

Some Concepts on Constant Interval Valued Intuitionistic Fuzzy Graphs

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Abstract: The fundamental characteristic of the IVIFS is that the values of its membership function and non-membership function are intervals rather than exact numbers. In this paper, we define degree, order and size of IVIFGs. Also constant Interval valued intuitionistic fuzzy graphs and totally constant IVIFGs are introduced and discussed some properties of constant IVIFG.

Keywords: Intuitionistic fuzzy graph, Interval valued intuitionistic fuzzy set, Interval valued intuitionistic fuzzy graph, Strong IVIFG, Constant IVIFG, totally constant IVIFG.

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I. Introduction:

Fuzzy set (FS) was introduced by Zadeh [16] in 1965, as a generalization of crisp sets. The concept of intuitionistic fuzzy sets (IFSs), as a generalization of fuzzy set was introduced by K. Atanassov [1] and defined new operations on intuitionistic fuzzy graphs. Later, K. Atanassov and G. Gargov [3] introduced the Interval valued intuitionistic fuzzy sets (IVIFSs) theory, as a generalization of both interval valued fuzzy sets (IVFSs) and intuitionistic fuzzy sets (IFSs). Hence, many scholars applied IFS and IVIFS to decision analysis and pattern recognition widely.

By introducing the degree of membership $M_A(x)$, the degree of non-membership $N_A(x)$ and the degree of hesitancy $H_A(x)$, the IFS theory and IVIFS theory established. According to the IVIFS definition, $M_A(x)$, $N_A(x)$ and $H_A(x)$ are intervals, where $M_A(x)$ denotes the range of support party, $N_A(x)$ denotes the range of opposition party and $H_A(x)$ denotes the range of absent party. Atanassov's IVIFS is based on point estimation, which means that these intervals can be regarded as estimation result of an experiment. K. Atanassov [2] in 1994 introduced some operations over interval valued fuzzy sets (IVFSs). In 2000, Some Operations on intuitionistic fuzzy sets were proposed by De S. K., Biswas. R and Roy A. R [6]. In 2006, some operators defined over Interval-valued intuitionistic fuzzy sets were proposed by N. K. Jana and M. Pal [8]. In 2008, properties of some IFS operators and operations were proposed by Q. Liu, C. Ma and X. Zhou [9]. In 2011, Intuitionistic fuzzy sets: Some new results were proposed by R. K. Verma and B. D. Sharma [12]. In 2014, An Interval-valued Intuitionistic Fuzzy Weighted Entropy (IVIFWE) Method for Selection of Vendor was proposed by D. Ezhilmaran and S. Sudharsan [7]. A. Nagoor Gani and S. Sajitha Begum [11] defined degree, Order and Size in intuitionistic fuzzy graphs and extend the properties. In 2014, M. Ismayil [10] et.al. introduced the notion of strong IVIFG and discussed some operations.

In this paper, we define the degree, order and size of IVIFGs with some results. Also, Constant IVIFGs and totally constant IVIFGs are introduced, and some properties are analyzed.

II. Preliminary

Definition 2.1: A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \rightarrow [0, 1]$ and $\mu : V \times V \rightarrow [0, 1]$, where for all $u, v \in V$, we have $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$.

Definition 2.2: An intuitionistic fuzzy graph is of the form $G = (V, E)$, where

- (i) $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1 : V \rightarrow [0, 1]$ and $\gamma_1 : V \rightarrow [0, 1]$ denote the degree of membership and non-membership of the element $v_i \in V$, respectively, and $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$ for every $v_i \in V$, ($i = 1, 2, \dots, n$),
- (ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \rightarrow [0, 1]$ and $\gamma_2 : V \times V \rightarrow [0, 1]$ are such that $\mu_2(v_i, v_j) \leq \min[\mu_1(v_i), \mu_1(v_j)]$ and $\gamma_2(v_i, v_j) \leq \max[\gamma_1(v_i), \gamma_1(v_j)]$ and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$, $i, j = 1, 2, \dots, n$.

