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Variational Inequalities Approach to Supply Chain Network Equilibrium

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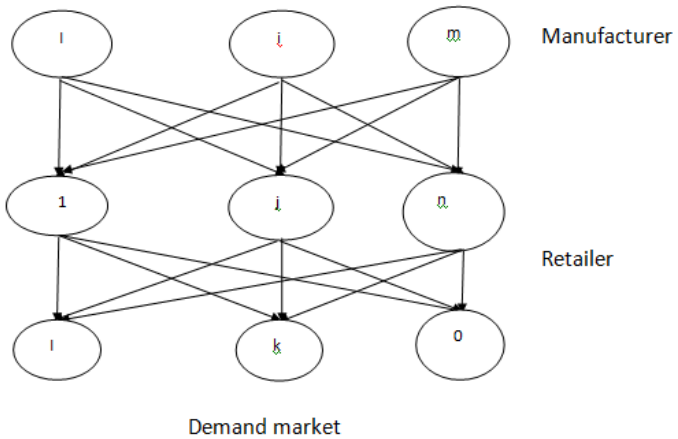
Abstract: *in this work, we examine a supply chain network consisting of manufacturer , retailer and the demand market , we study the behaviour of the various decision – maker and formulate an optimization problem based on the condition proposed in this decision maker and transcribed the optimization problem to variational inequality because of its handiness in solving equilibrium problem. and derived equilibrium condition that satisfy the manufacturer, retailers and the demand market, these conditions must be simultaneously satisfied so that no decision- maker has any incentive to alter his transaction.*

Key words: *supply chain network, variational inequality. Equilibrium ,Convex Optimization*

I. Introduction

Supply Chain Network: A Supply chain network is a goal – oriented network of processes used to deliver goods and service Anpindi & Bassok (1996). Supply chain network are critical infrastructure for the production, distributions and consumption of goods as well as services in our globalized network economy. Chopra & Meindl (2003) Supply chain has emerged as the backbones of economics activities in the modern world. Their importance to the timely and efficient delivery of products as varied as food, energy pharmaceuticals, clothing, and computer hardware has fuelled an immense interest in the analysis on the pact of both researchers and practitioners By necessity science is reductionist, all real world system are two complex to study in their totality. So scientist reduce them to a manageable size by restricting their slope and making simplify assumption to get anywhere with science of supply chain they exist only to support business activities and therefore must be evaluated in business terms, hence the fundamental objective of a supply chain is to contribute to long term profitability. A decentralize supply chain network model is one of the prevailing structure adopted which usually consist of different tiers of decision makers. Chopra & Meindi (2003) common definition of supply chain is a network of facilities that function to procure material , transform thatmaterialinto finished productand distribute to customer , a supply chain include all the companies and their associated business activities that are needed for the design, manufacture and distribution .

Figure 1. The Network structure of the supply chain



The objective of every supply chain is to maximize the overall value generated. the value a supply chain generate is the difference between what the final product is worth to the customer .Also supply chain is an integrated system or network which synchronize a series of inter – related business process in order to acquire row material and add value to the raw material by transforming them unto finish product and facilitate the flow of information among the various partner. Supply chain facility location problem are always considered as the most critical and most complex of issue to run an efficient supply chain, usually changes of othercomponent in

supply chain such as transportation and information system can be made with less hassle due to relatively bigger amount of choice. In other words.These types of decision can be readily re- optimized in response to changes.
A common definition of supply chain is a network of facilities that function to procure material , transform that material into finished product and distribute to customer , a supply chain include all the companies and their associated business activities that are needed for the design, manufacture and distribution .

Many classical economics equilibrium problems have been formulated as a system of equation, since market clearing conditions necessary equate the total supply with the total demand In terms of a variation inequality problem This chapter discusses the definition and analysis aspect that are necessary for a conceptualization of general applicability for supply chain network, and approach behind concept of optimality condition of manufacturer, retailer and consumer. Variational inequality formulation as a facilitator to supply chain network equilibrium.

