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Equi independent equitable domination number of cycle and bistar related graphs

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Abstract : *A subset D of $V(G)$ is called an equitable dominating set if for every $v \in V(G)-D$ there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|deg(u)-deg(v)| \leq 1$. A subset D of $V(G)$ is called an equitable independent set if for any $u \in D, v \notin N^e(u)$ for all $v \in D-\{u\}$. The concept of equi independent equitable domination is a combination of these two important concepts. An equitable dominating set D is said to be equi independent equitable dominating set if it is an equitable independent set. The minimum cardinality of an equi independent equitable dominating set is called equi independent equitable domination number which is denoted by i^e. We investigate equi independent equitable domination number of some cycle and bistar related graphs. Some graph families whose equitable domination number is equal to its equi independent equitable domination number are also reported.*

Keywords: *Domination number, equitable domination number, equi independent equitable domination number.*

AMS Subject Classification (2010): 05C38,05C69,05C76.

I. Introduction

We begin with simple, finite, connected and undirected graph G with vertex set $V(G)$ and edge set $E(G)$. For standard graph theoretic terminology we follow Harary [1] while the terms related to the theory of domination are used here in the sense of Haynes et al. [2]. A set $D \subseteq V(G)$ is called a dominating set if every vertex in $V(G)-D$ is adjacent to at least one vertex in D. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G. A subset D of $V(G)$ is an independent set if no two vertices in D are adjacent. A dominating set D which is also an independent set is called an independent dominating set. The independent domination number $i(G)$ is the minimum cardinality of an independent dominating set. The concept of independent domination was formalized by Berge [3], Ore [4], Cockayne and Hedetniemi [5,6]. A brief survey on independent domination is carried out in recent past by Goddard and Henning [7]. A subset D of $V(G)$ is called an equitable dominating set if for every $v \in V(G)-D$ there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|deg(u)-deg(v)| \leq 1$. The minimum cardinality of an equitable dominating set is equitable domination number of G which is denoted by γ^e. This concept was conceived by E. Sampathkumar.

Definition 1.1: For every vertex $v \in V(G)$ the open neighbourhood set $N(v)$ is the set of all vertices adjacent to v in G. That is, $N(v) = \{u \in V(G) / uv \in E(G)\}$.

Definition 1.2: The closed neighbourhood set $N[v]$ of v is defined as $N[v] = N(v) \bigcup \{v\}$.

Definition 1.3: A vertex $v \in D$ is an isolate of D if $N(v) \subseteq V(G)-D$.

Definition 1.4: A vertex $u \in V(G)$ is degree equitably adjacent or equitable adjacent with a vertex $v \in V(G)$ if $|deg(u)-deg(v)| \leq 1$ for $uv \in E(G)$.

Definition 1.5: A vertex $v \in V(G)$ is called equitable isolate if $|deg(u)-deg(v)| \geq 2$ for every $u \in N(v)$.

Definition 1.6: The equitable neighbourhood of v denoted by $N^e(v)$ is defined as $N^e(v) = \{u \in V(G) / u \in N(v), |deg(v)-deg(u)| \leq 1\}$.

The following concept known as equitable independent set was introduced by Swaminathan and Dharamlingam [8].

Definition 1.7: A subset D of $V(G)$ is called an equitable independent set if for any $u \in D, v \notin N^e(u)$ for all $v \in D-\{u\}$. The maximum cardinality of an equitable independent set is denoted by $\beta^e(G)$.

Remark 1.8: Every independent set is an equitable independent set.

Motivated through the concept of equitable dominating set and equitable independent set a new concept was conceived by Swaminathan and Dharamlingam [8]. This concept was formalized and named as equi independent equitable dominating set by Vaidya and Kothari [9,10,11].

Definition 1.9: An equitable dominating set D is said to be equi independent equitable dominating set if it is an equitable independent set. The minimum cardinality of an equi independent equitable dominating set is called equi independent equitable domination number which is denoted by i^e.

Illustration 1.10: In figure 1, $D=\{v_1,v_2,v_3,v_5,v_7,v_9\}$ is equitable independent set as well as equitable dominating set for graph G.

