^{(2)}\right| \to \infty} F_3^{(1,2,3)} = F_2^{(1,2)} F_1^{(3)}$$

And

$$\lim_{\left|x^{(4)} - x^{(3)}\right| \to \infty} F_4^{(1,2,3,4)} = F_3^{(1,2,3)} F_1^{(4)}$$

3.3. Co-Incidence Property

When two points coincide in the flow field, the components at these points should be obviously the same that is $F_2^{(1,2)}$ must be zero. Thus $v^{(2)} = v^{(1)}$, $\phi^{(2)} = \phi^{(1)}$ and $\psi^{(2)} = \psi^{(1)}$, but $F_2^{(1,2)}$ must also have the property. $\int F_2^{(1,2)} dv^{(2)} d\phi^{(2)} d\psi^{(2)} = F_1^{(1)}$

And hence it follows that

$$\lim_{\left|x^{(2)} - x^{(1)}\right| \to \infty} \int F_2^{(1,2)} = F_1^{(1)} \delta\left(v^{(2)} - v^{(1)}\right) \delta\left(\phi^{(2)} - \phi^{(1)}\right) \delta\left(\psi^{(2)} - \psi^{(1)}\right)$$

Similarly,

$$\lim_{\left|x^{(3)} - x^{(2)}\right| \to \infty} \int F_3^{(1,2,3)} = F_2^{(1,2)} \delta\left(v^{(3)} - v^{(1)}\right) \delta\left(g^{(3)} - g^{(1)}\right) \delta\left(\phi^{(3)} - \phi^{(1)}\right) \delta\left(\psi^{(3)} - \psi^{(1)}\right)$$

And

$$\lim_{\left|x^{(4)} - x^{(3)}\right| \to \infty} \int F_4^{(1,2,3,4)} = F_3^{(1,2,3)} \delta\left(v^{(4)} - v^{(1)}\right) \delta\left(\phi^{(4)} - \phi^{(1)}\right) \delta\left(\psi^{(4)} - \psi^{(1)}\right) \text{ etc.}$$

3.4. Symmetric Conditions

$$F_n^{(1,2,r,------s,-----n)} = F_n^{(1,2,-----s,---r,---n)}$$

3.5. Continuity Equation In Terms Of Distribution Functions

The continuity equations can be easily expressed in terms of distribution functions. An infinite number of continuity equations can be derived for the convective turbulent flow and are obtained directly by
Using div $u = 0$

Taking ensemble average of $\dfrac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}}$ we get

$$0=\left\langle \frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}} \right\rangle = \left\langle \ \frac{\partial}{\partial x_\alpha^{(1)}} u_\alpha^{(1)} \int F_1^{(1)} dv^{(1)} d\phi^{(1)} d\psi^{(1)} \ \right\rangle$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \left\langle \ u_\alpha^{(1)} \int F_1^{(1)} dv^{(1)} d\phi^{(1)} d\psi^{(1)} \ \right\rangle$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \int \left\langle \ u_\alpha^{(1)} \ \right\rangle \left\langle \ F_1^{(1)} \ \right\rangle dv^{(1)} d\phi^{(1)} d\psi^{(1)}$$

$$= \int \ \frac{\partial F_1^{(1)}}{\partial x_\alpha^{(1)}} v_\alpha^{(1)} dv^{(1)} d\phi^{(1)} d\psi^{(1)} \tag{8}$$

And similarly,

$$0 = \int \ \frac{\partial F_1^{(1)}}{\partial x_\alpha^{(1)}} \phi_\alpha^{(1)} dv^{(1)} d\phi^{(1)} d\psi^{(1)} \tag{9}$$

Which are the first order continuity equations in which only one-point distribution function is involved. For second-order continuity equations, if we multiply the continuity equation by

$\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right)$

And if we take the ensemble average, we obtain

$$0 = \left\langle \ \delta\left(u^{(2)} - v^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right) \frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}} \ \right\rangle$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \left\langle \ \delta\left(u^{(2)} - v^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right) u_\alpha^{(1)} \ \right\rangle$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \Big[\ \int \left\langle \ u_\alpha^{(1)} \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right) \times \delta\left(u^{(2)} - v^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right) \right.$$

$$\left. \delta\left(c^{(2)} - \psi^{(2)}\right) \ \right\rangle dv^{(1)} d\phi^{(1)} d\psi^{(1)} \ \Big]$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \int v_\alpha^{(1)} F_2^{(1,2)} dv^{(1)} d\phi^{(1)} d\psi^{(1)} \tag{10}$$

And similarly,

$$0 = \frac{\partial}{\partial x_\alpha^{(1)}} \int \phi_\alpha^{(1)} F_2^{(1,2)} dv^{(1)} d\phi^{(1)} d\psi^{(1)} \tag{11}$$

The Nth-order continuity equations are

$$0 = \frac{\partial}{\partial x_\alpha^{(1)}} \int v_\alpha^{(1)} F_N^{(1,2,---,N)} dv^{(1)} d\phi^{(1)} d\psi^{(1)} \tag{12}$$

And

$$0 = \frac{\partial}{\partial x_\alpha^{(1)}} \int \phi_\alpha^{(1)} F_N^{(1,2,.........,N)} dv^{(1)} d\phi^{(1)} d\psi^{(1)} \tag{13}$$

The continuity equations are symmetric in their arguments i.e.

$$\frac{\partial}{\partial x_\alpha^{(r)}} \left(v_\alpha^{(r)} F_N^{(1,2,.........;r.N)} dv^{(r)} d\phi^{(r)} d\psi^{(r)} \right) = \frac{\partial}{\partial x_\alpha^{(s)}} \int v_\alpha^{(s)} F_N^{(1,2,.....r,s,.....N)} dv^{(s)} d\phi^{(s)} d\psi^{(s)} \tag{14}$$

Since the divergence property is an important property and it is easily verified by the use of the property of distribution function as

$$\frac{\partial}{\partial x_\alpha^{(1)}} \int v_\alpha^{(1)} F_1^{(1)} dv^{(1)} d\phi^{(1)} d\psi^{(1)} = \frac{\partial}{\partial x_\alpha^{(1)}} \left\langle \ u_\alpha^{(1)} \ \right\rangle = \left\langle \ \frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}} \ \right\rangle = 0 \tag{15}$$

And all the properties of the distribution function obtained in the above section can also be verified.

IV. Equations For The Evolution Of Three –Point Joint Distribution Function ($F_3^{(1,2.3)}$)

The transport equation for three-point distribution function is obtained from the definition of $F_3^{(1,2.3)}$ and from the transport equations (1), (2) and (3).

