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On MarkovianModelling of Vehicular Traffic Congestion in Urban Areas at Kanyakumari District.

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Abstract: *Queue is the study of traffic behavior near a certain section where demand exceeds available capacity. Queue can be seen in many situations, boarding a bus, train or plane traffic signal etc. Mobility is an in dispensable activity of our daily lives and traffic congestion is one popular approach to mobility. In this paper we discuss the Markovian modelling of vehicular traffic congestion in Nagercoil junction and Marthandam junction at kanayakumari district. Queuing theory analytic methodologies, simulation and Chi-square distribution are applied and parameters such as average arrival rate and service rate are calculated based on the data obtained at the Marthandam and Nagercoil junctions on daily basis for six days in a week.*
Keywords: *Queuing system, Poisson process, Markov process, Chi - square distribution, exponential distribution*

I. Introduction

Human population growth in the district is doubled with increased road vehicle ownership is the main factor vehicular population increases .The volume of vehicular traffic on roads increases during rush hours. Here we discuss the traffic congestion problem in urban areas at kanayakumari district. In road expansion and road vehicular traffic increases, the positive impact of traffic wardens on road traffic congestive is likely to be inefficient of traffic flow and evolution for the unexpected congestions is not accurately. There are many applications of queuing theory to real life situations where congestions, queue, much traffic, delays, time wasting etc.

Queuing system is composed of units referred to as customers, needing some kind of service and who arrive at a service facility, join a queue service is not immediately available and eventually leave after a receiving the service. A server refers to mechanism that delivers service to the customer. If arrival of a customer, finds that server busy then she may from a queue, join it or leave the system without receiving any service even after waiting for some time.

Arrival pattern

The arrivals occur indicated by the inter arrival time between two consecutive arrivals .That is the inter arrival time may vary.

Arrival rate □

This is the average number of vehicles arriving per unit time.

The service pattern

The service is rendered and is specified by the time taken to complete a service. That is the distribution of service time must be specified, under stochastic modelling considerations.

Service time μ

This gives the average number of vehicles served per unit time. The server utilization is $\rho = \frac{\lambda}{\mu}$.

Stochastic processes

Let t be a parameter that assumes values in a set T and let X (t) represent a random variable for every t□T. Then X (t) gives the rule describing the traffic the observednon-deterministic behavior ofthe traffic system being modelled and any collection of $\{X(t), t \mathcal{E} T\}$constitutes a stochastic process, whose index t interprets the time element of the physical evolution of the system. In this case T is a linear set and may result in a discrete process or a continuous time process. That is$\{x_{n,}\ n = 0,1,2, \dots \dots .\}$can be described as a discrete- time process while that$\{X(t), t \geq 0\}$can also be described as a continuous the state space of the process. The set of all possible values that the random variable X (t) can assume constitute the state space of the process. As such a system may be described by any one of these four different categories of the space and time stochastic.

1) Discrete state space and discrete time.
2) Continuous state space and discrete time.
3) Discrete state space and continuous time.
4) Continuous state space and continuous time.

Here we discuss the discrete –time chains and continuous –time chains. Here relating this time evolution to vehicle queue at an urban areas as a result of traffic congestion constitute on observed stochastic processes.

Markov Chain

Suppose we observe the state of the vehicular traffic at discrete time points t = 0, 1, 2…for which successive time points define a set of random variables$X_{0,}X_1$,X_2……The values assumed by the random variables are the states of the system at time n. Suppose that X_n assumes the finite set of values 0, 1, 2,……..m, then $X_n = i$ means that the system's state at time n is i. The family of random variables $\{X_n , \geq 0\}$ is a stochastic process with discrete parameter space n = 0, 1, 2…and discrete state space S =$\{0,1,2, \dots \dots \dots m\}$.

A stochastic process $\{X_{n,} n \geq 0 \}$ is called a Markov chain if for every X□S,

Pr$\left\{ X_n = x_n / X_{n-1} = x_{n-1}, \dots \dots \dots \dots, X_0 = x_0 \right\}$ = Pr$\left\{ X_n = x_n / X_{n-1} = x_{n-1} \right\}$.It means that given the present state of the system, the future state is independent of that past state. Given that the state space S satisfies the Markov chain property of the above equation.

