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Fuzzy Inventory Model for Constantly Deteriorating Items with Power Demand and Partial Backlogging

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Abstract: *In this paper a fuzzy inventory model is developed for deteriorating items with power demand rate. Shortages are allowed and partially backlogged. The backlogging rate of unsatisfied demand is assumed to be a decreasing exponential function of waiting time. The cost components are considered as triangular fuzzy numbers. The objective of this paper is to develop an inventory model in a fuzzy environment, minimize the total cost and thereby derive optimal ordering policies. The total cost is defuzzified using Graded mean representation, signed distance and centroid methods. The values obtained by these methods are compared with the help of numerical examples. The convexity of the cost function is depicted graphically. Sensitivity analysis is performed to study the effect of change of some parameters.*

Keywords: *Centroid Method, Defuzzification, Deterioration, Graded mean representation method, Inventory, Partial backlogging, Power Demand, Shortages, Signed Distance Method, and Triangular Fuzzy Number.*

I. Introduction

Inventory control has a pivotal role in any business organization. Hence much of focus of any business is on inventory control. There are no standard strategies for inventory maintenance, since the conditions at each business or firm are unique and include many different features and limitations. There are many factors affecting the inventory, of which the main factors are deterioration, demand and the cost.

Deterioration of items in stock refers to the items in the stock that become decayed or damaged and hence cannot be used for supply to the customers. Deterioration of physical goods in an inventory is realistic and its impact on the inventory management is significant. Consequently, while determining the optimal inventory policy of deteriorating products, the loss due to deterioration cannot be ignored. In the literature of inventory theory, the deteriorating inventory models have been continually modified so as to accumulate more practical features of the real inventory systems.

The analysis of deteriorating inventory began with Ghare and Schrader [7], who established the classical no-shortage inventory model with a constant rate of decay. Shah and Jaiswal [20], proposed an order-level inventory model for a system with constant rate of deterioration. Dave and Patel [5] proposed inventory model for deteriorating items with time proportional demand. A Review on deteriorating inventory study was made by Ruxian Li et al., [11]. Later Vinod Kumar Mishra and Lal Sahab Singh [14] developed a deteriorating inventory model with time dependent demand and partial backlogging. Recently an inventory model for deteriorating items with time-dependent demand and time-varying holding cost under partial backlogging was proposed byVinod Kumar Mishra1 et.al.,[15].

Demand being a key factor of any inventory situation, has a high impact on inventory policies. Naddor [17] analyzed various components and properties of inventory systems and pointed out that demand process probably represents the most important properties of an inventory system. He introduced the power demand pattern. It is an interesting property to depict the evolution of the inventory level. This demand pattern recognizes and models numerous ways by which quantities are taken out of inventory. This pattern generally represents the behavior of demand when it is uniformly distributed throughout the period, and also models situations where a high percentage of units may be withdrawn either at the beginning or at the end of a period. Several research articles involving power demand have been published. Following Naddor, Goel and Aggawal [8] developed an order-level inventory model with power demand for deteriorating items. Datta and Pal [4] studied an inventory system with power demand and variable rate of deterioration. Lee and Wu [12] presented an EOQ model assuming deterioration, shortages and power demand pattern. Dye [6] extended the Lee and Wu model to a general class with time-proportional backlogging rate. Singh et al. [22] developed an EOQ model for perishable items with power demand pattern and partial backlogging. Rajeswari and Vanjikkodi [18] proposed an inventory model for deteriorating items with partial backlogging and power demand pattern. Rajeswari and Vanjikkodi [19] analyzed an inventory model for Weibull deteriorating items with power pattern time dependent demand. Mishra and Singh [16] presented an EOQ model for deteriorating items with power demand pattern and shortages partially backlogged. Recently, Sicilia et al. [21] studied an inventory model for deteriorating items with shortages and time varying demand following a power demand pattern. They assumed a complete backlogging of orders.

The next factor is the cost, that are varies from one cycle to another cycle. In such situation the Fuzzy set theory is a marvelous tool for modeling the kind of uncertainty associated with vagueness, with imprecision and/or with a lack of information regarding a particular element of the problem at hand. The fuzzy set theory was first introduced by Zadeh[25] and has now been applied in different inventory control systems to model their behavior more realistically.

Kacprzyk and Staniewski[10] applied the fuzzy set theory to the inventory problem and considered long term inventory policy through fuzzy decision making model. An algorithm was formulated to determine the optimal time-invariant strategy. Later several scholars have developed the Economic Order Quantity (EOQ) inventory problems using fuzzy sets .Chen et al [3] introduced a fuzzy inventory model with backorders by taking fuzzy demand, fuzzy ordering cost and fuzzy backorder cost. A modified fuzzy model for inventory with fuzzified back order quantity using triangular fuzzy number was presented by Chang *et al.*[1]. Yao and Chiang [24] introduced an inventory without back order with fuzzy total cost and fuzzy storing cost defuzzified by centroid and signed distance. They compared the optimal results obtained by both the defuzzification methods. Chang et al.[2] developed the mixture inventory model involving variable lead-time with backorders and lost sales. Fuzzify the total demand to be the triangular fuzzy number and derive the fuzzy total cost. By the centroid method of defuzzification, they estimate the total cost in the fuzzy sense. Lin [13] developed the inventory problem for a periodic review model with variable lead-time and fuzzified the expected demand shortage and backorder rate using signed distance method to defuzzify. C. K. Jaggi, S. Pareek, A. Sharma and Nidhi [9] presented a fuzzy inventory model for deteriorating items with time-varying demand and shortages. Very recently Sushil Kumar and Rajput [23] proposed fuzzy inventory model for deteriorating items with time dependent demand and partial backlogging.