Differentiating equation (6) we get

$$
\begin{aligned}
\frac{\partial F_3^{(1,2,3)}}{\partial t} = \frac{\partial}{\partial t}\Big\langle\ & \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right) \\
& \delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\Big\rangle \\
= \Big\langle\ & \delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right) \\
& \delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial}{\partial t}\delta\left(u^{(1)}-v^{(1)}\right)\Big\rangle + \Big\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta(c^{(1)}-\psi^{(1)})\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right) \\
& \delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial}{\partial t}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\Big\rangle + \Big\langle\ \delta\left(u^{(1)}-v^{(1)}\right) \\
& \delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right) \\
& \frac{\partial}{\partial t}\delta\left(c^{(1)}-\psi^{(1)}\right)\Big\rangle + \Big\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right) \\
& \delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial}{\partial t}\delta\left(u^{(2)}-v^{(2)}\right)\Big\rangle + \Big\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right) \\
& \delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial}{\partial t}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\Big\rangle \\
& +\Big\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right) \\
& \delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial}{\partial t}\delta\left(c^{(2)}-\psi^{(2)}\right)\Big\rangle + \Big\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right) \\
& \delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial}{\partial t}\delta\left(u^{(3)}-v^{(3)}\right)\Big\rangle + \Big\langle\ \delta\left(u^{(1)}-v^{(1)}\right) \\
& \delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right) \\
& \frac{\partial}{\partial t}\delta\left(\theta^{(3)}-\phi^{(3)}\right)\Big\rangle + \Big\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right) \\
& \delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\frac{\partial}{\partial t}\delta\left(c^{(3)}-\psi^{(3)}\right)\Big\rangle \\
= \Big\langle\ & \delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right) \\
& \delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial u^{(1)}}{\partial t}\frac{\partial}{\partial v^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\Big\rangle + \Big\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta(c^{(1)}-\psi^{(1)})\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right) \\
& \delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial \theta^{(1)}}{\partial t}\frac{\partial}{\partial \phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\Big\rangle + \Big\langle\ \delta\left(u^{(1)}-v^{(1)}\right) \\
& \delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right) \\
& \frac{\partial c^{(1)}}{\partial t}\frac{\partial}{\partial \psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\Big\rangle + \Big\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right) \\
& \delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial u^{(2)}}{\partial t}\frac{\partial}{\partial v^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\Big\rangle + \Big\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right) \\
& \delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial \theta^{(2)}}{\partial t}\frac{\partial}{\partial \phi^{(2)}} \\
& \delta\left(\theta^{(2)}-\phi^{(2)}\right)\Big\rangle + \Big\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)
\end{aligned}
$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial c^{(2)}}{\partial t}\frac{\partial}{\partial \psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\Big\rangle+\Big\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)$$

$$\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial u^{(3)}}{\partial t}\frac{\partial}{\partial v^{(3)}}$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\ \Big\rangle+\Big\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial \theta^{(3)}}{\partial t}\frac{\partial}{\partial \phi^{(3)}}\delta\left(\theta^{(3)}-\phi^{(3)}\right)\ \Big\rangle+\Big\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)$$

$$\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\frac{\partial c^{(3)}}{\partial t}\frac{\partial}{\partial \psi^{(3)}}\delta\left(c^{(3)}-\psi^{(3)}\right)\Big\rangle \qquad (16)$$

Using equation (1), (2) and (3) in equation (16) we have,

$$\therefore\quad \frac{\partial F_3^{(1,2,3)}}{\partial t}=\Big\langle-\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-\upsilon^{(3)}\right)$$

$$\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)[-u_\alpha^{(1)}\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}-\frac{\partial}{\partial x_\beta^{(1)}}\int\frac{1}{4\pi}\frac{\partial}{\partial x_\beta^{(2)}}\{u_\alpha^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}u_\alpha^{(2)}\}\frac{dx_\beta^{(2)}}{\left|x_\beta^{(1)}-x_\beta^{(2)}\right|}$$

$$+v\frac{\partial}{\partial x_\beta^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}}u_\alpha^{(1)}+f(u_\alpha^{(1)}-\upsilon_\alpha^{(1)})]\frac{\partial}{\partial v^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \Big\rangle+\Big\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\quad[-u_\alpha^{(1)}\frac{\partial \theta_\alpha^{(1)}}{\partial x_\beta^{(1)}}+\gamma\frac{\partial}{\partial x_\beta^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}}\theta_\alpha^{(1)}]$$

$$\frac{\partial}{\partial \phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \Big\rangle+\Big\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)[-u_\alpha^{(1)}\frac{\partial c_\alpha^{(1)}}{\partial x_\beta^{(1)}}+D\frac{\partial}{\partial x_\beta^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}}c^{(1)}-Rc^{(1)}]\frac{\partial}{\partial \psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\ \Big\rangle$$

$$+\Big\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)$$

$$\delta\left(c^{(3)}-\psi^{(3)}\right)[-u_\alpha^{(2)}\frac{\partial u_\alpha^{(2)}}{\partial x_\beta^{(2)}}-\frac{\partial}{\partial x_\beta^{(2)}}\int\delta\left(c^{(3)}-\psi^{(3)}\right)\quad[-u_\alpha^{(1)}\frac{\partial \theta_\alpha^{(1)}}{\partial x_\beta^{(1)}}+\gamma\frac{\partial}{\partial x_\beta^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}}\theta_\alpha^{(1)}]$$

$$\int\frac{1}{4\pi}\frac{\partial}{\partial x_\beta^{(3)}}\{u_\alpha^{(3)}\frac{\partial}{\partial x_\beta^{(3)}}u_\alpha^{(3)}\}\frac{dx_\beta^{(3)}}{\left|x_\beta^{(2)}-x_\beta^{(3)}\right|}+\upsilon\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}u_\alpha^{(2)}+f(u_\alpha^{(2)}-\upsilon_\alpha^{(2)})]\frac{\partial}{\partial v^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\ \Big\rangle$$

$$+\Big\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)$$

$$\delta\left(c^{(3)}-\psi^{(3)}\right)[-u_\alpha^{(2)}\frac{\partial \theta_\alpha^{(2)}}{\partial x_\beta^{(2)}}+\gamma\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}\theta_\alpha^{(2)}]\frac{\partial}{\partial \phi^{(2)}}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\ \Big\rangle+\Big\langle-\delta\left(u^{(1)}-v^{(1)}\right)$$

$$\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$[-u_\alpha^{(2)}\frac{\partial c_\alpha^{(2)}}{\partial x_\beta^{(2)}}+D\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}c^{(2)}-Rc^{(2)}]\frac{\partial}{\partial \psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\ \Big\rangle+\Big\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)$$

$$\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)[-u_\alpha^{(3)}\frac{\partial u_\alpha^{(3)}}{\partial x_\beta^{(3)}}-\frac{\partial}{\partial x_\beta^{(3)}}$$

$$\int\frac{1}{4\pi}\frac{\partial}{\partial x_\beta^{(4)}}\{u_\alpha^{(4)}\frac{\partial}{\partial x_\beta^{(4)}}u_\alpha^{(4)}\}\frac{dx_\beta^{(4)}}{\left|x_\beta^{(3)}-x_\beta^{(4)}\right|}+\upsilon\frac{\partial}{\partial x_\beta^{(3)}}\frac{\partial}{\partial x_\beta^{(3)}}u_\alpha^{(3)}+f(u_\alpha^{(3)}-\upsilon_\alpha^{(3)})]\frac{\partial}{\partial v^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\ \Big\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$[-u_\alpha^{(3)}\frac{\partial\theta_\alpha^{(3)}}{\partial x_\beta^{(3)}}+\gamma\frac{\partial}{\partial x_\beta^{(3)}}\frac{\partial}{\partial x_\beta^{(3)}}\theta_\alpha^{(3)}]\frac{\partial}{\partial\phi^{(3)}}\delta\left(\theta^{(3)}-\phi^{(3)}\right)\rangle+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)$$

$$\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)[-u_\alpha^{(3)}\frac{\partial c_\alpha^{(3)}}{\partial x_\beta^{(3)}}+D\frac{\partial}{\partial x_\beta^{(3)}}\frac{\partial}{\partial x_\beta^{(3)}}c^{(3)}-Rc^{(3)}]$$

$$\frac{\partial}{\partial\psi^{(3)}}\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle$$

$$=\langle\ \delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)$$

$$\delta\left(c^{(3)}-\psi^{(3)}\right)u_\alpha^{(1)}\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)u_\alpha^{(1)}\frac{\partial\theta_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)$$

$$\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$-u_\alpha^{(1)}\frac{\partial c_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)u_\alpha^{(2)}\frac{\partial u_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial v^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)$$

$$\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)u_\alpha^{(2)}\frac{\partial\theta_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial\phi^{(2)}}$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)$$