Definition 2.3: An edge $e = (x, y)$ of an IFG $G = (V, E)$ is called an effective edge if

$$\mu_2(x, y) = \mu_1(x) \wedge \mu_1(y) \text{ and } \gamma_2(x, y) = \gamma_1(x) \vee \gamma_1(y).$$

Definition 2.4: An Intuitionistic fuzzy graph is complete if $\mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$ and

$$\gamma_{2ij} = \max(\gamma_{2i}, \gamma_{2j}) \text{ for all } (v_i, v_j) \in V.$$

Definition 2.5: An Intuitionistic fuzzy graph G is said to be strong IFG if $\mu_2(x, y) = \mu_1(x) \wedge \mu_1(y)$ and $\gamma_2(x, y) = \gamma_1(x) \vee \gamma_1(y)$ for all $(v_i, v_j) \in E$. That is every edge is effective edge.

Definition 2.6: An Interval valued intuitionistic fuzzy set A in the finite universe X is defined as $A = \{ x, [\mu_A(x), \gamma_A(x)] \mid x \in X \}$, where $\mu_A: X \rightarrow [0,1]$ and $\gamma_A: X \rightarrow [0,1]$ with the condition $0 \leq \sup(\mu_A(x)) + \sup(\gamma_A(x)) \leq 1$, for any $x \in X$. The intervals $\mu_A(x)$ and $\gamma_A(x)$ denote the degree of membership function and the degree of non-membership of the element x to the set A. For every $x \in X$, $\mu_A(x)$ and $\gamma_A(x)$ are closed intervals and their Left and Right end points are denoted by $\mu_A^L(x), \mu_A^R(x)$ and $\gamma_A^L(x), \gamma_A^R(x)$. Let us denote $A = \{ [x, (\mu_A^L(x), \mu_A^R(x)), (\gamma_A^L(x), \gamma_A^R(x))] \mid x \in X \}$ where $0 \leq \mu_A^R(x) + \gamma_A^R(x) \leq 1, \mu_A^L(x) \geq 0, \gamma_A^L(x) \geq 0$. We call the interval $[1 - \mu_A^R(x) - \gamma_A^R(x), 1 - \mu_A^L(x) - \gamma_A^L(x)]$, abbreviated by $[\pi_A^L(x), \pi_A^R(x)]$ and denoted by $\pi_A(x)$, the interval-valued intuitionistic index of x in A, which is a hesitancy degree of x to A.

Especially if $\mu_A(x) = \mu_A^L(x) = \mu_A^R(x)$ and $\gamma_A(x) = \gamma_A^L(x) = \gamma_A^R(x)$ then the given IVIFS A is reduced to an ordinary IFS

III. Interval Valued intuitionistic fuzzy graph

Definition 3.1: An interval valued intuitionistic fuzzy graph (IVIFG) with underlying set V is defined to be a pair $G = (\mu, \gamma)$, Where

- (i) $\mu_1: V \rightarrow D[0,1]$ and $\gamma_1: V \rightarrow D[0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$, respectively, and $0 \leq \mu_1^R(v_i) + \gamma_1^R(v_i) \leq 1$ for every $v_i \in V, (i = 1, 2, \dots, n)$,
- (ii) The function $\mu_2: E \subseteq V \times V \rightarrow D[0,1]$ and $\gamma_2: E \subseteq V \times V \rightarrow D[0,1]$ where $\mu_2^L(v_i, v_j) \leq \min[\mu_1^L(v_i), \mu_1^L(v_j)]$ and $\mu_2^R(v_i, v_j) \leq \min[\mu_1^R(v_i), \mu_1^R(v_j)]$
 $\gamma_2^L(v_i, v_j) \leq \max[\gamma_1^L(v_i), \gamma_1^L(v_j)]$ and $\gamma_2^R(v_i, v_j) \leq \max[\gamma_1^R(v_i), \gamma_1^R(v_j)]$ such that $0 \leq \mu_2^R(v_i, v_j) + \gamma_2^R(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E, i, j = 1, 2, \dots, n$.