In this paper, we formulate a crisp inventory model involving constantly deteriorating items with power demand, where shortages are allowed and are partially backlogged. Thereafter the model is fuzzified. The average total inventory cost in a fuzzy sense is derived. All cost parameters are fuzzified as the triangular fuzzy numbers. The fuzzy model is then defuzzified by using Graded mean representation, signed distance and centroid methods. The solution for minimizing the fuzzy cost function has been derived. A numerical example is given in order to show the applicability of the proposed models. The convexity of the cost function is shown graphically. Sensitivity analysis is also carried out to study the effect of some model parameters of the system.

II. Preliminaries

Definition 2.1 For a set A, a **membership function** μ_A is defined as

$$\mu_A(x) = \begin{cases} 1 & \text{if and only if} \quad x \in A \\ 0 & \text{if and only if} \quad x \notin A \end{cases}$$

Also the function μ_A maps the elements in the universal set X to the interval [0,1] and is denoted by $\mu_A : X \to [0,1]$

Definition 2.2 A **fuzzy number** is represented with three points as follows: $\tilde{A} = (a_1, a_2, a_3)$

This representation is interpreted as a membership function and holds the following conditions

(i) a_1 to a_2 is increasing function
(ii) a_2 to a_3 is decreasing function
(iii) $a_1 \le a_2 \le a_3$.

$$\mu_A(x) = \begin{cases} 0, & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \le x \le a_3 \\ 0, & x > a_3 \end{cases}$$

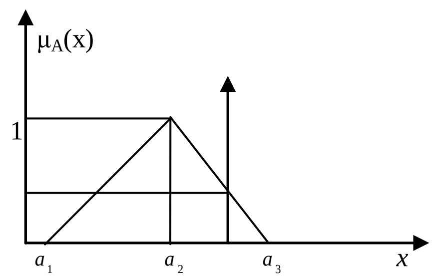

Figure 1. Triangular Fuzzy Number

Definition 2.3 The α - cut of $\tilde{A}$= (a, b, c)$\in F_N$, $0\le\alpha\le 1$, is defined by $A(\alpha) = [A_L(\alpha), A_R(\alpha)]$.
Where $A_L(\alpha) = a + (b-a)\alpha$ and $A_R(\alpha) = c - (c-b)\alpha$ are the left and right endpoints of $A(\alpha)$.

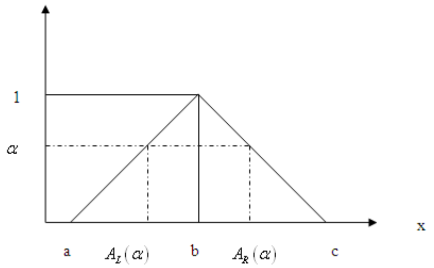

Figure 2. α –Cut of a Triangular Fuzzy Number

Definition 2.4 If $\tilde{A}$ = (a, b, c) is a triangular fuzzy number then the graded mean integration representation of $\tilde{A}$ is defined as

$$P(A) = \frac{\int_0^{w_A} h\left(\frac{L^{-1}(h)+R^{-1}(h)}{2}\right)dh}{\int_0^{w_A} h\,dh}$$

with $0 < h \le w_A$ and $0 < w_A \le 1$.

$$P(\tilde{A}) = (1/2)\frac{\int_0^1 h\left[a + h(b-a) + c - h(c-a)\right]dh}{\int_0^1 h\,dh} = \frac{a+4b+c}{6}$$

Definition 2.5 If $\tilde{A}$ = (a, b, c)is a triangular fuzzy number then the signed distance of $\tilde{A}$ is defined as

$$d(\tilde{A},\tilde{0}) = \int_0^1 d\left(\left[A_L(\alpha)_\alpha, A_R(\alpha)_\alpha\right],\tilde{0}\right) = \frac{1}{4}(a+2b+c)$$

Definition 2.6 The Centroid for a triangular fuzzy number $\tilde{A}$ = (*a*, *b*, *c)* is defined as

$$C(\tilde{A}) = \frac{a+b+c}{3}$$

III. Assumptions And Notations

The mathematical model is developed on the basis of the following assumptions and notations.

3.1 Notations

- A – Ordering cost per order.
- C – Purchase cost per unit.
- h – Inventory holding cost per unit per time unit.
- π_b – Backorder cost per unit short per time unit.
- π_l – Cost of lost sales per unit.
- π_d – Deterioration cost per unit
- t_1 – The time at which the inventory level reaches zero, $t_1 \ge 0$.
- t_2 – The length of period during which shortages are allowed, $t_2 \ge 0$.
- T – (=t_1+t_2) The length of cycle time.
- I_{MI} – The maximum inventory level during [0, T].
- I_{MB} – The maximum backordered units during stock out period.
- Q – (=I_{MI}+I_{MB}) The order quantity during a cycle of length T.
- $I_1(t)$ – The level of positive inventory at time t, $0 \le t \le t_1$.
- $I_2(t)$ – The level of negative inventory at time t, $t_1 \le t \le T$.
- TCUT– The total cost per time unit.
- $\tilde{C}$ – Fuzzy purchase cost per unit.
- $\tilde{h}$ – Inventory fuzzy holding cost per unit per time unit.
- $\tilde{\pi}_b$ – Fuzzy backorder cost per unit short per time unit.
- $\tilde{\pi}_l$ – Fuzzy cost of lost sales per unit.
- $\tilde{\pi}_d$ – Fuzzy deterioration cost per unit.
- $TCU\tilde{T}$ – Fuzzy total cost per time unit.