$$\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)-u_\alpha^{(2)}\frac{\partial c_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial\psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)$$

$$\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)u_\alpha^{(3)}\frac{\partial u_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial}{\partial v^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)u_\alpha^{(3)}\frac{\partial\theta_\alpha^{(3)}}{\partial x_\beta^{(3)}}$$

$$\frac{\partial}{\partial\phi^{(3)}}\delta\left(\theta^{(3)}-\phi^{(3)}\right)\rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)$$

$$\delta\left(\theta^{(3)}-\phi^{(3)}\right)u_\alpha^{(3)}\frac{\partial c_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial}{\partial\psi^{(3)}}\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle+\langle -\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)[-\frac{\partial}{\partial x_\beta^{(1)}}\int\frac{1}{4\pi}\frac{\partial}{\partial x_\beta^{(2)}}\{u_\alpha^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}u_\alpha^{(2)}\frac{dx_\beta^{(2)}}{\left|x_\beta^{(1)}-x_\beta^{(2)}\right|}\}$$

$$\frac{\partial}{\partial v^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)]\rangle+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)$$

$$\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)[-\frac{\partial}{\partial x_\beta^{(2)}}\int\frac{1}{4\pi}\frac{\partial}{\partial x_\beta^{(3)}}\{u_\alpha^{(3)}\frac{\partial}{\partial x_\beta^{(3)}}u_\alpha^{(3)}\frac{dx_\beta^{(3)}}{\left|x_\beta^{(2)}-x_\beta^{(3)}\right|}\}\frac{\partial}{\partial v^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)]\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)$$

$$\delta\left(c^{(3)}-\psi^{(3)}\right)[-\frac{\partial}{\partial x_\beta^{(3)}}\int\frac{1}{4\pi}\frac{\partial}{\partial x_\beta^{(4)}}\{u_\alpha^{(4)}\frac{\partial}{\partial x_\beta^{(4)}}u_\alpha^{(4)}\frac{dx_\beta^{(4)}}{\left|x_\beta^{(3)}-x_\beta^{(4)}\right|}\}\frac{\partial}{\partial v^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)]\rangle+\langle -\delta\left(\theta^{(1)}-\phi^{(1)}\right)$$

$$\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$(\upsilon \frac{\partial}{\partial x_\beta^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}} u_\alpha^{(1)})\frac{\partial}{\partial v^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle + \langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)(v\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}} u_\alpha^{(2)})\frac{\partial}{\partial v^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)$$

$$\delta\left(c^{(3)}-\psi^{(3)}\right)(v\frac{\partial}{\partial x_\beta^{(3)}}\frac{\partial}{\partial x_\beta^{(3)}} u_\alpha^{(3)})\frac{\partial}{\partial v^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\rangle + \langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\gamma\frac{\partial}{\partial x_\beta^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}}\theta_\alpha^{(1)}\frac{\partial}{\partial \phi^{(1)}}$$

$$\delta\left(\theta^{(1)}-\phi^{(1)}\right)\rangle + \langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\gamma\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}\theta_\alpha^{(2)}\frac{\partial}{\partial \phi^{(2)}}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\rangle + \langle -\delta\left(u^{(1)}-v^{(1)}\right)$$

$$\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$\gamma\frac{\partial}{\partial x_\beta^{(3)}}\frac{\partial}{\partial x_\beta^{(3)}}\theta_\alpha^{(3)}\frac{\partial}{\partial \phi^{(3)}}\delta\left(\theta^{(3)}-\phi^{(3)}\right)\rangle + \langle -\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)f(u_\alpha^{(1)}-v_\alpha^{(1)})\frac{\partial}{\partial v^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)$$

$$\delta\left(c^{(3)}-\psi^{(3)}\right)f(u_\alpha^{(2)}-v_\alpha^{(2)})\frac{\partial}{\partial v^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle + \langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)$$

$$\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)f(u_\alpha^{(3)}-v_\alpha^{(3)})\frac{\partial}{\partial v^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)$$

$$\delta\left(c^{(3)}-\psi^{(3)}\right)D\frac{\partial}{\partial x_\beta^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}}c^{(1)}\frac{\partial}{\partial \psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle + \langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)$$

$$\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)D\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}c^{(2)}\frac{\partial}{\partial \psi^{(2)}}$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle + \langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)D\frac{\partial}{\partial x_\beta^{(3)}}\frac{\partial}{\partial x_\beta^{(3)}}c^{(3)}\frac{\partial}{\partial \psi^{(3)}}\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle + \langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)$$

$$\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)Rc^{(1)}\frac{\partial}{\partial \psi^{(1)}}$$

$$\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle + \langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)$$

$$\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)Rc^{(2)}\frac{\partial}{\partial \psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle + \langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)$$

$$\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)Rc^{(3)}\frac{\partial}{\partial \psi^{(3)}}\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle$$

$$\Rightarrow \frac{\partial F_3^{(1,2,3)}}{\partial t} + \langle \ \delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(\theta^{(3)}-\phi^{(3)})$$

$$\delta(c^{(3)}-\psi^{(3)})u_\alpha^{(1)} \frac{\partial}{\partial x_\beta^{(1)}} \delta(u^{(1)}-v^{(1)}) \ \rangle + \langle \ \delta(u^{(1)}-v^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})$$

$$\delta(u^{(3)}-v^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})u_\alpha^{(1)} \frac{\partial}{\partial x_\beta^{(1)}} \delta(\theta^{(1)}-\phi^{(1)}) \ \rangle + \langle \ \delta(u^{(1)}-v^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(u^{(2)}-v^{(2)})$$

$$\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)}) - u_\alpha^{(1)} \frac{\partial}{\partial x_\beta^{(1)}} \delta(c^{(1)}-\psi^{(1)}) \ \rangle$$

$$+ \langle \ \delta(u^{(1)}-v^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(\theta^{(3)}-\phi^{(3)})$$

$$\delta(c^{(3)}-\psi^{(3)})u_\alpha^{(2)} \frac{\partial}{\partial x_\beta^{(2)}} \delta(u^{(2)}-v^{(2)}) \ \rangle + \langle \ \delta(u^{(1)}-v^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})$$

$$\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})u_\alpha^{(2)} \frac{\partial}{\partial x_\beta^{(2)}} \delta(\theta^{(2)}-\phi^{(2)}) \ \rangle + \langle \ \delta(u^{(1)}-v^{(1)})$$

$$\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)}) - u_\alpha^{(2)} \frac{\partial}{\partial x_\beta^{(2)}}$$

$$\delta(c^{(2)}-\psi^{(2)}) \ \rangle + \langle \ \delta(u^{(1)}-v^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})$$

$$\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})u_\alpha^{(3)} \frac{\partial}{\partial x_\beta^{(3)}} \delta(u^{(3)}-v^{(3)}) \ \rangle + \langle \ \delta(u^{(1)}-v^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})$$

$$\delta(u^{(2)}-v^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(c^{(3)}-\psi^{(3)})u_\alpha^{(3)} \frac{\partial}{\partial x_\beta^{(3)}} \delta(\theta^{(3)}-\phi^{(3)}) \ \rangle$$

$$+ \langle \ \delta(u^{(1)}-v^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})$$

$$\delta(\theta^{(3)}-\phi^{(3)})u_\alpha^{(3)} \frac{\partial}{\partial x_\beta^{(3)}} \delta(c^{(3)}-\psi^{(3)}) \ \rangle + \langle \ \delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$

$$\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)}) \ [-\frac{\partial}{\partial x_\beta^{(1)}} \int \frac{1}{4\pi} \frac{\partial}{\partial x_\beta^{(2)}} \{u_\alpha^{(2)} \frac{\partial}{\partial x_\beta^{(2)}} u_\alpha^{(2)} \frac{dx_\beta^{(2)}}{\left|x_\beta^{(1)}-x_\beta^{(2)}\right|}\}$$