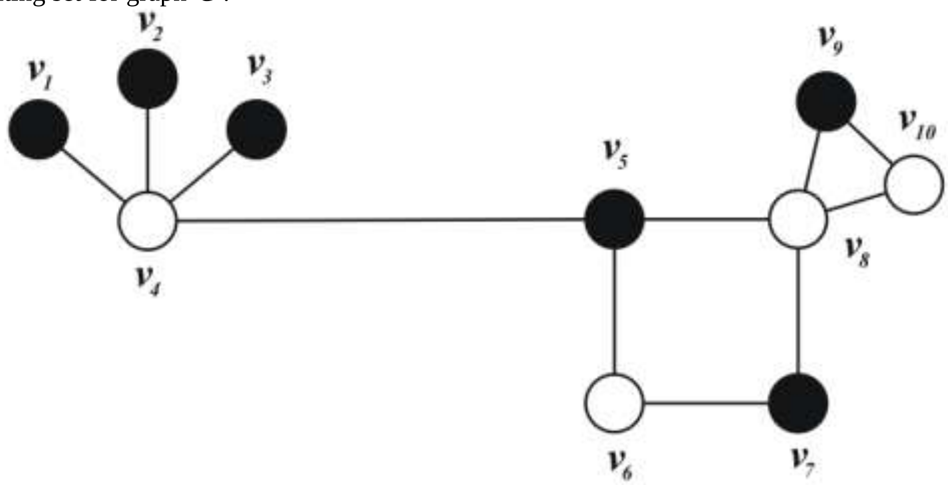

Figure 1

Definition 1.11: For a graph G the splitting graph $S'(G)$ of a graph G is obtained by adding a new vertex v' corresponding to each vertex v of G such that $N(v)=N(v')$.

Definition 1.12: The total graph $T(G)$ of a graph G is the graph whose vertex set is $V(G)\bigcup E(G)$ and two vertices are adjacent whenever they are either adjacent or incident in G.

Definition 1.13: The shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' and G''. Join each vertex u' in G' to the neighbours of the corresponding vertex u'' in G''.

Definition 1.14: The middle graph $M(G)$ of a graph G is the graph whose vertex set is $V(G)\bigcup E(G)$ and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident on it.

Definition 1.15:[12] Duplication of a vertex v_i by a new edge $e=v_i'v_i''$ in graph G produces a new graph G' such that $N(v_i')\bigcap N(v_i'')=\{v_i\}$.

Definition 1.16: The graph $K_{1,n}$ is called a star. It is the graph with $n+1$ vertices – a vertex of degree n called the apex and n pendant vertices.

Definition 1.17: Bistar is the graph obtained by joining the apex vertices of two copies of star $K_{1,n}$.

II. Main results

Theorem 2.1: $i^e(S'(C_n))=\gamma^e(S'(C_n))=\gamma(C_n)+n.$

Proof: Let $v_1,v_2,...,v_n$ be the vertices of path P_n and $u_1,u_2,\dots,u_n$ be the corresponding vertices which are added to obtain $S'(C_n)$. Here $d(u_i)=2,\ d(v_i)=4$ and vertex u_i is not adjacent to any other vertex u_j. This implies that vertices $u_i's$ are equitable isolates. Therefore they must belong to every equitable dominating set of $S'(C_n)$. While subgraph induced by $V(S'(C_n))-\{u_1,u_2,\dots,u_n\}$ is cycle C_n. Which implies that

$$\gamma^e(S'(C_n))\geq\gamma^e(C_n)+n$$

Now consider a set which satisfies above lower bound as $D=S\bigcup\{u_1,u_2,\dots,u_n\}$ where S is an equitable dominating set of C_n. Observe that D is an equitable dominating set of $S'(C_n)$ as $u_1,u_2,\dots,u_n$ are equitable isolates and other vertices are equitably dominated by set S. This implies that

$$\gamma^e(S'(C_n))=\gamma^e(C_n)+n=\gamma(C_n)+n$$

Also D is an equitable independent as vertices of S are not adjacent to each other and $u_1, u_2, \ldots, u_n$ are equitable isolates. Hence D is an equi independent equitable dominating set of $S'(C_n)$ and $i^e(S'(C_n)) = \gamma^e(S'(C_n)) = \gamma(S'(C_n)) + n$.