3.2 Assumptions

- The inventory system deals with single item.
- The demand rate $\frac{dt^{(1-n)/n}}{nT^{1/n}}$ at any time t, where d is a positive constant, n may be any positive number, T is the planning horizon. T=1.
- The deterioration rate θ is constant, 0 <θ<<1.
- The replenishment rate is infinite.
- The lead-time is zero or negligible.
- The planning horizon is infinite.
- When the inventory runs out of items, backlogging of orders take place. The backlogging rate is considered as a variable and depends on the length of the waiting time for the next replenishment. The proportion of the customers who are willing to receive the backlogged orders at time "t", is decreasing with the waiting time (T – t)for the next replenishment. In this case the backlogging rate is defined as $B(t) = \frac{1}{1+\delta(T-t)}$; δ> 0 denotes the backlogging parameter and $t_1 \le t \le T$.
- Purchase cost, holding cost, deterioration cost, back order cost and cost of lost sales are triangular fuzzy numbers.

IV. Mathematical Model

Suppose an inventory system consists of Q units of the product in the beginning of each cycle. Then the rate of change of inventory during positive stock period $[0,t_1]$ and shortage period $[t_1,T]$ are governed by differential equations.

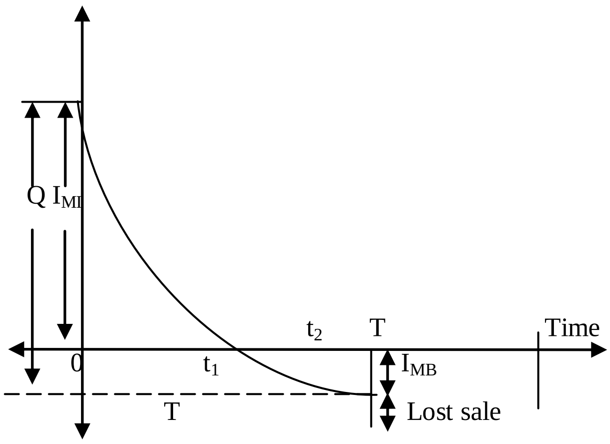

Figure 3. Representation of inventory system

4.1. Crisp Model

Inventory Level before Shortage Period

During the period $[0, t_1]$, the inventory depletes due to the demand and deterioration. Hence the inventory level $I_1(t)$ at any time t during the cycle $[0, t_1]$ is described by the differential equation

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -\frac{dt^{\frac{1-n}{n}}}{nT^{\frac{1}{n}}}, \qquad 0 \le t \le t_1 \tag{1}$$

With the boundary condition $I_1(t_1) = 0$ at $t = t_1$.

The solution of equation (1) is given by

$$I_1(t) = \frac{d}{T^{\frac{1}{n}}}\left[(1-\theta t)\left(t_1^{\frac{1}{n}} - t^{\frac{1}{n}}\right) + \frac{\theta}{1+n}\left(t_1^{\frac{1+n}{n}} - t^{\frac{1+n}{n}}\right)\right], \quad 0 \le t \le t_1 \tag{2}$$

Inventory level during shortage period

During the interval $[t_1,T]$ the inventory level depends on demand and a fraction of demand is backlogged. The state of inventory during $[t_1,T]$ can be represented by the differential equation,

$$\frac{dI_2(t)}{dt} = -\frac{\left(\dfrac{dt^{\frac{(1-n)}{n}}}{nT^{\frac{1}{n}}}\right)}{1+\delta(T-t)}, \qquad t_1 \le t \le T \tag{3}$$

With the boundary condition $I_2(t_1) = 0$ at $t = t_1$.

$$I_2(t) = -\frac{d}{nT^{\frac{1}{n}}}\left((1-\delta T)\left(t^{\frac{1}{n}} - t_1^{\frac{1}{n}}\right) + \frac{\delta}{1+n}\left(t^{\frac{1+n}{n}} - t_1^{\frac{1+n}{n}}\right)\right), \; t_1 \le t \le T \tag{4}$$

The maximum positive inventory is

$$I_{MI} = I_1(0) = \frac{d}{T^{\frac{1}{n}}}\left[t_1^{\frac{1}{n}} + \frac{\theta t_1^{\frac{1+n}{n}}}{1+n}\right] \tag{5}$$

The maximum backordered units are

$$I_{MB} = -I_2(T) = \frac{d}{T^{\frac{1}{n}}}\left[(1-\delta T)\left(T^{\frac{1}{n}} - t_1^{\frac{1}{n}}\right) + \frac{\delta}{1+n}\left(T^{\frac{1+n}{n}} - t_1^{\frac{1+n}{n}}\right)\right] \tag{6}$$

Hence, the order size during total time interval [0, T] is $Q = I_{MI} + I_{MB}$.

$$Q = \frac{d}{T^{\frac{1}{n}}}\left[T^{\frac{1}{n}} + \delta T t_1^{\frac{1}{n}} + \frac{1}{1+n}\left((\theta-\delta)t_1^{\frac{1+n}{n}} - \delta n T^{\frac{1+n}{n}}\right)\right] \tag{7}$$

Therefore the total cost per replenishment cycle consists of the following cost components.

Ordering cost per cycle

$$I_{OC} = A \tag{8}$$

Inventory holding cost per cycle

$$I_{HC} = h\int_0^{t_1} I_1(t)\,dt = \frac{hd}{T^{\frac{1}{n}}}\left[\frac{t_1^{\frac{1+n}{n}}}{1+n} + \frac{\theta t_1^{\frac{1+2n}{n}}}{2(1+2n)}\right] \tag{9}$$

Backordered cost per cycle

$$I_{BC} = \pi_b\int_{t_1}^{T}(-I_2(t))\,dt = \frac{\pi_b d}{T^{\frac{1}{n}}}\left[(\delta T^2 - T)t_1^{\frac{1}{n}} + \frac{nT^{\frac{1+n}{n}}}{1+n} + \frac{(1-2\delta T)t_1^{\frac{1+n}{n}}}{1+n} - \frac{2\delta n^2 T^{\frac{1+2n}{n}}}{(1+n)(1+2n)} + \frac{\delta t_1^{\frac{1+2n}{n}}}{1+2n}\right] \tag{10}$$