$$\frac{\partial}{\partial v^{(1)}} \delta(u^{(1)}-v^{(1)})]\rangle + \langle \ \delta(u^{(1)}-v^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})$$

$$\delta(u^{(3)}-v^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})[-\frac{\partial}{\partial x_\beta^{(2)}} \int \frac{1}{4\pi} \frac{\partial}{\partial x_\beta^{(3)}} \{u_\alpha^{(3)} \frac{\partial}{\partial x_\beta^{(3)}} u_\alpha^{(3)} \frac{dx_\beta^{(3)}}{\left|x_\beta^{(2)}-x_\beta^{(3)}\right|}\} \frac{\partial}{\partial v^{(2)}}$$

$$\delta(u^{(2)}-v^{(2)})]\rangle + \langle \ \delta(u^{(1)}-v^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})$$

$$\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})[-\frac{\partial}{\partial x_\beta^{(3)}} \int \frac{1}{4\pi} \frac{\partial}{\partial x_\beta^{(4)}} \{u_\alpha^{(4)} \frac{\partial}{\partial x_\beta^{(4)}} u_\alpha^{(4)} \frac{dx_\beta^{(4)}}{\left|x_\beta^{(3)}-x_\beta^{(4)}\right|}\} \frac{\partial}{\partial v^{(3)}} \delta(u^{(3)}-v^{(3)})]\rangle$$

$$+ \langle \ \delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(\theta^{(3)}-\phi^{(3)})$$

$$\delta\left(c^{(3)}-\psi^{(3)}\right)\left(\nu\frac{\partial}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial x_{\beta}^{(1)}}u_{\alpha}^{(1)}\right)\frac{\partial}{\partial v^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\left(\nu\frac{\partial}{\partial x_{\beta}^{(2)}}\frac{\partial}{\partial x_{\beta}^{(2)}}u_{\alpha}^{(2)}\right)\frac{\partial}{\partial v^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)$$
$$\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\left(\nu\frac{\partial}{\partial x_{\beta}^{(3)}}\frac{\partial}{\partial x_{\beta}^{(3)}}u_{\alpha}^{(3)}\right)$$
$$\frac{\partial}{\partial v^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)$$
$$\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\gamma\frac{\partial}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial x_{\beta}^{(1)}}\theta_{\alpha}^{(1)}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)$$
$$\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\gamma\frac{\partial}{\partial x_{\beta}^{(2)}}\frac{\partial}{\partial x_{\beta}^{(2)}}\theta_{\alpha}^{(2)}\frac{\partial}{\partial\phi^{(2)}}$$
$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$
$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\gamma\frac{\partial}{\partial x_{\beta}^{(3)}}\frac{\partial}{\partial x_{\beta}^{(3)}}\theta_{\alpha}^{(3)}\frac{\partial}{\partial\phi^{(3)}}\delta\left(\theta^{(3)}-\phi^{(3)}\right)\rangle+\langle\ \delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)$$
$$\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)f\left(u_{\alpha}^{(1)}-v_{\alpha}^{(1)}\right)$$
$$\frac{\partial}{\partial v^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)$$
$$\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)f\left(u_{\alpha}^{(2)}-v_{\alpha}^{(2)}\right)\frac{\partial}{\partial v^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)$$
$$\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)f\left(u_{\alpha}^{(3)}-v_{\alpha}^{(3)}\right)$$
$$\frac{\partial}{\partial v^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$
$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)D\frac{\partial}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial x_{\beta}^{(1)}}c^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle$$
$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)$$
$$\delta\left(c^{(3)}-\psi^{(3)}\right)D\frac{\partial}{\partial x_{\beta}^{(2)}}\frac{\partial}{\partial x_{\beta}^{(2)}}c^{(2)}\frac{\partial}{\partial\psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)$$
$$\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)D\frac{\partial}{\partial x_{\beta}^{(3)}}\frac{\partial}{\partial x_{\beta}^{(3)}}c^{(3)}\frac{\partial}{\partial\psi^{(3)}}$$
$$\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle+\langle\ -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)$$
$$\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)Rc^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)$$
$$\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)Rc^{(2)}\frac{\partial}{\partial\psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle$$
$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)$$
$$\delta\left(\theta^{(3)}-\phi^{(3)}\right)Rc^{(3)}\frac{\partial}{\partial\psi^{(3)}}\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle=0 \qquad (17)$$

Various terms in the above equation can be simplified using the properties of joint distribution function: We reduce 2nd, 3rd and 4th terms in equation (17) we get

$$\langle\ \delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)$$

$$\delta\left(c^{(3)}-\psi^{(3)}\right)u_\alpha^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)u_\alpha^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)$$

$$\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$u_\alpha^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\ \rangle$$

$$=\frac{\partial}{\partial x_\beta^{(1)}}\left\langle u_\alpha^{(1)}\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\right.$$

$$\left.\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\ \right\rangle$$

$$=\frac{\partial}{\partial x_\beta^{(1)}}v_\alpha^{(1)}F_3^{(1,2,3)}$$

$$=v_\alpha^{(1)}\frac{\partial F_3^{(1,2,3)}}{\partial x_\beta^{(1)}} \tag{18}$$

5th, 6th and 7th terms of equation (17) can be reduces as

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)u_\alpha^{(2)}$$

$$\frac{\partial}{\partial x_\beta^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\ \rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)$$

$$\delta\left(c^{(3)}-\psi^{(3)}\right)u_\alpha^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\ \rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)u_\alpha^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\ \rangle$$

$$=\frac{\partial}{\partial x_\beta^{(2)}}\langle\ u_\alpha^{(2)}\delta(u^{(1)}-v^{(1)})\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)$$

$$\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\ \rangle$$

$$=\frac{\partial}{\partial x_\beta^{(2)}}v_\alpha^{(2)}F_3^{(1,2,3)}$$

$$=v_\alpha^{(2)}\frac{\partial F_3^{(1,2,3)}}{\partial x_\beta^{(2)}} \tag{19}$$

8th, 9th and 10th terms of equation (17) can be reduces as

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$
$$u_\alpha^{(3)}\frac{\partial}{\partial x_\beta^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\ \rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$
$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)u_\alpha^{(3)}\frac{\partial}{\partial x_\beta^{(3)}}\delta\left(\theta^{(3)}-\phi^{(3)}\right)\ \rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)$$
$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)u_\alpha^{(3)}\frac{\partial}{\partial x_\beta^{(3)}}\delta\left(c^{(3)}-\psi^{(3)}\right)\ \rangle$$
$$=\frac{\partial}{\partial x_\beta^{(3)}}\langle\ u_\alpha^{(3)}\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$
$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\ \rangle$$
$$=\frac{\partial}{\partial x_\beta^{(3)}}v_\alpha^{(3)}F_3^{(1,2,3)}$$
$$=v_\alpha^{(3)}\frac{\partial F_3^{(1,2,3)}}{\partial x_\beta^{(3)}} \tag{20}$$

11th term of equation (17) can be reduces as

$$\langle\ \delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)$$
$$\delta\left(c^{(3)}-\psi^{(3)}\right)[-\frac{\partial}{\partial x_\beta^{(1)}}\int\frac{1}{4\pi}\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}u_\alpha^{(2)}u_\alpha^{(2)}\frac{dx_\beta^{(2)}}{\left|x_\beta^{(1)}-x_\beta^{(2)}\right|}\frac{\partial}{\partial\upsilon_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)]]\rangle$$
$$=\frac{\partial}{\partial v_\alpha^{(1)}}[-\frac{1}{4\pi}\int(\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\left|x_\beta^{(1)}-x_\beta^{(2)}\right|})(v_\alpha^{(2)}\frac{\partial}{\partial x_\beta^{(2)}})^2F_4^{(1,2,3,4)}dx^4dv^4d\phi^4d\psi^4] \tag{21}$$