Example 3.2:

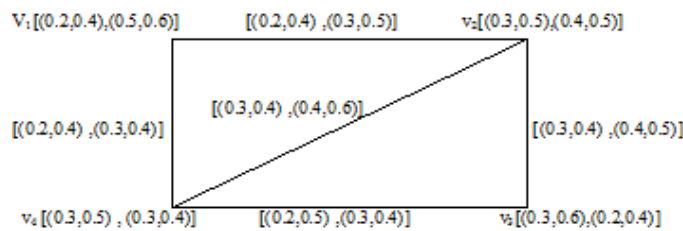


Fig -1: Interval Valued Intuitionistic Fuzzy Graph (IVIFG)

Definition 3.3: An IVIFG is called Strong Interval valued intuitionistic fuzzy graph (SIVIFG) if $\mu_2^R(v_i, v_j) = \min[\mu_1^R(v_i), \mu_1^R(v_j)]$, $\mu_2^L(v_i, v_j) = \min[\mu_1^L(v_i), \mu_1^L(v_j)]$ and $\gamma_2^L(v_i, v_j) = \max[\gamma_1^L(v_i), \gamma_1^L(v_j)]$, $\gamma_2^R(v_i, v_j) = \max[\gamma_1^R(v_i), \gamma_1^R(v_j)]$

Definition 3.4: Let $G = (V, E)$ be an IVIFG. Then the degree of a vertex v is defined by $d(v) = (d_\mu(v), d_\gamma(v))$ where $d_\mu(v) = (\sum_{u \neq v} \mu_2^L(v, u), \sum_{u \neq v} \mu_2^R(v, u))$ and $d_\gamma(v) = (\sum_{u \neq v} \gamma_2^L(v, u), \sum_{u \neq v} \gamma_2^R(v, u))$

Definition 3.5: The total degree of a vertex 'v' in IVIFG is defined as $t(v) = [(td_\mu(v), td_\gamma(v))]$, where

$$td_\mu(v) = (\sum_{u \neq v} \mu_2^L(v, u) + \mu_1^L(v), \sum_{u \neq v} \mu_2^R(v, u) + \mu_1^R(v))$$

$$td_\gamma(v) = (\sum_{u \neq v} \gamma_2^L(v, u) + \gamma_1^L(v), \sum_{u \neq v} \gamma_2^R(v, u) + \gamma_1^R(v))$$

Definition 3.6: Let G be an IVIFG and all vertices are of same degree then G is said to be regular IVIFG.

Definition 3.7: Let G be an IVIFG, then the order of G is defined to be

$$O(G) = [(\sum_{v \in V} \mu_1^L(v), \sum_{v \in V} \mu_1^R(v)), (\sum_{v \in V} \gamma_1^L(v) + \sum_{v \in V} \gamma_1^R(v))]$$

Definition 3.8: Let G be the IVIFG, then the size of G is defined to be

$$S(G) = [(\sum_{u \neq v} \mu_2^L(v, u), \sum_{u \neq v} \mu_2^R(v, u)), (\sum_{u \neq v} \gamma_2^L(v, u), \sum_{u \neq v} \gamma_2^R(v, u))]$$

Definition 3.9: In any IVIFG G,

$$\sum_{v \in V} d_G(v) = 2[(\sum_{u \neq v} \mu_2^L(v, u), \sum_{u \neq v} \mu_2^R(v, u)), (\sum_{u \neq v} \gamma_2^L(v, u), \sum_{u \neq v} \gamma_2^R(v, u))] = 2 S(G).$$

IV. Constant Interval Valued Intuitionistic Fuzzy Graph

Definition 4.1 : Let G be an IVIFG. If $d_\mu(v_i) = (K_i, K_j)$ and $d_\gamma(v_i) = (K_i, K_j)$ for all $v_i \in V$, then the graph G is constant IVIFG with degree $[(K_i, K_j), (K_i, K_j)]$.