II. Introduction Of Equilibrium Model.

In the classical circumstance we address there are agent called consumer, index by $k = 1 \dots \dots o$ along with agent called producer index by $= 1 \dots \dots . m$. both deal with goods indexed by $j = 1 \dots \dots . . n$, vector $R^{k;}$ having component that stand for quantities of these goods will be involved in both consumption and production for each consumer k there is a consumption set $X_i c \mathbf{R}^k$, whereas for each producer i there is a production set $Y_i C \mathbf{R}^k$ consumer k will chose a consumption vector $x_i \epsilon X_k$ and producer i will chose a production vector $y_i \epsilon Y_j$. consumption vector x_i are related by agent i according to their utility, which is described by a function $u_i on X_{i\,;}$ the higher the utility value $u_i(x_i)$ the better. Obviously the chosen $x_i s and y_i s$ must turn out to be such that the total consumption $\sum_{i=1}^{1} x_\mathrm{i}$ does not exceed the total (net) production $\sum_{i=1}^{j} y_\mathrm{j}$ plus the total endowment $\sum_{i=1}^{1} e_\mathrm{i}$. The coordination is to be achieved by a market in which goods can be traded at particular prices. The prices are not part of the given data, hence they must be determined from interaction of the consumer, retailer and producer over availabilities and preference .this is why finding equilibrium is much more than just a matter of optimization. In this study we developed a fixed demand version of the supply chain network model , the model consist of m manufacturer ,n retailer o demand market in figure (1) we denote a typical manufacturer by i and typical retailer by j and typical demand market by k .The links in the supply chain network represent transportation links. The equilibrium solution is denoted by $(*)$ all vector are assumed to be column vector. Therefore we first show the behaviour of the manufacturer and retailers, were then discus the behaviour of the consumer at the demand market. Finally we state the equilibrium condition for the supply chain network

The fundamental problem of optimization is to arrive at the best possible decision in any given set of circumstance. In real life problem situation may arise where the best is unattained .the first step in mathematical optimization is to set up a function whose variable are the variable interest. The function could be linear or non-linear. This function is better maximized or minimized subject to one or more constrained equation.

This is optimization problem in standard form.

Min/Max $f(x) \dots \dots \dots \dots \dots \dots \dots$........................ equation (1)

s.t $f_i(x) \leq 0 \;\; i = 1 \dots \dots \dots . . m$

$f_i(x) \quad = 0 \;\; i \; = 1 \dots \dots \dots . . p$

Therefore $x \in \mathbf{R}^n$ is the optimization variable

f_0 : $R^n \rightarrow \mathbf{R}$ is the objective or cost function.

f_i : $\mathbf{R}^n \rightarrow \mathbf{R}$; $i = 1 \dots \dots m$are the inequality constraint

h_i: $\mathbf{R}^n \rightarrow \mathbf{R}$ are the equality constraint.

An optimization problem is characterized by its given specific objective function that is maximize or minimized given set of constraint possible objective function include expression representing profit cost , market share , both constrained and unconstrained problem can be formulated as variational inequality problem the subsequent two preposition and theorem identify the relationship between an optimization problem and variational inequality problem.

The next theorem provides the basis of the relationship that exist between optimization problem and variational inequality problem.

Theorem 1

Let x^*be a solution to the optimization problem

Min$f(x) \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots . . (4)$

s.t$x \in K$

Where $f$is continuously differentiable and k is close and convex , then x^* is a solution of the variational inequality problem

$$\nabla f(x^*), (x - x^*) \geq 0 \forall x \in K \ldots\ldots\ldots\ldots\ldots. (5)$$