Cost due to lost sales per cycle

$$I_{LS} = \pi_l\int_{t_1}^{T}\left[1 - \frac{1}{1+\delta(T-t)}\right]\left(\frac{dt^{\frac{1-n}{n}}}{nT^{\frac{1}{n}}}\right)dt = \frac{\pi_l d\delta}{T^{\frac{1}{n}}}\left[\frac{nT^{\frac{1+n}{n}}}{1+n} - Tt_1^{\frac{1}{n}} + \frac{t_1^{\frac{1+n}{n}}}{1+n}\right] \tag{11}$$

Purchase cost per cycle

$$I_{PC} = C\times Q = \frac{Cd}{T^{\frac{1}{n}}}\left[T^{\frac{1}{n}} + \delta T t_1^{\frac{1}{n}} + \frac{1}{1+n}\left((\theta-\delta)t_1^{\frac{1+n}{n}} - \delta n T^{\frac{1+n}{n}}\right)\right] \tag{12}$$

Deteriorating cost per cycle

$$I_{DC} = \pi_d\left\{Q - \int_0^{t_1}\left(\frac{dt^{\frac{(1-n)}{n}}}{nT^{\frac{1}{n}}}\right)dt - \int_{t_1}^{T}\frac{1}{1+\delta(T-t)}\left(\frac{dt^{\frac{1-n}{n}}}{nT^{\frac{1}{n}}}\right)dt\right\} = \pi_d\frac{d\theta t_1^{\frac{1+n}{n}}}{(1+n)T^{\frac{1}{n}}} \tag{13}$$

Hence, the total cost per time unit is

$$TCUT = \frac{1}{T}\left[I_{OC} + I_{HC} + I_{BC} + I_{LS} + I_{PC} + I_{DC}\right]$$

$$\begin{aligned} TCUT = \frac{1}{T}\Bigg\{ & A + \frac{hd}{T^{\frac{1}{n}}}\left[\frac{t_1^{\frac{1+n}{n}}}{1+n} + \frac{\theta t_1^{\frac{1+2n}{n}}}{2(1+2n)}\right] + \frac{\pi_b d}{T^{\frac{1}{n}}}\left[(\delta T^2 - T)t_1^{\frac{1}{n}} + \frac{nT^{\frac{1+n}{n}}}{1+n} + \frac{(1-2\delta T)t_1^{\frac{1+n}{n}}}{1+n} - \frac{2\delta n^2 T^{\frac{1+2n}{n}}}{(1+n)(1+2n)} + \frac{\delta t_1^{\frac{1+2n}{n}}}{1+2n}\right] + \pi_d\frac{d\theta t_1^{\frac{1+n}{n}}}{(1+n)T^{\frac{1}{n}}} \\ & + \frac{\pi_l d\delta}{T^{\frac{1}{n}}}\left[\frac{nT^{\frac{1+n}{n}}}{1+n} - Tt_1^{\frac{1}{n}} + \frac{t_1^{\frac{1+n}{n}}}{1+n}\right] + \frac{Cd}{T^{\frac{1}{n}}}\left[T^{\frac{1}{n}} + \delta T t_1^{\frac{1}{n}} + \frac{1}{1+n}\left((\theta-\delta)t_1^{\frac{1+n}{n}} - \delta n T^{\frac{1+n}{n}}\right)\right]\Bigg\} \end{aligned} \tag{14}$$

To obtain the minimum the total cost TCUT per unit time the optimal value of $t_1$is determined by solving the equation

$$\frac{dTCUT}{dt_1} = 0 \tag{15}$$

The value of t_1 obtained from (15) is used to obtain the optimal values of Q and TCUT. Since the Equation (15) is nonlinear, it is solved using MATLAB.
The necessary condition for a minimum, $\frac{d^2TCUT}{dt_1^2}>0,$ is also satisfied for the value t_1 from(15).

By solving $\frac{dTCUT}{dt_1}=0$ the value of $t_1$can is obtained and if $t_1$and t_2 satisfy the $\frac{d^2TCUT}{dt_1^2}>0,$ then at these optimal values equation (14) provides minimum total inventory cost per unit time of the inventory system.

4.2 Fuzzy Model

Due to uncertainty in the environment it is not easy to define all the parameters precisely, accordingly it assumed that some of these parameters namely $\tilde{C},\tilde{h},\tilde{\pi}_b,\tilde{\pi}_1,\tilde{\pi}_d$ may change with in some limits.

Let $\tilde{\pi}_1=(\pi_{11},\pi_{12},\pi_{13})$, $\tilde{\pi}_b=(\pi_{b1},\pi_{b2},\pi_{b3})$, $\tilde{C}=(C_1,C_2,C_3)$, $\tilde{h}=(h_1,h_2,h_3)$, $\tilde{\pi}_d=(\pi_{d1},\pi_{d2},\pi_{d3})$ are triangular fuzzy numbers. Total cost of the system per unit time in fuzzy sense is given by

$$TCU\tilde{T}=\frac{1}{T}\Bigg\{A+\frac{\tilde{h}d}{T^{\frac{1}{n}}}\left[\frac{t_1^{\frac{1+n}{n}}}{1+n}+\frac{\tilde{\theta}t_1^{\frac{1+2n}{n}}}{2(1+2n)}\right]+\frac{\tilde{\pi}_bd}{T^{\frac{1}{n}}}\left[(\delta T^2-T)t_1^{\frac{1}{n}}+\frac{nT^{\frac{1+n}{n}}}{1+n}+\frac{(1-2\delta T)t_1^{\frac{1+n}{n}}}{1+n}-\frac{2\delta n^2T^{\frac{1+2n}{n}}}{(1+n)(1+2n)}+\frac{\delta t_1^{\frac{1+2n}{n}}}{1+2n}\right]+\tilde{\pi}_d\frac{d\theta t_1^{\frac{1+n}{n}}}{(1+n)T^{\frac{1}{n}}}$$
$$+\frac{\tilde{\pi}_1d\delta}{T^{\frac{1}{n}}}\left[\frac{nT^{\frac{1+n}{n}}}{1+n}-Tt_1^{\frac{1}{n}}+\frac{t_1^{\frac{1+n}{n}}}{1+n}\right]+\frac{\tilde{C}d}{T^{\frac{1}{n}}}\left[T^{\frac{1}{n}}+\delta Tt_1^{\frac{1}{n}}+\frac{1}{1+n}\left((\tilde{\theta}-\delta)t_1^{\frac{1+n}{n}}-\delta nT^{\frac{1+n}{n}}\right)\right]\Bigg\} \tag{16}$$