Theorem 2.2: $\gamma\left(T\left(C_n\right)\right) = \begin{cases} \dfrac{2n}{5} & n \equiv 0 \pmod 5 \\ \left\lceil \dfrac{2n}{5} \right\rceil & n \not\equiv 0 \pmod 5 \end{cases}$

Proof: Let $v_1, v_2, \ldots, v_n$ be the vertices of C_n and $e_1, e_2, \ldots, e_n$ be the edges of C_n. Then $V(T(C_n)) = \{v_1, v_2, \ldots, v_n, e_1, e_2, \ldots, e_n\}$ and $|V(T(C_n))| = 2n$. Observe that $T(C_n)$ is four regular graph. Therefore every vertex can dominate five vertices including itself. Which implies that

$$\gamma(T(C_n)) \geq \left\lceil \frac{2n}{5} \right\rceil$$

Now depending upon the number of vertices of C_n, consider following subsets of $T(C_n)$.

For $n \equiv 0 \pmod 5$, $D = \left\{ v_{5i+1}, u_{5j+3} / 0 \leq i < \dfrac{n}{5}, 0 \leq j < \dfrac{n}{5} \right\}$ with $|D| = \dfrac{2n}{5}$,

for $n \equiv 1, 2 \pmod 5$, $D = \left\{ v_{5i+1}, u_{5j+3} / 0 \leq i \leq \left\lfloor \dfrac{n}{5} \right\rfloor, 0 \leq j < \left\lfloor \dfrac{n}{5} \right\rfloor \right\}$ with $|D| = \left\lceil \dfrac{2n}{5} \right\rceil$,

for $n \equiv 3, 4 \pmod 5$, $D = \left\{ v_{5i+1}, u_{5j+3} / 0 \leq i \leq \left\lfloor \dfrac{n}{5} \right\rfloor, 0 \leq j \leq \left\lfloor \dfrac{n}{5} \right\rfloor \right\}$ with $|D| = \left\lceil \dfrac{2n}{5} \right\rceil$.

Here $N\left(v_{5i+1}\right) = \left\{ v_{5i}, v_{5i+2}, u_{5i}, u_{5i+1} \right\}$ and $N\left(u_{5i+3}\right) = \left\{ v_{5i+4}, v_{5i+4}, u_{5i+2}, u_{5i+4} \right\}$. This implies that

$N\left[D\right] = V\left(T\left(C_n\right)\right)$. Then D is dominating set of $T\left(C_n\right)$ and $\gamma\left(T\left(C_n\right)\right) = \begin{cases} \dfrac{2n}{5} & n \equiv 0 \pmod 5 \\ \left\lceil \dfrac{2n}{5} \right\rceil & n \not\equiv 0 \pmod 5 \end{cases}$.

Theorem 2.3: $i^e(T(C_n)) = \gamma^e(T(C_n)) = \gamma(T(C_n))$

Proof: To prove this result we continue with the terminology and notations used in Theorem 2.2. Here $T(C_n)$ is a four regular graph. This implies that

$$\gamma^e(T(C_n)) = \gamma(T(C_n))$$

Now consider D as γ-set of $T(C_n)$, as given in Theorem 2.2. Observe that vertices of D are not adjacent to each other. Then D is an independent set of $T(C_n)$ consequently D is an equitable independent set of $T(C_n)$. Hence D is an equi independent equitable dominating set of $T(C_n)$ and $i^e(T(C_n)) = \gamma^e(T(C_n)) = \gamma(T(C_n))$

Theorem 2.4: $\gamma^e\left(D_2\left(C_n\right)\right) = \gamma\left(D_2\left(C_n\right)\right) = \begin{cases} \dfrac{n}{2} & \text{for } n \equiv 0 \pmod 4 \\ \left\lceil \dfrac{n}{2} \right\rceil & \text{for } n \equiv 1, 3 \pmod 4 \\ \dfrac{n}{2} + 1 & \text{for } n \equiv 2 \pmod 4 \end{cases}$