Proof.
Let $\emptyset(t) = f(x^* + t(x - x^*))\ fort\varepsilon[0\ 1]$
Since $\emptyset(t)$ achieve its minimum at $t = 0$

$$0 \leq \emptyset^{!}(0) = \nabla f(x^*).(x - x^*) ie$$

x^*Is a solution of$\nabla f(x^*)T.(x - x^*) \geq 0$
Theorem 2
If $f(x)$ is a convex function and x^* is a solution to $VI(\nabla f, k)$ then x^* is a solution to the optimization problem
Min $f(x) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$
s.t$x\epsilon K$
Proof: since $f(x)$ is convex

$$f(x) \geq f(x^*) + \nabla f(x^*).(x - x^*) \forall x \epsilon k \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

But $\nabla f(x^*)^{T}.(x - x^*) \geq 0$
Since x^* is solution to $VI(\nabla f, k)$
Therefore from (6) we conclude that $f(x) \geq f(x^*) \forall x \epsilon k$

Variational inequality theory : (Variational Inequality). The finite – dimensional variational inequality problem $VI\ (f, k)$ is to determine a vector $x^* \epsilon kc\mathbf{R}^N$ such that
$\langle f(X^*), X - X^* \rangle \geq 0\ \forall x \epsilon K \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$

Where f is a given continuous function from $KtoR^N$ K is a given closed convex set and $\langle .. \rangle$ denote the inner product in $\mathbf{R}$ Or equivalently$\langle F(x^*)T, (x - x^*) \rangle \geq 0\ \forall x \epsilon k$ where f is a given continuous function from K is close and convex set and $\langle .. \rangle$ denote the inner product in $n - dimensionalspace$. Variational inequality are closely related with many general non linear analysis. Such as complementarity, fixed point and optimization problem. The simplest example is variational inequality is the problem of solving a system of equation, the variational inequality is the problem of finding a point $u^* \epsilon K$ such that.$\langle G(u^*), u - u^* \rangle \geq 0\ \forall u \epsilon K \ldots\ldots\ldots. (9)$

2.1 The **Manufacturer Behaviour And Optimality Condition**

Let p^*_{1ij} denote the price charge for the product by manufacturer i in transacting with retailer j the price P^*_{1ij} is an endogenous variable and will be determined once the entire supply chain network equilibrium model is solved .we assume that the quantity produce by manufacturer i must satisfy the following conservation of flow of equation

$$q_i = \sum_{j=1}^{n} q_{ij} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Which state that the quantity of product produced by manufacturer i is exactly equal to the sum of quantities transacted between a manufacturer and the retailer, the production cost function f_i for each manufacturer i ; $i \ldots\ldots\ldots. m$ be expresses as a function of the flows? Q^1, Assuming that the manufacturer are profit maximize, we can express the optimization problem as

$$\max \sum_{j=1}^{n} p^*{}_{ij} q_{ij} - f_i(Q^1) - \sum_{j=1}^{n} c_{ij}(q_{ij})$$

$$s.t : q_{ij} \geq 0 \text{ for all } j; j = 1, \ldots.. n$$

the first term in equation (16) represent price charge , and subsequent terms the product cost and transaction cost for each manufacturer are continuously differentiable and convex , the optimality condition for all manufacturer i can be expressed as the following variational inequality determine $Q^1 \epsilon R^{mn}_{+}\ satisfy$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial f_i(Q^{1*})}{\partial q_{ij}} + \frac{\partial c_{ij}(q^*_{ij})}{\partial q_{ij}} - p^*_{1ij}\right] \times \left[q_{ij} - q^*_{ij}\right] \geq 0 \forall Q^1 \in R^{mn}_{+} \ldots..(17)$$