The fuzzy total cost $TCU\tilde{T}(t_1,T)$, is defuzzified by graded mean representation, signed distance and centroid methods.

i. By Graded Mean Representation Method, Total cost is given by

$$TCUT_{dG}=\left[TCUT_{dG_1},TCUT_{dG_2},TCUT_{dG_3}\right]$$

Where

$$TCUT_{dG_i}=\frac{1}{T}\Bigg\{A+\frac{h_id}{T^{\frac{1}{n}}}\left[\frac{t_1^{\frac{1+n}{n}}}{1+n}+\frac{\theta_it_1^{\frac{1+2n}{n}}}{2(1+2n)}\right]+\frac{\pi_{bi}d}{T^{\frac{1}{n}}}\left[(\delta T^2-T)t_1^{\frac{1}{n}}+\frac{nT^{\frac{1+n}{n}}}{1+n}+\frac{(1-2\delta T)t_1^{\frac{1+n}{n}}}{1+n}-\frac{2\delta n^2T^{\frac{1+2n}{n}}}{(1+n)(1+2n)}+\frac{\delta t_1^{\frac{1+2n}{n}}}{1+2n}\right]+\pi_{di}\frac{d\theta t_1^{\frac{1+n}{n}}}{(1+n)T^{\frac{1}{n}}}$$
$$+\frac{\pi_{1i}d\delta}{T^{\frac{1}{n}}}\left[\frac{nT^{\frac{1+n}{n}}}{1+n}-Tt_1^{\frac{1}{n}}+\frac{t_1^{\frac{1+n}{n}}}{1+n}\right]+\frac{C_id}{T^{\frac{1}{n}}}\left[T^{\frac{1}{n}}+\delta Tt_1^{\frac{1}{n}}+\frac{1}{1+n}\left((\theta_i-\delta)t_1^{\frac{1+n}{n}}-\delta nT^{\frac{1+n}{n}}\right)\right]\Bigg\}\quad \text{where } i=1,2,3 \tag{17}$$

$$TCUT_{dG}=\frac{1}{6}\left[TCUT_{dG_1}+4TCUT_{dG_2}+TCUT_{dG_3}\right] \tag{18}$$

To minimize total cost function per unit time $TCUT_{dG}$, the optimal value of t_1 can be obtained by solving the following equation:

$$\frac{dTCUT_{dG}}{dt_1}=0 \tag{19}$$

Equation (19) is equivalent to

$$\frac{1}{6T}\Bigg\{\frac{h_1d}{T^{\frac{1}{n}}}\left[\frac{t_1^{\frac{1}{n}}}{n}+\frac{\theta t_1^{\frac{1+n}{n}}}{2n}\right]+\frac{\pi_{b1}d}{T^{\frac{1}{n}}}\left[(\delta T^2-T)\frac{t_1^{\frac{1-n}{n}}}{n}+\frac{(1-2\delta T)t_1^{\frac{1}{n}}}{n}+\frac{\delta t_1^{\frac{1+n}{n}}}{n}\right]+\frac{\pi_{11}d\delta}{T^{\frac{1}{n}}}\left[-\frac{Tt_1^{\frac{1-n}{n}}}{n}+\frac{t_1^{\frac{1}{n}}}{n}\right]+\frac{C_1d}{T^{\frac{1}{n}}}\left[\frac{\delta Tt_1^{\frac{1-n}{n}}}{n}+\frac{(\theta-\delta)t_1^{\frac{1}{n}}}{n}\right]+\frac{\pi_{d1}d\theta t_1^{\frac{1+n}{n}}}{(1+n)T^{\frac{1}{n}}}$$
$$+\frac{4h_2d}{T^{\frac{1}{n}}}\left[\frac{t_1^{\frac{1}{n}}}{n}+\frac{\theta t_1^{\frac{1+n}{n}}}{2n}\right]+\frac{4\pi_{b2}d}{T^{\frac{1}{n}}}\left[(\delta T^2-T)\frac{t_1^{\frac{1-n}{n}}}{n}+\frac{(1-2\delta T)t_1^{\frac{1}{n}}}{n}+\frac{\delta t_1^{\frac{1+n}{n}}}{n}\right]+\frac{4\pi_{12}d\delta}{T^{\frac{1}{n}}}\left[-\frac{Tt_1^{\frac{1-n}{n}}}{n}+\frac{t_1^{\frac{1}{n}}}{n}\right]+\frac{4C_2d}{T^{\frac{1}{n}}}\left[\frac{\delta Tt_1^{\frac{1-n}{n}}}{n}+\frac{(\theta-\delta)t_1^{\frac{1}{n}}}{n}\right]$$
$$+\frac{4\pi_{d2}d\theta t_1^{\frac{1+n}{n}}}{(1+n)T^{\frac{1}{n}}}+\frac{\pi_{d3}d\theta t_1^{\frac{1+n}{n}}}{(1+n)T^{\frac{1}{n}}}+\frac{h_3d}{T^{\frac{1}{n}}}\left[\frac{t_1^{\frac{1}{n}}}{n}+\frac{\theta t_1^{\frac{1+n}{n}}}{2n}\right]+\frac{\pi_{13}d\delta}{T^{\frac{1}{n}}}\left[-\frac{Tt_1^{\frac{1-n}{n}}}{n}+\frac{t_1^{\frac{1}{n}}}{n}\right]+\frac{\pi_{b3}d}{T^{\frac{1}{n}}}\left[(\delta T^2-T)\frac{t_1^{\frac{1-n}{n}}}{n}+\frac{(1-2\delta T)t_1^{\frac{1}{n}}}{n}+\frac{\delta t_1^{\frac{1+n}{n}}}{n}\right]+\frac{C_3d}{T^{\frac{1}{n}}}\left[\frac{\delta Tt_1^{\frac{1-n}{n}}}{n}+\frac{(\theta-\delta)t_1^{\frac{1}{n}}}{n}\right]\Bigg\}=0 \tag{20}$$