Definition 4.2: Let G be an IVIFG. If $td_\mu(v_i) = (c_1, c_2)$ and $td_\gamma(v_i) = (c_1, c_2)$ for all $v_i \in V$, then the IVIFG is called totally constant IVIFG with degree $[(c_1, c_2), (c_1, c_2)]$

Theorem 4.3: Let G be the IVIFG and if $\mu_1^L = \gamma_1^L$ and $\mu_1^R = \gamma_1^R$ for all vertices if and only if the following are equivalent

- (i) G is constant IVIFG
- (ii) G is totally constant IVIFG

Proof: Let $\mu_1^L = \gamma_1^L = a$, and $\mu_1^R = \gamma_1^R = b$ for all vertices, where a and b are constants. Assume that G is constant IVIFG with constant degree $[(c_1, c_2), (c_1, c_2)]$. Then $d_\mu(v_i) = (c_1, c_2)$,

$$d_\gamma(v_i) = (c_1, c_2) \text{ for all } v_i \in V. \text{ So } td_\mu(v_i) = (c_1 + a, c_2 + b), td_\gamma(v_i) = (c_1 + a, c_2 + b) \text{ for all } v_i \in V.$$

Hence G is totally constant IVIFG. Thus (i) \Rightarrow (ii) is proved.

Now, suppose that G is totally constant IVIFG, then $td_\mu(v_i) = (k_1, k_2)$, $td_\gamma(v_i) = (k_1, k_2)$ for all $v_i \in V$. We get $d_\mu(v_i) + (a, b) = (k_1, k_2) \Rightarrow d_\mu(v_i) = (k_1 - a, k_2 - b)$ and similarly, $d_\gamma(v_i) = (k_1 - a, k_2 - b)$. So G is constant IVIFG. Thus (ii) \Rightarrow (i) is proved. Hence (i) and (ii) are equivalent.

Conversely, Assume G is both constant IVIFG and Totally constant IVIFG.

Suppose $\mu_1^L \neq \gamma_1^L$ and $\mu_1^R \neq \gamma_1^R$ for at least one pair of vertices say $v_1, v_2 \in V$.

Let G be constant IVIFG, then $d_\mu(v_1) = d_\mu(v_2) = (c_1, c_2)$. So $td_\mu(v_1) = (c_1, c_2) + (\mu_1^L(v_1), \mu_1^R(v_1)) = (c_1 + \mu_1^L(v_1), c_2 + \mu_1^R(v_1))$. Similarly $td_\gamma(v_1) = (c_1 + \gamma_1^L(v_1), c_2 + \gamma_1^R(v_1))$. Also, $td_\mu(v_2) = (c_1, c_2) + (\mu_1^L(v_2), \mu_1^R(v_2)) = (c_1 + \mu_1^L(v_2), c_2 + \mu_1^R(v_2))$ and $td_\gamma(v_2) = (c_1 + \gamma_1^L(v_2), c_2 + \gamma_1^R(v_2))$. Since $\mu_1^L \neq \gamma_1^L$ and $\mu_1^R \neq \gamma_1^R$, then $td_\mu(v_1) \neq td_\mu(v_2)$, $td_\gamma(v_1) \neq td_\gamma(v_2)$.

So G is not totally constant IVIFG, which is a contradiction to our assumption.

Now, we take G is totally constant IVIFG, we get $d_\mu(v_1) \neq d_\mu(v_2)$, $d_\gamma(v_1) \neq d_\gamma(v_2)$. So, G is not constant IVIFG, which is a contradiction to our assumption. Hence, we get, $\mu_1^L = \gamma_1^L$ and $\mu_1^R = \gamma_1^R$.

Theorem 4.4: The size of a constant IVIFG with degree $[(k_1, k_2), ((k_1, k_2))]$ is $[(\frac{pk_1}{2}, \frac{pk_2}{2}), (\frac{pk_1}{2}, \frac{pk_2}{2})]$

where p is order of IVIFG.