2.2 Retailer Behaviour And Their Optimality Condition.

The retailers in turn purchase the product from the manufacturer and transact with the customer at the demand market. Thus a retailer is involved in transacting both with the manufacturer as well as with the demand market. Let p_{2j}^* denote the price charge by he retailer j ,for the product. This price will be determined endogenously after the model is solved. We assume also that the retailer are profit maximize, hence the optimization problem faced by a retailer $j$is given by

$$\max \sum_{k=0}^{n} p_{2j}^* \, q_{jk} - c_j(Q^1) - \sum_{i=1}^{m} p_{1ij}^* \, q_{ij} \; \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

$$s.t \sum_{k=1}^{0} q_{jk} \leq \sum_{i=1}^{m} q_{ij} \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Where$q_{ij} \geq 0 \; and q_{jk} \geq 0 \; for all i ; i = 1 \ldots\ldots m and k ; k = 1 \ldots 0$ the first term in (18) represent price charge and the second and third term represent handling cost and payout cost to the manufacturer . hence constrain (19) expresses that total quantity of the product transacted with the demand market by a retailer cannot exceed the amount that the retailer has obtained from the manufacturer. Also handling cost for each retailer is continuously differentiable and convex. Then the optimality condition for all the retailer can be expressed as the following variation inequality

Determine $(Q^{1*}, Q^{2*}, \gamma^*) \in R^{mn+no+n} satisfying$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial c_j(Q^{1*})}{\partial q_{ij}} + p^*_{1ij}\right]\times\left[q_{ij} - q^*_{ij}\right] + \sum_{j=1}^{n}\sum_{k=1}^{o}\left[-p^*_{2j} + \gamma_j^*\right]\times\left[\left[q_{jk} - q^*_{jk}\right]\right] +$$

$$\sum_{j=1}^{n}\left[\sum_{i=1}^{m} q_{ij}^* - \sum_{k=1}^{o} q_{\partial jk}^*\right]\times\left[\gamma_j - \gamma_j^*\right] \geq 0 \forall (Q^1, Q^2, \gamma) \in R^{mn+no+n} \ldots\ldots\ldots.(20)$$

The term γ_j is the Lagrange multiplier or shadow price associated with constraint (19) for retailer j and γ is the n – dimensional vector of all the shadow prices.

2.3 **The Consumer Demand Market And Equilibrium Condition.**

The consumer take account the price charged by the retailer and the unit transaction cost incurred to obtain the product in making their consumption decision, we assume dynamic model in which we allow the demand to be time varying. The following conservation of flow equation must hold

$$d_k = \sum_{j=1}^{n} q_{jk}, k = 1, \ldots\ldots o$$

Where d_k is fixed for each demand market K. We assume the unit transaction cost function c_{jk} are continuous function for $j ; j = 1 \ldots\ldots n and k; k = 1 \ldots\ldots 0$

The equilibrium condition for consumer at the demand market k then take the form for each retailer $j ; j = 1 \ldots\ldots\ldots n$

$$p_{2j}^* + c_{jk}(Q^{2*}) \begin{cases} = p_{3k}^* , if \, q_{jk}^* \geq 0 \\ \geq p_{3k}^* if \, q_{jk}^* = 0 \end{cases} \ldots\ldots\ldots\ldots\ldots. (22)$$

Equation (22) state that in equilibrium, if the consumer at demand market k purchase the product from retailer j then the price the consumer pay is exactly equal to the price charge by retailer plus the unit transaction cost .however if the sum of the prince charge by the retailer and the unit transaction cost exceed the price that the consumer is willing to pay at the demand market, there will be no transaction. In equilibrium condition (22) must hold simultaneously for all demand market, we express the equilibrium condition as the variation inequality determine $Q^{2*} \varepsilon k^1$ such that $\sum_{j=1}^{n}\sum_{k=1}^{o}\left[p_{2j}^* + c_{jk}(Q^{2*})\right]\times\left[q_{\partial jk} - q_{jk}^*\right] \geq 0$

$$\forall Q^2 \in K^1,$$

Where $K^1 \equiv Q^2 \in R_+^{no} and (21) hold \ldots\ldots\ldots\ldots(23)$

In equilibrium the optimality condition of the entire manufacturer, the optimality condition for all the retailers and the equilibrium condition for all the demand market must be simultaneously satisfied so that no decision maker has any incentive to alter his transaction.