Further, for the total cost function $TCUT_{dG}$ to be convex, the following condition must be satisfied.

$$\frac{d^2TCUT_{dG}}{dt_1^2}>0 \tag{21}$$

The second derivatives of the total cost function $TCUT_{dG}$ are complicated and it is very difficult to prove the convexity mathematically. Hence, the convexity of total cost function can be established graphically.

ii. By Signed Distance Method, Total cost is given by

$TCUT_{dS}=\left[TCUT_{dS_1},TCUT_{dS_2},TCUT_{dS_3}\right]$

where $TCUT_{dS_i}$ are defined by (17).

$$TCUT_{dS}=\frac{1}{4}\left[TCUT_{dS_1}+2TCUT_{dS_2}+TCUT_{dS_3}\right] \tag{22}$$

The total cost function $TCUT_{dS}$ has been minimized following the same process as has been stated in case(i).To minimize total cost function per unit time $TCUT_{dS}$, the optimal value of t_1 can be obtained by solving the following equation:

$$\frac{dTCUT_{dS}}{dt_1}=0 \tag{23}$$

Equation (23) is equivalent to

$$\begin{aligned}&\frac{1}{4T}\left\{\frac{h_1d}{T^{\frac{1}{n}}}\left[\frac{t_1^{\frac{1}{n}}}{n}+\frac{\theta t_1^{\frac{1+n}{n}}}{2n}\right]+\frac{\pi_{b1}d}{T^{\frac{1}{n}}}\left[\left(\delta T^2-T\right)\frac{t_1^{\frac{1-n}{n}}}{n}+\frac{\left(1-2\delta T\right)t_1^{\frac{1}{n}}}{n}+\frac{\delta t_1^{\frac{1+n}{n}}}{n}\right]+\frac{\pi_{l1}d\delta}{T^{\frac{1}{n}}}\left[-\frac{Tt_1^{\frac{1-n}{n}}}{n}+\frac{t_1^{\frac{1}{n}}}{n}\right]+\frac{C_1d}{T^{\frac{1}{n}}}\left[\frac{\delta Tt_1^{\frac{1-n}{n}}}{n}+\frac{\left(\theta-\delta\right)t_1^{\frac{1}{n}}}{n}\right]+\frac{\pi_{d1}d\theta t_1^{\frac{1}{n}}}{nT^{\frac{1}{n}}}\right.\\
&+\frac{2h_2d}{T^{\frac{1}{n}}}\left[\frac{t_1^{\frac{1}{n}}}{n}+\frac{\theta t_1^{\frac{1+n}{n}}}{2n}\right]+\frac{2\pi_{b2}d}{T^{\frac{1}{n}}}\left[\left(\delta T^2-T\right)\frac{t_1^{\frac{1-n}{n}}}{n}+\frac{\left(1-2\delta T\right)t_1^{\frac{1}{n}}}{n}+\frac{\delta t_1^{\frac{1+n}{n}}}{n}\right]+\frac{2\pi_{l2}d\delta}{T^{\frac{1}{n}}}\left[-\frac{Tt_1^{\frac{1-n}{n}}}{n}+\frac{t_1^{\frac{1}{n}}}{n}\right]+\frac{2C_2d}{T^{\frac{1}{n}}}\left[\frac{\delta Tt_1^{\frac{1-n}{n}}}{n}+\frac{\left(\theta-\delta\right)t_1^{\frac{1}{n}}}{n}\right]\\
&\left.+\frac{2\pi_{d2}d\theta t_1^{\frac{1}{n}}}{nT^{\frac{1}{n}}}+\frac{\pi_{d3}d\theta t_1^{\frac{1}{n}}}{nT^{\frac{1}{n}}}+\frac{h_3d}{T^{\frac{1}{n}}}\left[\frac{t_1^{\frac{1}{n}}}{n}+\frac{\theta t_1^{\frac{1+n}{n}}}{2n}\right]+\frac{\pi_{l3}d\delta}{T^{\frac{1}{n}}}\left[-\frac{Tt_1^{\frac{1-n}{n}}}{n}+\frac{t_1^{\frac{1}{n}}}{n}\right]+\frac{\pi_{b3}d}{T^{\frac{1}{n}}}\left[\left(\delta T^2-T\right)\frac{t_1^{\frac{1-n}{n}}}{n}+\frac{\left(1-2\delta T\right)t_1^{\frac{1}{n}}}{n}+\frac{\delta t_1^{\frac{1+n}{n}}}{n}\right]+\frac{C_3d}{T^{\frac{1}{n}}}\left[\frac{\delta Tt_1^{\frac{1-n}{n}}}{n}+\frac{\left(\theta-\delta\right)t_1^{\frac{1}{n}}}{n}\right]\right\}=0\end{aligned} \tag{24}$$

Further, for the total cost function $TCUT_{dS}$ to is convex, the following conditions must be satisfied

$$\frac{d^2TCUT_{dS}}{dt_1^2}>0 \tag{25}$$

The second derivatives of the total cost function $TCUT_{dS}$ are complicated and it is very difficult to prove the convexity mathematically. Hence, the convexity of total cost function can be established graphically.