12th term of equation (17) can be reduces as

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)$$
$$\delta\left(c^{(3)}-\psi^{(3)}\right)[-\frac{\partial}{\partial x_\beta^{(2)}}\int\frac{1}{4\pi}\frac{\partial}{\partial x_\beta^{(3)}}\{u_\alpha^{(3)}\frac{\partial}{\partial x_\beta^{(3)}}u_\alpha^{(3)}\frac{dx_\beta^{(3)}}{\left|x_\beta^{(2)}-x_\beta^{(3)}\right|}\}\frac{\partial}{\partial v^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)]]\rangle$$
$$=\frac{\partial}{\partial v_\alpha^{(2)}}[-\frac{1}{4\pi}\int(\frac{\partial}{\partial x_\beta^{(3)}}\frac{\partial}{\left|x_\beta^{(2)}-x_\beta^{(3)}\right|})(v_\alpha^{(3)}\frac{\partial}{\partial x_\beta^{(3)}})^2F_4^{(1,2,3,4)}dx^4dv^4d\phi^4d\psi^4] \tag{22}$$

13th term of equation (17) can be reduces as

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)$$
$$\delta\left(c^{(3)}-\psi^{(3)}\right)[-\frac{\partial}{\partial x_\beta^{(3)}}\int\frac{1}{4\pi}\frac{\partial}{\partial x_\beta^{(4)}}\{u_\alpha^{(4)}\frac{\partial}{\partial x_\beta^{(4)}}u_\alpha^{(4)}\frac{dx_\beta^{(4)}}{\left|x_\beta^{(3)}-x_\beta^{(4)}\right|}\}\frac{\partial}{\partial v^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)]]\rangle$$
$$=\frac{\partial}{\partial v_\alpha^{(3)}}[-\frac{1}{4\pi}\int(\frac{\partial}{\partial x_\beta^{(4)}}\frac{\partial}{\left|x_\beta^{(3)}-x_\beta^{(4)}\right|})(v_\alpha^{(4)}\frac{\partial}{\partial x_\beta^{(4)}})^2F_4^{(1,2,3,4)}dx^4dv^4d\phi^4d\psi^4] \tag{23}$$

14th term of equation (17) can be reduces as

$$\langle\ \delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)$$

$$\delta\left(c^{(3)}-\psi^{(3)}\right)\left(v\frac{\partial}{\partial x_\beta^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}}u_\alpha^{(1)}\right)\frac{\partial}{\partial v^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$=\langle v\frac{\partial}{\partial v^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}}u_\alpha^{(1)}\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle$$

$$=v\frac{\partial}{\partial v^{(1)}}\langle\frac{\partial}{\partial x_\beta^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}}u_\alpha^{(1)}[\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)]\rangle$$

$$=v\frac{\partial}{\partial v^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}}\langle u_\alpha^{(1)}\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle$$

$$=v\frac{\partial}{\partial v^{(1)}}\lim_{x^{(4)}\to x^{(1)}}\frac{\partial}{\partial x_\beta^{(4)}}\frac{\partial}{\partial x_\beta^{(4)}}\langle u_\alpha^{(4)}\{\delta\left(u^{(4)}-v^{(4)}\right)\delta\left(\theta^{(4)}-\phi^{(4)}\right)\delta\left(c^{(4)}-\psi^{(4)}\right)\delta\left(u^{(2)}-v^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\}\times\{\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)$$

$$\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\}$$

$$dv^4 d\phi^4 d\psi^4\rangle$$

$$=v\frac{\partial}{\partial v_\alpha^{(1)}}\lim_{x^{(4)}\to x^{(1)}}\frac{\partial}{\partial x_\beta^{(4)}}\frac{\partial}{\partial x_\beta^{(4)}}\int \upsilon_\alpha^4 F_4^{(1,2,3,4)}dv^4 d\phi^4 d\psi^4 \tag{24}$$

15th term of equation(17) can be reducesas

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)$$

$$\delta\left(c^{(3)}-\psi^{(3)}\right)\left(v\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}u_\alpha^{(2)}\right)\frac{\partial}{\partial v^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\ \rangle$$

$$=\langle v\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial v^{(2)}}u_\alpha^{(2)}\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle$$

$$=v\frac{\partial}{\partial v_\alpha^{(2)}}\langle\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}u_\alpha^{(2)}\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle$$

$$=v\frac{\partial}{\partial v_\alpha^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}\langle u_\alpha^{(2)}\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle$$

$$=v\frac{\partial}{\partial v_\alpha^{(2)}}\lim_{x^{(4)}\to x^{(2)}}\frac{\partial}{\partial x_\beta^{(4)}}\frac{\partial}{\partial x_\beta^{(4)}}\langle\{\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(4)}-v^{(4)}\right)\delta\left(\theta^{(4)}-\phi^{(4)}\right)$$

$$\delta\left(c^{(4)}-\psi^{(4)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\}\times\{\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)$$

$$\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\}\rangle$$

$$=v\frac{\partial}{\partial v_\alpha^{(2)}}\lim_{x^{(4)}\to x^{(2)}}\frac{\partial}{\partial x_\beta^{(4)}}\frac{\partial}{\partial x_\beta^{(4)}}\int v_\alpha^{(4)}F_4^{(1,2,3,4)}dv^4 d\phi^4 d\psi^4 \tag{25}$$

Similarly 16th term of equation (17) can be reduces as

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)$$

$$\delta\left(c^{(3)}-\psi^{(3)}\right)\left(v\frac{\partial}{\partial x_\beta^{(3)}}\frac{\partial}{\partial x_\beta^{(3)}}u_\alpha^{(3)}\right)\frac{\partial}{\partial v^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\ \rangle$$

$$=v\frac{\partial}{\partial \upsilon_\alpha^{(3)}}\lim_{x^{(4)}\to x^{(3)}}\frac{\partial}{\partial x_\beta^{(4)}}\frac{\partial}{\partial x_\beta^{(4)}}\int v_\alpha^{(4)}F_4^{(1,2,3,4)}dv^4d\phi^4d\psi^4 \tag{26}$$

17th term of equation (17) written as

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)$$

$$\delta\left(c^{(3)}-\psi^{(3)}\right)\gamma\frac{\partial}{\partial x_\beta^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}}\theta_\alpha^{(1)}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \rangle$$

$$=\frac{\partial}{\partial\phi^{(1)}}\lim_{x^{(4)}\to x^{(1)}}\gamma\frac{\partial}{\partial x_\beta^{(4)}}\frac{\partial}{\partial x_\beta^{(4)}}\int\phi^{(4)}F_4^{(1,2,3,4)}dv^4d\phi^4d\psi^4 \tag{27}$$

18th term of equation (17) written as

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)$$

$$\delta\left(c^{(3)}-\psi^{(3)}\right)\gamma\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}\theta_\alpha^{(2)}\frac{\partial}{\partial\phi^{(2)}}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\ \rangle$$

$$=\frac{\partial}{\partial\phi^{(2)}}\lim_{x^{(4)}\to x^{(2)}}\gamma\frac{\partial}{\partial x_\beta^{(4)}}\frac{\partial}{\partial x_\beta^{(4)}}\int\phi^{(4)}F_4^{(1,2,3,4)}dv^4d\phi^4d\psi^4 \tag{28}$$