Definition (3) Supply Chain Network Equilibrium.

The equilibrium state of the supply chain network is one where the product flows between the tiers of network coincide and the product flows satisfy the conservation of flow equation of the sum of optimality condition of manufacturer, the retailer and the equilibrium condition at the demand markets.

3.4 Variational Inequality Formulation Of Supply Chain Network Equilibrium: The equilibrium condition governing the supply chain network according to definition 3 coincide with the solution of the (finite – dimensional) Variational inequality given by:

determine, $(Q^{1*}, Q^{2*}, \gamma^*) \in K^2 satisfying$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial f_i(Q^{1*})}{\partial q_{ij}} + \frac{\partial c_{ij}(q^*_{ij})}{\partial q_{\partial ij}} + \frac{\partial c_j(Q^{1*})}{\partial q_{ij}} - \gamma^*_j\right]\times\left[q_{ij} - q^*_{ij}\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\left[c_{jk}(Q^{2*}) + \gamma^*_j\right] + \left[q_{jk} - q^*_{jk}\right] + \sum_{j=1}^{n}\left[\sum_{i=1}^{m} q_{ij}{}^* - \sum_{k=1}^{o} q^*_{jk}\right]\times\left[\gamma_j - \gamma^*_j\right] \geq 0$$

$$\forall (Q^1, Q^2, \gamma) \in K^2$$

Where

$$K^2 \equiv \left\{Q^1, Q^2, \gamma\right\} \in R^{mn+no+n} and (21) holds........(24)$$

Proof : We first demonstrate that an equilibrium pattern according to definition (3) satisfies the variational equality, summing up inequalities 17, 20 and 23, after algebraic simplication we obtain (24). We now show the converse, that is a solution to Variation inequality (24) satisfies the sum of condition 17, 20, & 23 and is therefore a supply chain network equilibrium pattern first we add the term $-p^*_{1ij} + p^*_{1ij}$ to the first term in the first summed expression in (24), then, we add the term $-p^*_{2j} + p^*_{2j}$ to the first term in the second summed expression in (24) because these term are all equal to zero , they do not change(24) we obtain the following inequality

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial f_i(Q^{1*})}{\partial q_{ij}} + \frac{\partial c_{ij}(q^*_{ij})}{\partial q_{ij}} + \frac{\partial c_j(Q^{1*})}{\partial q_{ij}} - \gamma^*_j + p^*_{1ij} - p^*_{1ij}\right]\times\left[q_{ij} - q^*_{ij}\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\left[c_{jk}(Q^{2*}) + \gamma^*_j + p^*_{2j} - p^*_{2j}\right]\times\left[q_{jk} - q^*_{jk}\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m} q^*_{ij} - \sum_{k=1}^{o} q^*_{jk}\right]\times\left[\gamma_j - \gamma^*_j\right] \geq 0 \forall (Q^1, Q^2, \gamma) \in K^2........(25)$$

Which can be rewritten as?

$$\sum_{i=1}^{m}\sum_{i=1}^{n}\left[\frac{\partial f_i(Q^{1*})}{\partial q_{ij}} + \frac{\partial c_{ij}(q^*_{ij})}{\partial q_{ij}} - p_{1ij}{}^*\right] X\left[q_{ij} - q_{ij}{}^*\right] +$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial c_j(Q^{1*})}{\partial q_{ij}} + p_{1ij}{}^* - \gamma^*_j\right] X\left[q_{ij} - q_{ij}{}^*\right] +$$

$$\sum_{j=1}^{n}\sum_{k=1}^{o}\left[-p_{2ij}{}^* + \gamma^*_j\right] X\left[q_{jk} - q_{jk}{}^*\right] +$$

$$\sum_{j=1}^{n}\left[\sum_{i=1}^{m} q^*_{ij} - \sum_{k=1}^{o} q_{jk}{}^*\right] + \sum_{j=1}^{n}\sum_{k=1}^{o}\left[p_{2j} + c_{jk}(Q^{2*})\right] X\left[q_{jk} - q^*_{jk}\right] \geq 0$$

$$\forall (Q^1, Q^2, \gamma) \in K^2....................(26)$$

Clearly (26) is equal to the sum of the optimality conditions of manufacturer, retailer and the consumer demand market (23) and is, hence variational inequality governing the supply chain network equilibrium in definition (3) Hence the Variational inequality problem of (24) can be written in standard variational inequality form as follows: determine $X^{*}\epsilon k$ satisfying