iii. By Centroid Method, Total cost is given by

$TCUT_{dC}=\left[TCUT_{dC_1},TCUT_{dC_2},TCUT_{dC_3}\right]$

where $TCUT_{dC_i}$ are defined by (17).

$$TCUT_{dC}=\frac{1}{3}\left[TCUT_{dC_1}+TCUT_{dC_2}+TCUT_{dC_3}\right] \tag{26}$$

The total cost function $TCUT_{dC}$ has been minimized following the same process as has been stated in case(i). To minimize total cost function per unit time $TCUT_{dC}$, the optimal value of t_1 can be obtained by solving the following equation:

$$\frac{dTCUT_{dC}}{dt_1}=0 \tag{27}$$

Equation (27) is equivalent to

$$\begin{aligned}&\frac{1}{3T}\left\{\frac{h_1d}{T^{\frac{1}{n}}}\left[\frac{t_1^{\frac{1}{n}}}{n}+\frac{\theta t_1^{\frac{1+n}{n}}}{2n}\right]+\frac{\pi_{b1}d}{T^{\frac{1}{n}}}\left[\left(\delta T^2-T\right)\frac{t_1^{\frac{1-n}{n}}}{n}+\frac{\left(1-2\delta T\right)t_1^{\frac{1}{n}}}{n}+\frac{\delta t_1^{\frac{1+n}{n}}}{n}\right]+\frac{\pi_{l1}d\delta}{T^{\frac{1}{n}}}\left[-\frac{Tt_1^{\frac{1-n}{n}}}{n}+\frac{t_1^{\frac{1}{n}}}{n}\right]+\frac{C_1d}{T^{\frac{1}{n}}}\left[\frac{\delta Tt_1^{\frac{1-n}{n}}}{n}+\frac{\left(\theta-\delta\right)t_1^{\frac{1}{n}}}{n}\right]+\frac{\pi_{d1}d\theta t_1^{\frac{1}{n}}}{nT^{\frac{1}{n}}}\right.\\
&+\frac{h_2d}{T^{\frac{1}{n}}}\left[\frac{t_1^{\frac{1}{n}}}{n}+\frac{\theta t_1^{\frac{1+n}{n}}}{2n}\right]+\frac{\pi_{b2}d}{T^{\frac{1}{n}}}\left[\left(\delta T^2-T\right)\frac{t_1^{\frac{1-n}{n}}}{n}+\frac{\left(1-2\delta T\right)t_1^{\frac{1}{n}}}{n}+\frac{\delta t_1^{\frac{1+n}{n}}}{n}\right]+\frac{\pi_{l2}d\delta}{T^{\frac{1}{n}}}\left[-\frac{Tt_1^{\frac{1-n}{n}}}{n}+\frac{t_1^{\frac{1}{n}}}{n}\right]+\frac{C_2d}{T^{\frac{1}{n}}}\left[\frac{\delta Tt_1^{\frac{1-n}{n}}}{n}+\frac{\left(\theta-\delta\right)t_1^{\frac{1}{n}}}{n}\right]\\
&\left.+\frac{\pi_{d2}d\theta t_1^{\frac{1}{n}}}{nT^{\frac{1}{n}}}+\frac{\pi_{d3}d\theta t_1^{\frac{1}{n}}}{nT^{\frac{1}{n}}}+\frac{h_3d}{T^{\frac{1}{n}}}\left[\frac{t_1^{\frac{1}{n}}}{n}+\frac{\theta t_1^{\frac{1+n}{n}}}{2n}\right]+\frac{\pi_{l3}d\delta}{T^{\frac{1}{n}}}\left[-\frac{Tt_1^{\frac{1-n}{n}}}{n}+\frac{t_1^{\frac{1}{n}}}{n}\right]+\frac{\pi_{b3}d}{T^{\frac{1}{n}}}\left[\left(\delta T^2-T\right)\frac{t_1^{\frac{1-n}{n}}}{n}+\frac{\left(1-2\delta T\right)t_1^{\frac{1}{n}}}{n}+\frac{\delta t_1^{\frac{1+n}{n}}}{n}\right]+\frac{C_3d}{T^{\frac{1}{n}}}\left[\frac{\delta Tt_1^{\frac{1-n}{n}}}{n}+\frac{\left(\theta-\delta\right)t_1^{\frac{1}{n}}}{n}\right]\right\}=0\end{aligned} \tag{28}$$

Further, for the total cost function $TCUT_{dC}$ to is convex, the following conditions must be satisfied

$$\frac{d^2TCUT_{dC}}{dt_1^2}>0 \tag{29}$$

The second derivatives of the total cost function $TCUT_{dC}$ are complicated and it is very difficult to prove the convexity mathematically. Hence, the convexity of total cost function can be established graphically.

To illustrate and validate the proposed model, appropriate a numerical data is considered and the optimal values are found in the following section. Sensitivity analysis is carried out with respect to backlogging parameter and deterioration rate.

V. Numerical Example And Sensitivity Analysis:

Consider an inventory system with following parametric values:

Crisp Model:

d=50units, n=2units, A = \$250/order, C =\$ 8/unit, h = \$ 0.50 /unit/year, π_b=\$12/unit/year, π_d=\$11/unit, π_l=\$15/unit, δ=0.8 units, θ=0.05, T=1.

The solution of crisp model is

TCUT=\$672.8320≅\$ 673, t_1 = 0.9201 years, Q = 50.6699 units ≅ 51 units.