19th term of equation (17) written as

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)$$

$$\delta\left(c^{(3)}-\psi^{(3)}\right)\gamma\frac{\partial}{\partial x_\beta^{(3)}}\frac{\partial}{\partial x_\beta^{(3)}}\theta_\alpha^{(3)}\frac{\partial}{\partial\phi^{(3)}}\delta\left(\theta^{(3)}-\phi^{(3)}\right)\ \rangle$$

$$=\frac{\partial}{\partial\phi^{(3)}}\lim_{x^{(4)}\to x^{(3)}}\gamma\frac{\partial}{\partial x_\beta^{(4)}}\frac{\partial}{\partial x_\beta^{(4)}}\int\phi^{(4)}F_4^{(1,2,3,4)}d\upsilon^4d\phi^4d\psi^4 \tag{29}$$

20th term of equation (17) written as

$$\langle\ \delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)$$

$$\delta\left(c^{(3)}-\psi^{(3)}\right)f(u_\alpha^{(1)}-v_\alpha^{(1)})\frac{\partial}{\partial v^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$=\langle\ f(u_\alpha^{(1)}-v_\alpha^{(1)})\frac{\partial}{\partial v^{(1)}}[\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)]\rangle$$

$$=f(u_\alpha^{(1)}-v_\alpha^{(1)})\frac{\partial}{\partial v^{(1)}}F_3^{(1,2,3)} \tag{30}$$

21th term of equation (17) written as

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)$$

$$\delta\left(c^{(3)}-\psi^{(3)}\right)f(u_\alpha^{(2)}-v_\alpha^{(2)})\frac{\partial}{\partial v^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\ \rangle$$

$$=\langle f(u_\alpha^{(2)}-v_\alpha^{(2)})\frac{\partial}{\partial v^{(2)}}[\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)]\rangle$$

$$=f(u_\alpha^{(2)}-v_\alpha^{(2)})\frac{\partial}{\partial v^{(2)}}F_3^{(1,2,3)} \tag{31}$$

22th term of equation (17) written as

$$\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right) f(u_\alpha^{(3)}-v_\alpha^{(3)})\frac{\partial}{\partial v^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\right\rangle$$

$$=\left\langle f(u_\alpha^{(3)}-v_\alpha^{(3)})\frac{\partial}{\partial v^{(3)}}[\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)]\right\rangle$$

$$=f(u_\alpha^{(3)}-v_\alpha^{(3)})\frac{\partial}{\partial v^{(3)}}F_3^{(1,2,3)} \tag{32}$$

23th term of equation (17) written as

$$\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)D\frac{\partial}{\partial x_\beta^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}}c^{(1)}\frac{\partial}{\partial \psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle$$

$$=\frac{\partial}{\partial \psi^{(1)}}\lim_{x^{(4)}\to x^{(1)}}D\frac{\partial}{\partial x_\beta^{(4)}}\frac{\partial}{\partial x_\beta^{(4)}}\int \psi^{(4)}F_4^{(1,2,3,4)}dv^4 d\phi^4 d\psi^4 \tag{33}$$

24th term of equation (17) written as

$$\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)D\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}c^{(2)}\frac{\partial}{\partial \psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\right\rangle$$

$$=\frac{\partial}{\partial \psi^{(2)}}\lim_{x^{(4)}\to x^{(2)}}D\frac{\partial}{\partial x_\beta^{(4)}}\frac{\partial}{\partial x_\beta^{(4)}}\int \psi^{(4)}F_4^{(1,2,3,4)}dv^4 d\phi^4 d\psi^4 \tag{34}$$

25th term of equation (17) written as

$$\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)D\frac{\partial}{\partial x_\beta^{(3)}}\frac{\partial}{\partial x_\beta^{(3)}}c^{(3)}\frac{\partial}{\partial \psi^{(3)}}\delta\left(c^{(3)}-\psi^{(3)}\right)\right\rangle$$

$$=\frac{\partial}{\partial \psi^{(3)}}\lim_{x^{(4)}\to x^{(3)}}D\frac{\partial}{\partial x_\beta^{(4)}}\frac{\partial}{\partial x_\beta^{(4)}}\int \psi^{(4)}F_4^{(1,2,3,4)}dv^4 d\phi^4 d\psi^4 \tag{35}$$

26th term of equation (17) written as

$$\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)Rc^{(1)}\frac{\partial}{\partial \psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle$$

$$=-R\psi^{(1)}\frac{\partial}{\partial \psi^{(1)}}F_3^{(1,2,3)} \tag{36}$$

27th term of equation (17) written as

$$\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)Rc^{(2)}\frac{\partial}{\partial \psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\right\rangle$$

$$=-R\psi^{(2)}\frac{\partial}{\partial \psi^{(2)}}F_3^{(1,2,3)} \tag{37}$$

28th term of equation (17) written as

$$\langle -\delta(u^{(1)}-v^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})$$
$$\delta(\theta^{(3)}-\phi^{(3)})Rc^{(3)}\frac{\partial}{\partial\psi^{(3)}}\delta(c^{(3)}-\psi^{(3)})\rangle$$
$$=-R\psi^{(3)}\frac{\partial}{\partial\psi^{(3)}}F_3^{(1,2,3)} \tag{38}$$

Using equation (18)-(38) in equation (17) we get the required equation

$$\frac{\partial F_3^{(1,2,3)}}{\partial t}+(v_\alpha^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}+v_\alpha^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}+v_\alpha^{(3)}\frac{\partial}{\partial x_\beta^{(3)}})F_3^{(1,2,3)}+\frac{\partial}{\partial v_\alpha^{(1)}}[-\frac{1}{4\pi}\int(\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\left|x_\beta^{(1)}-x_\beta^{(2)}\right|})(v_\alpha^{(2)}\frac{\partial}{\partial x_\beta^{(2)}})^2$$
$$F_4^{(1,2,3,4)}dx^4dv^4d\phi^4d\psi^4]+\frac{\partial}{\partial v_\alpha^{(2)}}[-\frac{1}{4\pi}\int(\frac{\partial}{\partial x_\beta^{(3)}}\frac{\partial}{\left|x_\beta^{(2)}-x_\beta^{(3)}\right|})(v_\alpha^{(3)}\frac{\partial}{\partial x_\beta^{(3)}})^2F_4^{(1,2,3,4)}dx^4dv^4d\phi^4d\psi^4]$$
$$+\frac{\partial}{\partial v_\alpha^{(3)}}[-\frac{1}{4\pi}\int(\frac{\partial}{\partial x_\beta^{(4)}}\frac{\partial}{\left|x_\beta^{(3)}-x_\beta^{(4)}\right|})(v_\alpha^{(4)}\frac{\partial}{\partial x_\beta^{(4)}})^2F_4^{(1,2,3,4)}dx^4dv^4d\phi^4d\psi^4]+v[\frac{\partial}{\partial v_\alpha^{(1)}}\lim_{x^{(4)}\to x^{(1)}}$$
$$+\frac{\partial}{\partial v_\alpha^{(2)}}\lim_{x^{(4)}\to x^{(2)}}+\frac{\partial}{\partial v_\alpha^{(3)}}\lim_{x^{(4)}\to x^{(3)}}]\frac{\partial}{\partial x_\beta^{(4)}}\frac{\partial}{\partial x_\beta^{(4)}}\int v_\alpha^4F_4^{(1,2,3,4)}dv^4d\phi^4d\psi^4+\gamma[\frac{\partial}{\partial\phi^{(1)}}\lim_{x^{(4)}\to x^{(1)}}$$
$$+\frac{\partial}{\partial\phi^{(2)}}\lim_{x^{(4)}\to x^{(2)}}+\frac{\partial}{\partial\phi^{(3)}}\lim_{x^{(4)}\to x^{(3)}}]\frac{\partial}{\partial x_\beta^{(4)}}\frac{\partial}{\partial x_\beta^{(4)}}\int\phi^{(4)}F_4^{(1,2,3,4)}dv^4d\phi^4d\psi^4+D[\frac{\partial}{\partial\psi^{(1)}}\lim_{x^{(4)}\to x^{(1)}}$$
$$+\frac{\partial}{\partial\psi^{(2)}}\lim_{x^{(4)}\to x^{(2)}}+\frac{\partial}{\partial\psi^{(3)}}\lim_{x^{(4)}\to x^{(3)}}]\frac{\partial}{\partial x_\beta^{(4)}}\frac{\partial}{\partial x_\beta^{(4)}}\int\psi^{(4)}F_4^{(1,2,3,4)}dv^4d\phi^4d\psi^4+[f(u_\alpha^{(1)}-v_\alpha^{(1)})$$
$$\frac{\partial}{\partial v^{(1)}}+f(u_\alpha^{(2)}-v_\alpha^{(2)})\frac{\partial}{\partial v^{(2)}}+f(u_\alpha^{(3)}-v_\alpha^{(3)})\frac{\partial}{\partial v^{(3)}}]F_3^{(1,2,3)}-[R\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}+R\psi^{(2)}\frac{\partial}{\partial\psi^{(2)}}$$
$$+R\psi^{(3)}\frac{\partial}{\partial\psi^{(3)}}]F_3^{(1,2,3)}=0 \tag{39}$$