Proof: Consider two copies of C_n. Let $v_1, v_2, \ldots, v_n$ be the vertices of first copy of C_n and $u_1, u_2, \ldots, u_n$ be the vertices of second copy of C_n. Then $|D_2(C_n)| = 2n$. Observe that $D_2(C_n)$ is four regular graph. This implies that minimum $\dfrac{2n}{4} = \dfrac{n}{2}$ vertices are essential to dominate $V(D_2(C_n))$ and

$$\gamma^e(D_2(C_n)) = \gamma(D_2(C_n)) \geq \frac{n}{2}$$

Now depending upon the number of vertices of C_n, consider following subsets of $V\left(D_2\left(C_n\right)\right)$, For $n \equiv 0 \pmod 4$ $D = \left\{ v_{4i+1}, u_{4i+2} \,/\, 0 \le i \le \frac{n}{4} \right\}$,

for $n \equiv 1 \pmod 4$ $D = \left\{ v_{4i+1}, u_{4j+2} \,/\, 0 \le i \le \left\lfloor \frac{n}{4} \right\rfloor, 0 \le j < \left\lfloor \frac{n}{4} \right\rfloor \right\}$,

for $n \equiv 2,3 \pmod 4$ $D = \left\{ v_{4i+1}, u_{4i+2} \,/\, 0 \le i \le \left\lfloor \frac{n}{4} \right\rfloor \right\}$.

Here $N(v_{4i+1}) = \{v_{4i}, v_{4i+2}, u_{4i}, u_{4i+2}\}$ and $N(u_{4i+2}) = \{v_{4i+1}, v_{4i+3}, u_{4i+1}, u_{4i+3}\}$. Therefore $N[D] = V(D_2(C_n))$. Thus D is dominating set as well as equitable dominating set of $D_2(C_n)$ and

$$\gamma^e(D_2(C_n)) = \gamma(D_2(C_n)) = \begin{cases} \frac{n}{2} & \text{for } n \equiv 0 \pmod 4 \\ \left\lceil \frac{n}{2} \right\rceil & \text{for } n \equiv 1,3 \pmod 4 \\ \frac{n}{2} + 1 & \text{for } n \equiv 2 \pmod 4 \end{cases}$$

Theorem 2.5: $i^e(D_2(C_n)) = \begin{cases} \frac{2n}{3} & \text{for } n \equiv 0 \pmod 3 \\ \left\lceil \frac{2n}{3} \right\rceil + 1 & \text{for } n \equiv 1 \pmod 3 \\ \left\lfloor \frac{2n}{3} \right\rfloor + 1 & \text{for } n \equiv 2 \pmod 3 \end{cases}$

Proof: To prove this result we continue with the terminology and notations used in Theorem 2.4. Observe that vertex v_i cannot dominate u_i and vice versa. On other hand vertex v_i is adjacent to all the vertices of $N(u_i)$ and vertex u_i is adjacent to all the vertices of $N(v_i)$. This implies that if vertex v_i belongs to any equitable independent set of $D_2(C_n)$ then u_i must belong to that equitable independent set. Therefore

$$i^e(D_2(C_n)) > \gamma^e(D_2(C_n))$$

Now depending upon the number of vertices of C_n, consider following subsets of $V(D_2(C_n))$,

For $n \equiv 0 \pmod 3$ $D = \left\{ v_{3i+1}, u_{3i+1} \,/\, 0 \le i < \frac{n}{3} \right\}$,

for $n \equiv 1 \pmod 3$ $D = \left\{ v_{3i+1}, u_{3i+1}, v_{n-1}, u_{n-1} \,/\, 0 \le i < \left\lfloor \frac{n}{3} \right\rfloor \right\}$,

for $n \equiv 2 \pmod 3$ $D = \left\{ v_{3i+1}, u_{3i+1} \,/\, 0 \le i < \left\lceil \frac{n}{3} \right\rceil \right\}$.

Here $N(v_{3i+1}) = \{v_{3i}, v_{3i+2}, u_{3i}, u_{3i+2}\}$ and $N(u_{3i+1}) = \{v_{3i}, v_{3i+2}, u_{3i}, u_{3i+2}\}$. This implies that vertices v_{3i+1} and u_{3i+1} are not adjacent to each other. Therefore D is an independent set of $D_2(C_n)$. Consequently D is an equitable independent set of $D_2(C_n)$. Also $N^e[D] = V(D_2(C_n))$ implies that D is an equitable dominating set of $D_2(C_n)$. Hence D is an equi independent equitable dominating set of $D_2(C_n)$ and

$$i^e(D_2(C_n)) = \begin{cases} \frac{2n}{3} & \text{for } n \equiv 0 \pmod 3 \\ \left\lceil \frac{2n}{3} \right\rceil + 1 & \text{for } n \equiv 1 \pmod 3 \\ \left\lfloor \frac{2n}{3} \right\rfloor + 1 & \text{for } n \equiv 2 \pmod 3 \end{cases}$$