Proof:

$$\text{The size of IVIFG is } S(G) = [(\sum_{u \neq v} \mu_2^L(v, u), \sum_{u \neq v} \mu_2^R(v, u)), (\sum_{u \neq v} \gamma_2^L(v, u), \sum_{u \neq v} \gamma_2^R(v, u))]$$

Since G is $[(k_1, k_2), ((k_1, k_2))]$ - regular IVIFG. Then, $d_\mu(v) = (k_1, k_2)$, $d_\gamma(v) = (k_1, k_2)$ for all $v \in V$.

We have $(\sum_{v \in V} d_{\mu}^L(v), \sum_{v \in V} d_{\mu}^R(v)), (\sum_{v \in V} d_{\gamma}^L(v), \sum_{v \in V} d_{\gamma}^R(v))$
 $= 2 [(\sum_{u \neq v} \mu_2^L(v, u), \sum_{u \neq v} \mu_2^R(v, u)), (\sum_{u \neq v} \gamma_2^L(v, u), \sum_{u \neq v} \gamma_2^R(v, u))] = 2 S(G)$

That is, $2 S(G) = [(\sum_{v \in V} k_1, \sum_{v \in V} k_2), (\sum_{v \in V} k_1, \sum_{v \in V} k_2)] = [(pk_1, pk_2), ((pk_1, pk_2))]$

Therefore, $S(G) = [(\frac{pk_1}{2}, \frac{pk_2}{2}), (\frac{pk_1}{2}, \frac{pk_2}{2})]$

Theorem 4.5: Let G be an IVIFG and G is totally constant IVIFG with $[(c_1, c_2)(c_1, c_2)]$ as total degree for all vertices, then $2 S(G) + O(G) = [(pc_1, pc_2)(pc_1, pc_2)]$. Where $p = O(G)$.

Proof:

Given G is $[(c_1, c_2)(c_1, c_2)]$ - totally constant IVIFG ,
 then $(c_1, c_2) = (\sum_{u \neq v} \mu_2^L(v, u) + \mu_1^L(v), \sum_{u \neq v} \mu_2^R(v, u) + \mu_1^R(v))$.

Also $(c_1, c_2) = (\sum_{u \neq v} \gamma_2^L(v, u) + \gamma_1^L(v), \sum_{u \neq v} \gamma_2^R(v, u) + \gamma_1^R(v))$

Therefore,

$$[(\sum_{v \in V} (c_1, c_2), \sum_{v \in V} (c_1, c_2))] =$$

$$\sum_{v \in V} [(\sum_{u \neq v} \mu_2^L(v, u) + \mu_1^L(v), \sum_{u \neq v} \mu_2^R(v, u) + \mu_1^R(v)), (\sum_{u \neq v} \gamma_2^L(v, u) + \gamma_1^L(v), \sum_{u \neq v} \gamma_2^R(v, u) + \gamma_1^R(v))]$$

$$= \sum_{v \in V} [(\sum_{u \neq v} \mu_2^L(v, u), \sum_{u \neq v} \mu_2^R(v, u)) + (\sum_{v \in V} \mu_1^L(v), \sum_{v \in V} \mu_1^R(v)),$$

$$\sum_{v \in V} (\sum_{u \neq v} \gamma_2^L(v, u), \sum_{u \neq v} \gamma_2^R(v, u)) + (\sum_{v \in V} \gamma_1^L(v), \sum_{v \in V} \gamma_1^R(v))]$$

That is $[(pc_1, pc_2), (pc_1, pc_2)] = 2 S(G) + O(G)$ where $p = O(G)$

Theorem 4.6: Let G be Strong IVIFG , $\mu_1^L + \gamma_1^L$ and $\mu_1^R + \gamma_1^R$ are constant functions for all vertices then $\mu_2^L + \gamma_2^L$ and $\mu_2^R + \gamma_2^R$ are also with same constant functions for all edges, but the converse need not true.

Proof:

Suppose, $\mu_1^L + \gamma_1^L = c_1$ and $\mu_1^R + \gamma_1^R = c_2$ for all the vertices.