II. M/M/1 queuing model

Let p_n be the probability that there are n vehicles in the system at time zero .Suppose that N(t) gives the number of vehicles in the system at any time t measured from a fixed initial moment t = 0 and its probability distribution is given by $p_n(t) = pr\{N(t) = M\}$,n = 0,1,2,…………..For this practical problem of vehicular traffic congestion a complete description of $\{N(t),\ t \geq 0 \}$ is necessary in order to find a time – dependent solution ,p_n(t) , n≥ 0 for equilibrium state . The limiting behavior of p_n(t) as t ->∞, denoted by $\lim_{n\to\infty} p_n$ (t), n=0, 1, 2, 3, 4… Whenever the limit exists the system is said to reach a steady state. Let p_0 denotes the proportion of time $\{a_n , n \geq 0 \}$and $\{d_n, n \geq 0\}$ such that a_n be the probability that arriving vehicles find n units in the congestion when they arrive and d_n be the probability that departing vehicles leave n units in the system when they depart. The three terms p_n, a_n, d_n may not always be all equal and so for such vehicular system in equilibrium in which arrivals and departures occur one by one independently that is$a_n = d_n$for n ≥ 0.For a queuing system of many vehicles and one server, suppose N (t) is the number of individual vehicles at time t, in front of a server waiting to be served. Let N(t) has two possible states of 1 and 0 respectively for the states of being served and completed being served. That is N (t) = 1 implies a vehicles is being served and N (t) = 0 implies the vehicles has been served. Let $X_t = \begin{pmatrix} p(t) \\ q(t) \end{pmatrix}$ be the time independent distribution vector for the states being served and finished served so that p (t) = p (N (t) = 1) and q (t) = p(N(t) =0).Then it follows that q(t) = 1-p(t) and so $\lim_{t\to\infty} p(t) = 0$.Let P(served in $\Delta\, t) = \mu \Delta t$ and P(not served in ∆t)= 1-μ∆t then the transition matrix for each ∆t time is given by

A = $\begin{bmatrix} 1 - \mu \Delta t & 0 \\ \mu \Delta t & 1 \end{bmatrix}$. Assuming the initial conditions p (0) =1 and solving the matrix equation $X_{t+\Delta t}$ = AX_t, we get p (t) = $e^{-\lambda t}$ and q (t) = 1- $e^{-\lambda t}$.

III. Chi-square distribution

The chi-square test is one of the simplest and most widely used non parametric test in statiscal work. It makes no assumptions about the population being sampled. The quantity χ^2 –describes the magnitude of discrepancy between theory and observation, that is with the help of χ^2-test we can know whether a given discrepancy between theory and observation can be attributed to chance or whether it results from the inadequacy of the theory to fit the observed facts .If χ^2 is zero, it means that the observed and expected frequencies completely coincide.

The formula for computing chi-square is $\chi^2 = \frac{\sum (O-E)^2}{E}$

Where O - observed frequency

E – Expected or theoretical frequency

The degrees of freedom is v = (r-1) (c-1).

The calculated value of χ^2 is compared with the table value of χ^2 for given degrees of freedom at specified level of significance. If the calculated value of is χ^2 is greater than the table value, the null hypothesis is rejected .If the calculated value of χ^2 is less than the table value at a specified level of significance the null holds true.

The level of service of an intersection is a qualitative measure of capacity and operating conditions and is directly related to vehicle delay.

The level of service for the urban junctions in kanayakumari district.

Level of service (LOS)	Average control Delay (sec/vehicle)	Conclusion
A	≤ 10	No delays of at junctions and continuous traffic flow
B	10.1 to 20	Same as LOS A
C	20.1 to 30	Medium traffic flow
D	30.1 to 40	Heavy traffic flow conditions
E	≥ 40	Unstable traffic flow

Six days vehicle traffic count for marthandam junction

Time	Day1	Day2	Day3	Day4	Day5	Day6	Total
8.00 – 8.10	124	116	102	98	115	93	648
8.10 – 8.20	115	75	120	101	107	90	641
8.20 – 8.30	105	108	92	93	110	85	593
8.30 – 8.40	108	85	93	120	99	97	602
8.40 – 8.50	118	97	111	85	94	89	594
8.50 – 9.00	120	103	118	120	104	101	666
total	650	604	636	630	629	555	3744

Analysis of Service time

Service time (X)	Observed frequency (F)	(XF)
3	174	522
4	238	952
5	240	1200
6	245	1470
7	208	1456
8	116	1044
9	264	2376
10	380	3800
11	504	5544
12	256	3072
13	211	2743
14	310	4340
15	289	4335
16	309	4944
total	3744	37798