Theorem 2.6: $i^e(M(C_n)) = \gamma^e(M(C_n)) = \gamma(C_n) + n.$

Proof: Let $v_1, v_2, \ldots, v_n$ be the vertices of C_n and $e_1, e_2, \ldots, e_n$ be the edges of C_n. Then $V(M(C_n)) = \{v_1, v_2, \ldots, v_n, e_1, e_2, \ldots, e_n\}$. Here $d(e_i) = 4, d(v_i) = 2$ in $M(C_n)$ and vertices v_i are not adjacent to each other. Then vertices $v_1, v_2, \ldots, v_n$ are equitable isolates. Therefore they must belong to every equitable dominating set of $M(C_n)$. While subgraph induced by $V(M(C_n)) - \{v_1, v_2, \ldots, v_n\}$ is cycle C_n. This implies that

$$\gamma^e\left(M\left(C_n\right)\right) \geq \gamma^e\left(C_n\right) + n = \gamma\left(C_n\right) + n$$

Now consider a set which satisfies above lower bound as $D = S \bigcup \{v_1, v_2, \ldots, v_n\}$ where S is an equitable dominating set of C_n. Observe that D is an equitable dominating set of $M(C_n)$ as $v_1, v_2, \ldots, v_n$ are equitable isolates and other vertices are equitably dominated by set S. This implies that

$$\gamma^e(M(C_n)) = \gamma^e(C_n) + n = \gamma(C_n) + n$$

Also D is an equitable independent as vertices of S are not adjacent to each other and $v_1, v_2, \ldots, v_n$ are equitable isolates. Hence D is an equi independent equitable dominating set of $M(C_n)$ and $i^e(M(C_n)) = \gamma^e(M(C_n)) = \gamma(M(C_n)) + n$.

Theorem 2.7: The graph obtained by duplication of each vertex of C_n by an edge then $i^e(G) = \gamma^e(G) = \gamma(C_n) + n$

Proof: Let $v_1, v_2, \ldots, v_n$ be the vertices of C_n and G be a obtained by duplication of each vertex v_i of C_n by an edge $u_i u_{i+1}$. Then $V(G) = \{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_{2n}\}$. Here vertices u_{2i-1} and u_{2i} are equitably adjacent to each other but they are not equitably adjacent to vertex v_i. Therefore at least one vertex from $\{u_{2i-1}, u_{2i}\}$ must belong to every equitable dominating set of G. While subgraph induced by $V(G) - \{u_1, u_2, \ldots, u_{2n}\}$ is cycle C_n. This implies that

$$\gamma^e(G) \geq \gamma^e(C_n) + n = \gamma(C_n) + n$$

Now consider a set which satisfies above lower bound as $D = S \bigcup \{u_1, u_2, \ldots, u_{2n}\}$, where S is $\gamma^e -$ set of C_n. Then D is an equitable dominating set of G, as vertices $u_1, u_2, \ldots, u_{2n}$ are equitable isolates and remaining vertices are dominated by set S. This implies that

$$\gamma^e(G) = \gamma^e(C_n) + n = \gamma(C_n) + n$$

Also D is an equitable independent set as vertices $u_1, u_2, \ldots, u_{2n}$ are equitably non adjacent to vertices of S and vertices of S are not adjacent to each other. Hence D is an equi independent equitable dominating set of G and $i^e(G) = \gamma^e(G) = \gamma(C_n) + n$.

Theorem 2.8: $i^e(B^2{}_{n,n}) = \gamma(B^2{}_{n,n}) = \begin{cases} 1 & \text{for } n = 2 \\ 3 & \text{for } n \geq 3 \end{cases}$

Proof: Consider $B_{n,n}$ with vertex set $\{u, v, u_i, v_i\}$ where u, v are apex vertices and u_i, v_i are pendant vertices. Then $V(B_{n,n}^2) = V(B_{n,n})$ and $E(B_{n,n}^2) = E(B_{n,n}) \bigcup \{u_i u_j, u_i v, v_i v_j, v_i u / 1 \leq i, j, \leq n, i \neq j\}$. Note that $d(u_i) = d(v_i) = n+1$ and $d(u) = d(v) = 2n+1$.