Since given is strong IVIFG, we get $\mu_2^L(v_i, v_j) = \min[\mu_1^L(v_i), \mu_1^L(v_j)]$,
 $\mu_2^R(v_i, v_j) = \min[\mu_1^R(v_i), \mu_1^R(v_j)]$ and $\gamma_2^L(v_i, v_j) = \text{Max}[\gamma_1^L(v_i), \gamma_1^L(v_j)]$,
 $\gamma_2^R(v_i, v_j) = \text{Max}[\gamma_1^R(v_i), \gamma_1^R(v_j)]$

So, we get $\mu_2^L + \gamma_2^L = c_1$ and $\mu_2^R + \gamma_2^R = c_2$ for all the edges.

Conversely, for the edges, the maximum is taken from either two vertices and then the maximum value may be differ in the one of the vertices and $\mu_2^R + \gamma_2^R = c_2$, but $\mu_1^R + \gamma_1^R$ may not equal to c_2 for all vertices. Also

$\mu_1^L + \gamma_1^L$ may not equal to c_1 for all vertices.

Therefore, the converse need not true.

Example 4.7:

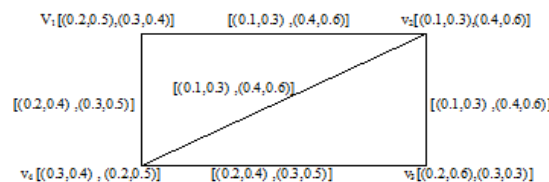


Fig -2: Strong IVIFG with $\mu_1^L + \gamma_1^L$ and $\mu_1^R + \gamma_1^R$ are constant functions

Here, $\mu_1^L + \gamma_1^L = 0.5$ and $\mu_1^R + \gamma_1^R = 0.9$ for all vertices, Then we get for all edges, $\mu_2^L + \gamma_2^L = 0.5$ and $\mu_2^R + \gamma_2^R = 0.9$.

Example 4.8:

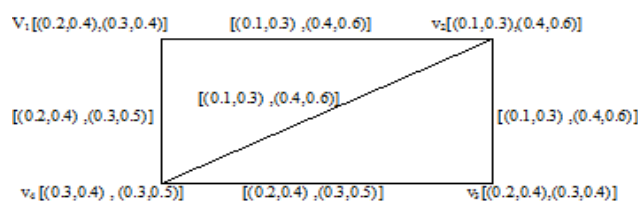


Fig -2: Strong IVIFG with $\mu_2^L + \gamma_2^L$ and $\mu_2^R + \gamma_2^R$ are constant functions

Here, $\mu_2^L + \gamma_2^L = 0.5$ and $\mu_2^R + \gamma_2^R = 0.9$ for all edges, but $\mu_1^L + \gamma_1^L \neq 0.5$ and $\mu_1^R + \gamma_1^R \neq 0.9$ for all vertices.

Proposition 4.9:

For an Interval valued intuitionistic fuzzy graph G, It is impossible that $\mu_1^L + \gamma_1^L$ and $\mu_1^R + \gamma_1^R$ are equal for any vertices, Also $\mu_2^L + \gamma_2^L$ and $\mu_2^R + \gamma_2^R$ are not also equal for any edges.

Proof: In any IVIFG, membership and non-membership values are intervals and in the form $\mu_1^L < \mu_1^R$ and $\gamma_1^L < \gamma_1^R$ respectively. So, $\mu_1^L + \gamma_1^L$ and $\mu_1^R + \gamma_1^R$ are not equal for any intervals. Similarly, for the edges also we get $\mu_2^L + \gamma_2^L$ and $\mu_2^R + \gamma_2^R$ are not equal.

V. Conclusion

Here we defined order and size of IVIFG and verify the property that sum of degrees of vertices is equal to twice the size of the graph. Also, constant IVIFGs and totally constant IVIFGs are defined and some results are proved which is very rich both in theoretical developments and applications. In future, more theorems and results of IVIFGs will be proposed.

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