$\left\langle f(x^{*}), x-x^{*}\right\rangle \geq 0 \forall x \in K.............(27)$ where$X \equiv (Q^1, Q^2, \gamma,)$ and $f(x) \equiv (f_{ij}, f_{jk}, f_j, f_k) i = 1 ..... m; j = 1 ..... n; k = 1 ... o.$

3.41 Corrolary

This corollary established that in equilibrium, the supply chain with fixed demand, since we are interested in the market clearing ,the market for the product clears for each retailer in the supply chain network equilibrium that is

$$\sum_{i=1}^{m} q_{ij}^{*} = \sum_{j=1}^{n} q_{jk}^{*} \, for j = 1 \dots\dots n$$

Clearly from equation (19) if $\gamma_j^{*} > 0$ $then \sum_{i=1}^{m} q_{ij}{}^{*} = \sum_{k=1}^{o} q_{jk}{}^{*}$ h$olds$ now we consider the case where $\gamma_j^{*} = 0$ for some retailer j we examine the first term in inequality (19) since we assume that the production cost function , the transaction cost function and the handling cost function are convex , it is therefore to assume that either the marginal production cost or the marginal transaction cost or the marginal handling cost for each manufacturer /retailer is strictly positive at equilibrium.

Then $\dfrac{\partial f_i(Q^{1*})}{\partial q_{ij}} + \dfrac{\partial c_{ij}(q_{ij}{}^{*})}{\partial q_{ij}} + \dfrac{\partial c_j(Q^{1*})}{\partial q_{ij}} > 0$

Which implies that $q_{ij}^{*} = 0$ $forall i for that j,$ it follows then from the third term in (19) that $\sum_{k=1}^{0} q^{*}{}_{jk} = 0$ Hence we have that $\sum_{k=1}^{0} q^{*}{}_{jk} = 0 = \sum_{i=1}^{m} q_{ij}{}^{*}$. for any j such that $\gamma^{*}{}_{j} = 0$ there we conclude that the market clears for retailer in the supply chain equilibrium, the price that exist when a market is in equilibrium, the equilibrium price equate the quantity demanded and quantity supply, more over the equilibrium price is simultaneously equal to both the demand price and supply price, the equilibrium price is also commonly referred to as the market clearing price. Since we are interested in the determination of equilibrium flow and price, we transform constrain (19) into

$$\sum_{k=1}^{o} q_{jk} = \sum_{i=1}^{m} q_{ij} ....(28)$$

Now we can define the feasible set as $K^{3} \equiv (Q^1, Q^1) \in R_{+}^{mn+no}$ such that (27) holds

For notational convenience we let $S_j \equiv \sum_{k=1}^{o} q_{jk}, \; j = 1, ...n$

Note that, special case of demand function and supply function that are separable, the jacobians of these function are symmetric since they are diagonal and given, respectively by

$$\nabla s(p) = \begin{bmatrix} \frac{\partial s_1}{dp_1} & 0 & 0 & 0 \\ 0 & \frac{\partial s_2}{dp_2} & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial s_n}{dp_n} \end{bmatrix}$$

$$\nabla \mathrm{d}(p) = \begin{bmatrix} \frac{\partial d_1}{dp_1} & 0 & 0 & 0 \\ 0 & \frac{\partial d_2}{dp_2} & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial d_n}{dp_n} \end{bmatrix}$$