Case 1: $n = 2$

In this case vertex u is equitably adjacent to all the vertices. Hence $i^e(B_{n,n}^2) = \gamma^e(B_{n,n}^2) = 1$.

Case 2: $n \geq 3$

Here vertex u_i is equitably adjacent to all the vertices u_j where $i \neq j$ and similarly vertex v_i is equitably adjacent to all the vertices v_j where $i \neq j$. Therefore one vertex from $u_i's$ say u_1 and one vertex from $v_i's$ say v_1 must belong to every equitable dominating set of $B_{n,n}^2$. While vertex u is equitably adjacent to v only. This implies that

$$\gamma^e(B_{n,n}^2) \geq 3$$

Now consider a set which satisfies above lower bound as $D = \{u, u_1, v_1\}$. Here $N^e(u) \bigcup N^e(u_1) \bigcup N^e(v_1) = V(B_{n,n}^2)$. Therefore D is an equitable dominating set of $B_{n,n}^2$ and $\gamma^e(B_{n,n}^2) = 3$. Also vertices u, u_1 and v_1

are not equitably adjacent to each other. This implies that D is an equitable independent set of $B^2_{n,n}$. Hence D is an equi independent equitable dominating set of $B^2_{n,n}$ and $i^e(B^2_{n,n}) = \gamma^e(B^2_{n,n}) = 3$.

Theorem 2.9: $i^e(D_2(B_{n,n})) = \gamma^e(D_2(B_{n,n})) = 4n+2$

Proof: Consider two copies of $B_{n,n}$. Let $A = \{u, v, u_i, v_i / 1 \le i \le n\}$ and $B = \{u', v', u'_i, v'_i / 1 \le i \le n\}$ be the corresponding vertex sets of each copy of $B_{n,n}$. Then $V(D_2(B_{n,n})) = A \bigcup B$ and $E(D_2(B_{n,n})) = \{uu_i, uu'_i, u'u_i, u'u'_i, vv_i, vv'_i, v'v_i, v'v'_i, uv, uv', v'u, v'u'\}$. Note that $d(u_i) = d(u'_i) = d(v_i) = d(v'_i) = 2$, $d(u) = d(u') = d(v) = d(v') = 2n+2$ and vertices u_i, u'_i, v_i, v'_i are not adjacent to each other. This implies that vertices u_i, v_i, u'_i, v'_i are note adjacent to each other. Then vertices u_i, v_i, u'_i, v'_i are equitable isolates. Therefore they must belong to every equitable dominating set of $D_2(B_{n,n})$. While rest of vertices u, u', v, v' can be equitably dominate by either u, u' or v, v'. This implies that

$$\gamma^e(D_2(B_{n,n})) \ge 4n+2$$

Now consider a set which satisfies above lower bound as $D = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n, u'_1, u'_2, \ldots, u'_n, v'_1, v'_2, \ldots, v'_n \quad u, u'\}$. Then $N^e[D] = V(D_2(B_{n,n}))$ with $|D| = 4n+2$. Therefore $\gamma^e(D_2(B_{n,n})) = 4n+2$. While vertices of D other than u, u' are equitable isolates and u, u' are not adjacent to each other. This implies that D is an equitable independent set of $D_2(B_{n,n})$. Hence D is an equi independent equitable dominating set of $D_2(B_{n,n})$ and $i^e(D_2(B_{n,n})) = \gamma^e(D_2(B_{n,n})) = 4n+2$.

Theorem 2.10: $i^e(S'(B_{n,n})) = \gamma^e(S'(B_{n,n})) = \begin{cases} 3 & \text{for } n = 2 \\ 4n+3 & \text{for } n \ge 3 \end{cases}$

Proof: Consider $B_{n,n}$ with vertex set $\{u, v, u_i, v_i / 1 \le i \le n\}$ where u_i, v_i are pendant vertices. In order to obtain $S'(B_{n,n})$ add u', v', u'_i, v'_i vertices corresponding to u, v, u_i, v_i, where $1 \le i \le n$.