Fuzzy Model:

$\tilde{C}$ = (7.5, 8, 8.8), $\tilde{h}$ = (0.4, 0.5,0.65), $\tilde{\pi}_b$ = (11, 12, 14), $\tilde{\pi}_l$ = (13.5, 15, 15.5), $\tilde{\pi}_d$ = (10.5, 11, 11.7)

The solution of fuzzy model is determined by three different methods. Computations are given below:

Table 1 Computations with fuzzy parameters

Method	Fuzzy Number	t_1 (years)	$TCUT_{dG}$(\$)	Q_1(units)
Grade Mean Representation Method	$\tilde{C}, \tilde{h}, \tilde{\pi}_b, \tilde{\pi}_l, \tilde{\pi}_d$	0.9194	675.5163	50.6678
	$\tilde{h}, \tilde{\pi}_b, \tilde{\pi}_l, \tilde{\pi}_d$	0.9197	672.9829	50.6687
	$\tilde{\pi}_b, \tilde{\pi}_l, \tilde{\pi}_d$	0.9201	672.8587	50.6699
	$\tilde{\pi}_l, \tilde{\pi}_d$	0.9194	672.8455	50.6678
	$\tilde{\pi}_d$	0.9200	672.8565	50.6696
Signed Distance method	$\tilde{C}, \tilde{h}, \tilde{\pi}_b, \tilde{\pi}_l, \tilde{\pi}_d$	0.9189	676.8584	50.6664
	$\tilde{h}, \tilde{\pi}_b, \tilde{\pi}_l, \tilde{\pi}_d$	0.9195	673.0583	50.6681
	$\tilde{\pi}_b, \tilde{\pi}_l, \tilde{\pi}_d$	0.9201	672.8720	50.6699
	$\tilde{\pi}_l, \tilde{\pi}_d$	0.9190	672.8521	50.6667
	$\tilde{\pi}_d$	0.9199	672.8688	50.6693
Centroid Method	$\tilde{C}, \tilde{h}, \tilde{\pi}_b, \tilde{\pi}_l, \tilde{\pi}_d$	0.9417	678.6384	50.6749
	$\tilde{h}, \tilde{\pi}_b, \tilde{\pi}_l, \tilde{\pi}_d$	0.9193	673.1337	50.6676
	$\tilde{\pi}_b, \tilde{\pi}_l, \tilde{\pi}_d$	0.9201	672.8854	50.6699
	$\tilde{\pi}_l, \tilde{\pi}_d$	0.9187	672.8587	50.6658
	$\tilde{\pi}_d$	0.9199	672.8810	50.6693

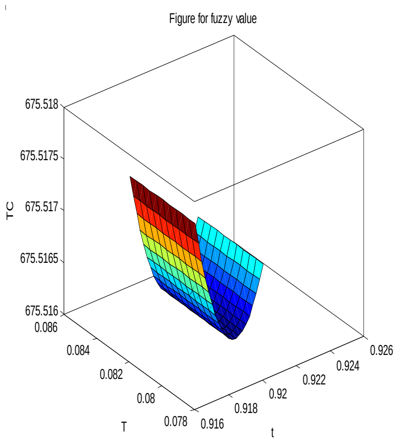

Fig.4 Grade Mean Representation Method

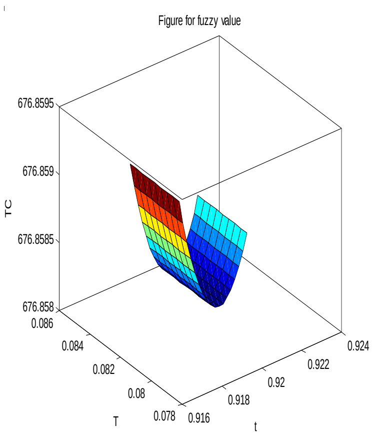

Fig.5 Signed Distance method

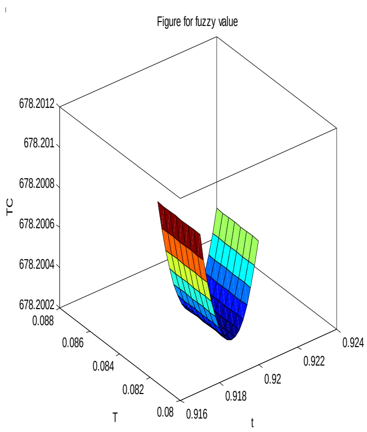

Fig.6 Centroid Method

Fig.4, Fig.5 and Fig.6 depict the total cost function (convex).

Effect of backlogging parameter (δ):

Crisp value of the backlogging parameter is 0.8. Now varying the backlogging parameter from 0.7 to 0.9 the following table is obtained.

Table 2 Variation in backlogging parameter 'δ'

Value of δ	t_1(years)	$TCUT_{dG}$($)	Q(units)
0.70	0.9166	675.4635	50.6687
0.75	0.9180	675.4903	50.6681
0.80	0.9194	675.5163	50.6678
0.85	0.9207	675.5415	50.6675
0.90	0.9219	675.5659	50.6672

Effect of Deterioration parameter (θ):

Crisp value of the deterioration parameter is taken as 0.05. Now varying the backlogging parameter from 0.025 to 0.075 the following table is obtained.

Table 3 Variation in deterioration parameter 'θ'

Value of θ	t_1(years)	$TCUT_{dG}$($)	Q(units)
0.0250	0.9451	668.3067	50.3521
0.0375	0.9321	671.9490	50.5152
0.0500	0.9194	675.5163	50.6678
0.0625	0.9068	679.0103	50.8098
0.0750	0.8944	682.4324	50.9416

VI. Conclusion

This paper presents a fuzzy inventory model for deteriorating items with allowable shortages and power demand. Deterioration rate, inventory holding cost, unit cost and shortage cost back order cost and cost of last sale are represented by triangular fuzzy numbers. For defuzzification, graded mean, signed distance and centroid methods are used to evaluate the optimal time period of positive stock and order quantity which minimizes the total cost. Numerical example reveals that the graded mean representation method gives minimum cost as compared to signed distance and centroid methods. Sensitivity analysis is also conducted on the parameters to explore the effects of variation in the parameters.

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