Input parameters

Arrival rate λ = 10
Service rate μ = 15
Experiment duration = 2580 minutes
Maximum queue length =3744

Result	Computed value	Simulated value
Customers in queue L_s	1.999	2.027
Customers in the queueL_q	1.33	1.36
Time in system w_s	0.2	0.25025
Time in queue w_q	0.133	0.1367
Idle probability p_0	0.333	0.336
Server utilization ρ	.666	0.6646

Number of vehicles that arrived = 3744
Time within which vehicles arrived = 360 minutes
Arrival rate λ =$\frac{\text{number of arrivals}}{\text{time taken}}$
=10.4 vehicles per minutes = 10 vehicles approximately

Mean of service distribution

Mean =$\frac{\sum f_i X_i}{\sum f} = \frac{37798}{3744}$ = 10.09= 10 vehicles
Number of vehicles served = 3744.
Time with in which vehicles where served = 2580 minutes
Service rate μ = 14.65 =15 vehicles

Inter departure time $=\frac{1}{\mu}$ = 0.068 minutes.
Using Chi- square distribution calculated value is 23.5648
Degrees of freedom = (6-1) (6-1) = 25.
At 5% level of significant table value is 37.7. Here calculated value is less than table value. That is null hypothesis is accepted and it is conclude that arrivals at the Marthandam junction road follows the null hypothesis of Poisson distribution. Thus the process is Markovian nature.

Six days vehicle traffic count for Nagercoil junction

Time	Day1	Day2	Day3	Day4	Day5	Day6	Total
8.00 – 8.10	172	135	130	133	131	120	830
8.10 – 8.20	124	129	132	141	135	110	771
8.20 – 8.30	137	148	130	129	145	132	821
8.30 – 8.40	138	135	141	131	142	126	813
8.40 – 8.50	145	130	129	124	133	139	810
8.50 – 9.00	148	138	137	145	130	129	827
total	864	815	808	823	816	756	4872

Analysis of Service time

Service time (X)	Observed frequency (F)	(XF)
3	94	282
4	183	732
5	242	1210
6	85	510
7	204	1428
8	166	1328
9	271	2439
10	270	2700
11	302	3322
12	388	4656
13	255	3315
14	267	3738
15	293	4395
16	252	4032
17	290	4930
18	271	4878
19	337	6403
20	270	5400
21	112	2352
22	48	10546
23	120	2760
More than 24	152	3648
Total	4872	65514

Input parameters
Arrival rate λ = 14
Service rate μ = 18
Experiment duration = 360 minutes
Maximum queue length = 4872

Result	Computed value	Simulated value
Customers in queue L_s	3.5	3.45
Customers in the queue L_q	2.72	2.67
Time in system w_s	0.25	.2477
Time in queue w_q	0.194	.1915
Idle probability p_0	0.222	.2245
Server utilization ρ	0.722	.7824

Number of vehicles that arrived = 4872
Time within which vehicles arrived = 360 minutes
Arrival rate $\lambda = \frac{\text{number of arrivals}}{\text{time taken}}$
=13.5 vehicles per minutes=14 vehicles approximately

IV. Mean of service distribution

Mean = $\frac{\sum f_i X_i}{\sum f}$ = $\frac{65514}{4872}$ = 13.5 vehicles
Number of vehicles served = 13.5 =13 approximately
Time with in which vehicles where served = 3600 minutes
Service rate μ = 18.19 vehicles
Inter departure time = 0.0547 minutes.
Using Chi- square distribution calculated value = 20.37
Degrees of freedom = (6-1)(6-1) = 25

At 5% level of significant table value is 37.7. Here calculated value is less than table value. That is null hypothesis is accepted and it is conclude that arrivals at the Nagercoil junction road follows the null hypothesis of Poisson distribution. Thus the process is Markovian nature.

V. Conclusion

In this work, measured the traffic flow at marthandam and at Nagercoil junction of urban road during the morning rush hours and have demonstrated features of the queue built up at the junctions with data and modified traffic flow as M/M/1, Chi square distribution and simulation result .The current queue system will continue to develop heavy traffic, during the peak hours. Based on the results of the analysis it can be concluded that the level of service for the traffic flow at junctions falls A with no delay and continuous flow of traffic. Traffic flow at the junctions are found to be consistent and ideal expect for the identified problems are like bad road surfaces, absence of modern traffic utilities, increase in the road infrastructure and signal time adjustments.

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