Which is the transport equation for three-point distribution function undergoing a 1st order chemical reaction in convective dusty fluid turbulent flow.

V. Result And Discussions

If we want to calculate for two-point joint distribution function in convective turbulent flow due to first order reaction in presence of dust particles, we apply the above technique and similarly from equation no. (39) We get

$$\frac{\partial F_2^{(1,2)}}{\partial t}+(v_\alpha^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}+v_\alpha^{(2)}\frac{\partial}{\partial x_\beta^{(2)}})F_2^{(1,2)}+\frac{\partial}{\partial v_\alpha^{(1)}}[-\frac{1}{4\pi}\int(\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\left|x_\beta^{(1)}-x_\beta^{(2)}\right|})(v_\alpha^{(2)}\frac{\partial}{\partial x_\beta^{(2)}})^2F_3^{(1,2,3)}$$
$$dx^3dv^3d\phi^3d\psi^3]+\frac{\partial}{\partial v_\alpha^{(2)}}[-\frac{1}{4\pi}\int(\frac{\partial}{\partial x_\beta^{(3)}}\frac{\partial}{\left|x_\beta^{(2)}-x_\beta^{(3)}\right|})(v_\alpha^{(3)}\frac{\partial}{\partial x_\beta^{(3)}})^2F_3^{(1,2,3)}dx^3dv^3d\phi^3d\psi^3]$$
$$+v[\frac{\partial}{\partial v_\alpha^{(1)}}\lim_{x^{(3)}\to x^{(1)}}+\frac{\partial}{\partial v_\alpha^{(2)}}\lim_{x^{(3)}\to x^{(2)}}]\frac{\partial}{\partial x_\beta^{(3)}}\frac{\partial}{\partial x_\beta^{(3)}}\int v_\alpha^3F_3^{(1,2,3)}dv^3d\phi^3d\psi^3+\gamma[\frac{\partial}{\partial\phi^{(1)}}\lim_{x^{(3)}\to x^{(1)}}$$
$$+\frac{\partial}{\partial\phi^{(2)}}\lim_{x^{(3)}\to x^{(2)}}]\frac{\partial}{\partial x_\beta^{(3)}}\frac{\partial}{\partial x_\beta^{(3)}}\int\phi^{(3)}F_3^{(1,2,3)}dv^3d\phi^3d\psi^3+D[\frac{\partial}{\partial\psi^{(1)}}\lim_{x^{(3)}\to x^{(1)}}+\frac{\partial}{\partial\psi^{(2)}}\lim_{x^{(3)}\to x^{(2)}}]\frac{\partial}{\partial x_\beta^{(3)}}\frac{\partial}{\partial x_\beta^{(3)}}$$
$$\int\psi^{(3)}F_3^{(1,2,3)}dv^3d\phi^3d\psi^3+[f(u_\alpha^{(1)}-v_\alpha^{(1)})\frac{\partial}{\partial v^{(1)}}+f(u_\alpha^{(2)}-v_\alpha^{(2)})\frac{\partial}{\partial v^{(2)}}]F_2^{(1,2)}-[R\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}$$
$$+R\psi^{(2)}\frac{\partial}{\partial\psi^{(2)}}]F_2^{(1,2)}=0 \tag{40}$$

Which is same as equation number (28) of Azad et al [2013].

Same as the above procedure we get transport equation for one-point joint distribution function in convective turbulent flow due to first order reaction in presence of dust particles as

$$\frac{\partial F_1^{(1)}}{\partial t}+v_\alpha^{(1)}\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}+\frac{\partial}{\partial v_\alpha^{(1)}}[-\frac{1}{4\pi}\int(\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\left|x_\beta^{(1)}-x_\beta^{(2)}\right|})(v_\alpha^{(2)}\frac{\partial}{\partial x_\beta^{(2)}})^2F_2^{(1,2)}dx^2dv^2d\phi^2d\psi^2]+v\frac{\partial}{\partial v_\alpha^{(1)}}\lim_{x^{(2)}\to x^{(1)}}$$
$$\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}\int v_\alpha^{(2)}F_2^{(1,2)}dv^2d\phi^2d\psi^2+\gamma\frac{\partial}{\partial\phi^{(1)}}\lim_{x^{(2)}\to x^{(1)}}\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}\int\phi^{(2)}F_2^{(1,2)}dv^2d\phi^2d\psi^2$$
$$+D\frac{\partial}{\partial\psi^{(1)}}\lim_{x^{(2)}\to x^{(1)}}\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}\int\psi^{(2)}F_2^{(1,2)}dv^2d\phi^2d\psi^2+f(u_\alpha^{(1)}-v_\alpha^{(1)})\frac{\partial}{\partial v^{(1)}}F_1^{(1)}-R\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}F_1^{(1)}=0 \qquad (41)$$

Which is same as equation number (27) of Azad et al [2013].

If the constant reaction rate R=0 then, the transport equation for one-point distribution function $F_1^{(1)}$ in turbulent flow equation (41) becomes

$$\frac{\partial F_1^{(1)}}{\partial t}+v_\alpha^{(1)}\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}+\frac{\partial}{\partial v_\alpha^{(1)}}[-\frac{1}{4\pi}\int(\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\left|x_\beta^{(1)}-x_\beta^{(2)}\right|})(v_\alpha^{(2)}\frac{\partial}{\partial x_\beta^{(2)}})^2F_2^{(1,2)}dx^2dv^2d\phi^2d\psi^2]+v\frac{\partial}{\partial v_\alpha^{(1)}}\lim_{x^{(2)}\to x^{(1)}}$$
$$\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}\int v_\alpha^{(2)}F_2^{(1,2)}dv^2d\phi^2d\psi^2+\gamma\frac{\partial}{\partial\phi^{(1)}}\lim_{x^{(2)}\to x^{(1)}}\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}\int\phi^{(2)}F_2^{(1,2)}dv^2d\phi^2d\psi^2$$
$$+D\frac{\partial}{\partial\psi^{(1)}}\lim_{x^{(2)}\to x^{(1)}}\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}\int\psi^{(2)}F_2^{(1,2)}dv^2d\phi^2d\psi^2+f(u_\alpha^{(1)}-v_\alpha^{(1)})\frac{\partial}{\partial v^{(1)}}F_1^{(1)}=0 \qquad (42)$$

Which was obtained earlier by Azad et al [2012].