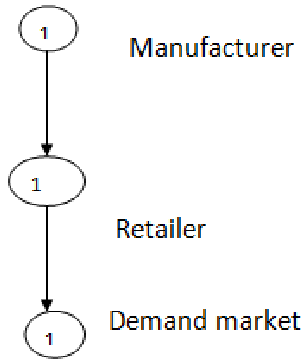
In this special case model the supply of a product or commodity depends only upon the price of that commodity while the demand for a commodity depends only upon the price of that commodity. Hence the price vector p^* that satisfies the equilibrium condition can be obtained by solving the following optimization problem

$max/min \sum_{i=1}^{n} \int_0^{p_i} S_i(x)dx - \sum_{i=1}^{n} \int_0^{p_i} d_i(y)dy$......................(34)

$subjecttop_i \geq 0 \ , i = 1 \ldots \ldots . n$

Hence the quantity of product demanded from demand market is equal to the quantity of product supply.

Example 1, We consider a network structure consist of single manufacturer, a single Retailer and Single demand market as show below



The manufacturer production cost function is given by $f_i(q) = q_1^2 + 5q_1$

The transaction cost between the manufacturer and retailer pair is $c_{11}(q_{11}) = q_{11}^2 + 2q_{11}$

The handling cost at the Retailer is $c_1(Q^1) = 2(q_{11})^2$ and the unit transaction cost associated with the transaction between the retailer and demand market is $c_{11}(Q^1) = q_{11} + 1$ with the demand function given by $d_1(p_3) = -p_{31} + 100$ view of these function , the corollary which show that the equilibrium product flow between the manufacturer and the retailer is equal to the equilibrium product between the retailer and the demand market , therefore Variational inequality (24) of this example takes the form determine $(q_{11}^*, \gamma_1^*, \rho_{31}^*)$ (all the value being nonnegative) and satisfying

$[2q_{11}^* + 5 + 2q_{11}^* + 2 + 4q_{11}^* - q_{11}^*] \times [q_{11} - q_{11}^*]$ $+ [q_{11}^* + 1 + \gamma_1^* - p_{31}^*] \times [q_{11} - q_{11}^*] +$ $[q_{11}^* - q_{11}^*] \times [\gamma_1 - \gamma_1^*] + [q_{11}^* + \rho_{31}^* - 100] \times [\rho_{31} - \rho_{31}^*] \geq 0 \ \forall q_{11} \geq 0 \ \forall \gamma_1 \geq 0 \ \forall \rho_{31} \geq 0$

Combining the like terms in this inequality, after algebraic simplication yields

$$[9q_{11}^* + 8 - p_{31}^*][q_{11} - q_{11}^*] + [q_{11}^* + p_{31}^* - 100] \times [p_{31} - p_{31}^*] \geq 0$$

Now if we let $q_{11} = q_{11}^*$, we assume that the equilibrium demand market price p_{31}^* is positive , then substituting into the inequality above gives

$$p_{31}^* = 1\,00 - q_{11}^*$$

Therefore letting $p_{31} = p_{31}^*$, substituting now into the same inequality, with$p_{31}^* = 100 - q_{11}^*$ gives us $[9q_{11}^* + 8 - 100 + q_{11}^*] \times [q_{11} - q_{11}^*] \geq 0$ Assuming that the product flow q_{11}^* is positive (so that the manufacturer produces the product) we can conclude that $10q_{11}^* - 92 = 0$ hence $q_{11}^* = 9.20$

Then the demand market price is $p_{31}^* = 90.80$, the shadow price γ_1^* can be determined from the above variational inequality , we obtain $\gamma_1^* = 80.60$

III. Conclusion:

In conclusion this seminar work has succeeded in clarifying the use of variational inequality as a tool in supply chain network for finding the optimality conditions for these three tiers of decision- maker and their equilibrium condition. Such that each must be simultaneously satisfied so that no decision- maker has any incentive to alter his transaction and provides a market clearing condition.

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