Case 1: $n = 2$

In this case vertices u'_1, u'_2, v'_1, v'_2 are equitable isolates. Therefore they must belong to every equitable dominating set of $S'(B_{2,2})$. Observe that $N^e(u') = \{u_1, u_2, v\}$, $N^e(v') = \{v_1, v_2, u\}, N^e(u) = \{u_1, u_2, u'_1, u'_2, v\}$, $N^e(v) = \{v_1, v_2, v'_1, v'_2, u\}$. From this consider $D = \{u, u', v', u'_1, u'_2, v'_1, v'_2\}$. Then $N^e[D] = V(S'(B_{2,2}))$. Therefore D is an equitable dominating set of $S'(B_{2,2})$ and $\gamma^e(S'(B_{2,2})) = 7$. While vertices u'_1, u'_2, v'_1, v'_2 are equitable isolates and other vertices u', v', u are note equitably adjacent to each other. This implies that D is an equitable independent set of $S'(B_{2,2})$. Hence D is an equi independent equitable dominating set of $S'(B_{2,2})$ and $i^e(S'(B_{2,2})) = \gamma^e(S'(B_{2,2})) = 7$.

Case 2: $n \ge 3$

In this case $d(u'_i) = d(v'_i) = 1$, $d(u') = d(v') = n+1$, $d(u_i) = d(v_i) = 2$, $d(u) = d(v) = 2n+2$. Note that vertices u_i, v_i are not adjacent to each other and not adjacent to u'_i, v'_i. Then vertices $u_i, v_i, u'_i, v'_i, u', v'$ are equitable isolates. Therefore they must belong to every equitable dominating set of $S'(B_{n,n})$. While remaining vertices u and v are equitably adjacent to each other. Therefore at least one vertex from $\{u, v\}$ must belong to every equitable dominating set of $S'(B_{n,n})$. This implies that

$$\gamma^e(S'(B_{n,n})) \ge 4n+3$$

Now consider a set which satisfies above lower bound as $D = \{u_i, v_i, u'_i, v'_i, u', v', u / 1 \le i \le n\}$ with $|D| = 4n+3$. Here $N^e[D] = V(S'(B_{n,n}))$. Therefore D is an equitable dominating set of $S'(B_{n,n})$. Also D is an equitable independent set as all the vertices of D other than u are equitable isolates. Hence D is an equi independent equitable dominating set of $S'(B_{n,n})$ and $i^e(S'(B_{n,n})) = \gamma^e(S'(B_{n,n})) = 4n+3$.

III. Conclusion

The equi independent equitable dominating set is a combination of two concepts – equitable dominating set and equitable independent set. We derive equi independent equitable domination number of some cycle and bistar related graphs

References

[1] F. Harary, Graph Theory (Addison Wesley, Massachusetts, 1972).
[2] T. W. Haynes, S. T. Hedetniemi, and P. J. Slater, Fundamentals of domination in graphs (Marcel Dekker, New York, 1998).
[3] C. Berge, Theory of Graphs and its Applications (Methuen, London, 1962).
[4] O. Ore, Theory of graphs, Amer. Math. Soc. Transl. **38**, (1962), 206-212.
[5] E. J. Cockayne and S.T. Hedetniemi, Independence graphs, Congr. Numer., X, (1974), 471-491.
[6] E. J. Cockayne and S.T. Hedetniemi, Towards a theory of domination in graphs, Networks, 7, (1977), 247-261.
[7] W. Goddard and M. A. Henning, Independent domination in graphs: A survey and recent results, Discrete Mathematics, 313, (2013), 839-854.
[8] V. Swaminathan and K. M. Dharmalingam, Degree equitable domination on graphs, Kragujevac Journal of Mathematics, 35(1), (2011), 191-197.
[9] S. K. Vaidya and N. J. Kothari, Equi independent equitable dominating sets in graphs (Accepted for publication in Proyecciones Journal of Mathematics).
[10] S. K. Vaidya and N. J. Kothari, Some Results on Equi Independent Equitable Dominating Sets in Graphs, Journal of Scientific Research, 7(3), (2015), 77-85.
[11] S. K. Vaidya and N. J. Kothari, On equi independent equitable dominating sets in graphs (Accepted for publication in International Journal of Mathematics and Soft Comuting).
[12] S.K.Vaidya and Lekha Bijukumar, Some New Families of Mean Graphs, Journal of Mathematics Research, 2(3), 2010, 169-176.