If the fluid is clean i.e. f=0 then, the transport equation for one-point distribution function $F_1^{(1)}$ in turbulent flow equation (41) becomes

$$\frac{\partial F_1^{(1)}}{\partial t}+v_\alpha^{(1)}\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}+\frac{\partial}{\partial v_\alpha^{(1)}}[-\frac{1}{4\pi}\int(\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\left|x_\beta^{(1)}-x_\beta^{(2)}\right|})(v_\alpha^{(2)}\frac{\partial}{\partial x_\beta^{(2)}})^2F_2^{(1,2)}dx^2dv^2d\phi^2d\psi^2]+v\frac{\partial}{\partial v_\alpha^{(1)}}\lim_{x^{(2)}\to x^{(1)}}$$
$$\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}\int v_\alpha^{(2)}F_2^{(1,2)}dv^2d\phi^2d\psi^2+\gamma\frac{\partial}{\partial\phi^{(1)}}\lim_{x^{(2)}\to x^{(1)}}\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}\int\phi^{(2)}F_2^{(1,2)}dv^2d\phi^2d\psi^2$$
$$+D\frac{\partial}{\partial\psi^{(1)}}\lim_{x^{(2)}\to x^{(1)}}\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}\int\psi^{(2)}F_2^{(1,2)}dv^2d\phi^2d\psi^2-R\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}F_1^{(1)}=0 \qquad (43)$$

Which was obtained by Kishore and Shing [1984].

For closing the system of equations for the joint distribution function, some approximations are required. Closure scheme can be used here and closure can be obtained by decomposing $F_2^{(1,2)}$ as

$$F_2^{(1,2)}=(1+\varepsilon)F_1^{(1)}F_1^{(2)}$$
$$F_3^{(1,2,3)}=(1+\varepsilon)^2F_1^{(1)}F_1^{(2)}F_1^{(3)}$$

Where, ε is the correlation coefficient between the particles. The transport equation for the joint distribution function of velocity, temperature and concentration field have been presented to provide the advantageous basis for modeling the turbulent flow for concentration undergoing a first order reaction in presence of dust particles.

Acknowledgements

Authors are grateful to the Department of Applied Mathematics, University of Rajshahi, Bangladesh for giving all facilities and support to carry out this work.

References

[1] Ta-You W., 1966. Kinetic Theory of Gases and Plasma. Addision Wesley Phlelishing.
[2] Lundgren T.S., 1967. Hierarchy of coupled equations for multi-point turbulence velocity distribution functions. Phys. Fluid., 10: 967.
[3] Kumar P. and Patel S .R .Phys. of Fluids.17,1362(1974).
[4] Kumar P.S.R and Patel S .R . , Int. J. Eng. Sci.,13.305-315(1975).
[5] Patel S .R . , Int. J. Eng. Sci.,14,75 (1976).
[6] Kishore N., 1978. Distribution functions in the statistical theory of MHD turbulence of an incompressible fluid. J. Sci. Res., BHU, 28(2): 163.
[7] Pope S. B., 1979. The statistical theory of turbulence flames. Phil. Trans Roy. Soc. London A, 291: 529.
[8] Pope S.B., 1981.The transport equation for the joint probability density function of velocity and scalars in turbulent flow. Phys. Fluid., 24: 588.

[9] Dixit. And B.N. Upadhyay, 1989. Distribution functions in the statistical theory of MHD turbulence of an incompressible fluid in the presence of the Coriolis force, Astrophysics and Space Sci., 153, 297-309.

[10] Kishore N. and S. R. Singh, 1984. Transport equation for the bivariate joint distribution function of velocity and temperature in turbulent flow. Bull. Tech. Univ. Istambul, 37, 91-100.

[11] Kishore N. and S. R. Singh, 1985. Transport equation for the joint distribution function of velocity, temperature and concentration in convective turbulent flow. Prog. of Maths. 19(1&2):13-22.

[12] Sarker M.S.A. and N. Kishore, 1991. Distribution functions in the statistical theory of convective MHD turbulence of an incompressible fluid. Astrophys. Space Sci., 181: 29.

[13] Sarker M.S.A. and N. Kishore, 1999. Distribution functions in the statistical theory of convective MHD turbulence of mixture of a miscible incompressible fluid. Prog. Math., BHU India, 33(1-2): 83.

[14] Islam M.A and Sarker M.S.A. ,Indian J .Pure.Appl.Math.,32(8).1173-1184(2001).

[15] Azad M. A. K. and M .S.A. Sarker, 2004. Statistical theory of certain distribution functions in MHD turbulence in a rotating system in presence of dust particles. Rajshahi university studies. Part -B. J. Sci., 32: 193-210.

[16] M. A. K. Azad, M. A. Aziz and M. S. A. Sarker, First Order Reactant in Magneto-hydrodynamic Turbulence before the Final Period of Decay in presence of dust particles in a Rotating System, Journal of Physical Sciences, 13 , 175-190, 2009.

[17] M. S. Alam Sarker, M. A. K.Azad and M. A. Aziz, First Order Reactant in MHD Turbulence before the Final Period of Decay for the Case of Multi-Point and Multi-Time in Presence of Dust Particles. J. Phy. Sci. , 13, 21-38, 2009a.

[18] Aziz, M. A., M. A. K. Azad and M. S. A. Sarker (2010). Statistical Theory of Distribution Functions in Magneto-hydrodynamic Turbulence in a Rotating System Undergoing a First Order Reaction in Presence of Dust Particles, Res. J. Math.and Stat., 2(2), 37-55.

[19] Azad, M. A. K., M. A. Aziz and M. S. A. Sarker (2011). Statistical Theory of certain Distribution Functions in MHD Turbulent flow for Velocity and Concentration Undergoing a First Order Reaction in a Rotating System, Bangladesh J. Sci. Ind. Res., 46(1), 59-68.

[20] Azad, M. A. K., M. H. U. Molla and M. Z. Rahman, 2012. Transport Equatoin for the Joint Distribution Function of Velocity, Temperature and Concentration in Convective Tubulent Flow in Presence of Dust Particles, Res. J. Appl.Sci., Engng. Tech., 4(20), 4150-4159.

[21] M. A. Bkar Pk, M. A. K. Azad and M. S. A. Sarker, First-order reactant in homogeneou dusty fluid turbulence prior to the ultimate phase of decay for four-point correlation in a rotating system. Res. J. Math. & Stat., 4(2): 30-38, 2012a.

[22] Azad, M. A. K., M. H. U. Molla and M. Z. Rahman, 2013. Transport Equatoin for the Joint Distribution Functions in Convective Tubulent Flow in Presence of Dust Particles undergoing a first order chemical reaction, Rajshahi Uni. J. Sci. Eng. Vol.40-42:31-43,2012-2014.

[23] M. H. U. Molla M. A. K. Azad and M. Z. Rahman, Transport equation for the joint distribution functions of velocity, temperature and concentration in convective turbulent flow in presence of coriolis force, Int. J. Scholarly Res. Gate, 1(4): 42-56, 2013a.

[24] M. A. K. Azad, M. N. Islam, and Mst. Mumtahinah, Transport Equations of Three- point Distribution Functions in MHD Turbulent Flow for Velocity, Magnetic Temperature and Concentration, Res. J. Appl. Sci. Engng. & Tech. 7(24): 5184-5220, 2014(a).

[25] M. A. K. Azad and M. N. Islam, Transport Equations of Three Point Distribution Functions in MHD Turbulent Flow for Velocity, Magnetic Temperature and Concentration in a Rotating System, Int. J. Scholarly Res. Gate, 2(3): 75-120, 2014(b).

[26] M. A. K. Azad, Mst. Mumtahinah, and M. A. Bkar Pk, 2015. Transport equation for the joint distribution functions of certain variables in convective dusty fluid turbulent flow in a rotating system undergoing a first order reaction, American Journal of Applied Mathematics, 3(1): 21-30.