IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 6 Ver. 1 (Nov. - Dec. 2015), PP 37-43
www.iosrjournals.org

A Mathematical Model for Unemployment with effect of self employment.

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Abstract: *In this paper we have indicated and analyzed a mathematical model of unemployment. In this nonlinear mathematical model we focused on three dynamic variables (i) Number of unemployed persons ,(ii) Number of employed persons and (iii) Number of newly created vacancies by intermediation of government and private sector. We assumed the situation that unemployed persons are also trying for their independent work and create chances for self employment. We studied stability of equilibrium point of the model. Numerical simulation is also given to compare with analytical result.*

Keywords: *dynamic variables, employed person, self employment, Unemployed person.*

I. Introduction

In present, Unemployment is one of the most serious issue for every country. Since last few decades population is constantly grow up and with this uncontrollable population unemployment is also raised. Unemployment affects economically as well as socially to a person and the country.

Nikolopoulos and Tzanetis developed a model for housing allocation of homeless families due to natural disaster [1]. Based on some concept of this paper Misra and Singh presented a nonlinear mathematical model for unemployment. In [3] they considered three dynamic variables (i) Number of unemployed persons denoted by *U*, (ii) Number of employed persons denoted *E*, (iii) Number of newly created vacancies through government intervention denoted by *V*. Time delay is considered proportional to newly created vacancies.

Inspired by ([2,3]) we considered a new model for better understanding of unemployment problem and it's possible solution. In this model we have assumed that there is no time delay by government and private sector in creating new vacancies. Also, assumed that unemployed persons try for independent work and create chances for self employment which is necessary to survive in their life.

The paper is organized as follows: Model of unemployment is described in section 2, An equilibrium analysis is described in section 3, The stability of equilibrium point is verify in section 4, Effects (characterization) of different variables on unemployment is presented in section 5, Numerical simulation is described in section 6 and conclusion is given in section 7.

II. Mathematical Model

In the process of making model we assume that all entrants of the category unemployment are fully qualified to do any job at any time *t*. Number of unemployed persons, U ,increases with constant rate a_1. Number of present jobs in the market provided by government and private sector is constant denoted by *P*. Also, government and private sector both try to create new vacancies, *V*, proportional to number of unemployed person without any delay but they unsuccessful at some rate due to lack of funds and less demand of human load. Also, there is possibility that employed persons, *E*, fired from their job or leave the job and joint to unemployed class. It is assume that rate of migration proportional to number of unemployed person. The rate of self employment is also assumed proportional to number of unemployed. So, number of available vacancies in the market is *(P+V-E)* and rate of movement from unemployed class to employed class is jointly proportional to *U* and *(P+V-E).*

$$\left.\begin{aligned}\frac{dU}{dt} &= a_1 - a_2U(P+V-E) - a_3U + a_4E - a_5U \\ \frac{dE}{dt} &= a_2U(P+V-E) - a_4E + a_5U - a_6E \\ \frac{dV}{dt} &= \alpha U - \beta V\end{aligned}\right\} \qquad (1)$$

a_2 = Rate of movement of unemployed persons to joint employed class,

a_3 = Rate of migration as well as death of unemployed person,

a_4 = Rate of employed persons who become unemployed because of losing their job (Either fired or leave the job)

a_5 = Rate of unemployed person who start their own independent work and become self employed,

a_6 = Rate of death and retirement of employed person,

α , β = Rate of newly created vacancies and diminution of newly created vacancies by government and private sector.

III. Equilibrium point

To obtain equilibrium point of system (1) we have to solve algebraic equations

$$a_1 - a_2U(P+V-E) - a_3U + a_4E - a_5U = 0 \qquad \text{\_\_\_\_\_\_\_\_ (2)}$$
$$a_2U(P+V-E) - a_4E + a_5U - a_6E = 0 \qquad \text{\_\_\_\_\_\_\_\_(3)}$$
$$\alpha U - \beta V = 0 \qquad \text{\_\_\_\_\_\_\_\_(4)}$$

From (3) we get

$$E = \frac{U[a_2(P+V)+a_5]}{a_2U + a_4 + a_6} \qquad \text{\_\_\_\_\_\_\_\_(5)}$$

From (4) we get $V = \dfrac{\alpha U}{\beta}$ ________(6)

Put the values of (5) and (6) in (2)

$$a_1 - a_2U\left(P + \frac{\alpha U}{\beta}\right) + \frac{a_2U^2\left[a_2\left(P + \frac{\alpha U}{\beta}\right) + a_5\right]}{a_2U + a_4 + a_6} - a_3U + \frac{a_4U\left[a_2\left(P + \frac{\alpha U}{\beta}\right) + a_5\right]}{a_2U + a_4 + a_6} - a_5U = 0$$

By taking L.C.M. and simplify the above equation we get

$$AU^2 + BU - C = 0$$

Where, $A = a_2a_6\alpha + a_2a_3\beta$, $B = \beta[a_2a_6P + a_3a_4 + a_3a_6 + a_5a_6 - a_1a_2]$,

$C = a_1a_2 + a_1a_6$

Since a_i, α, β *i=1,2,3,4,5,6* all are positive so, *A,B,C* are also positive.

Let, $h(U) = AU^2 + BU - C$

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Number of changes in signs of $h(U)$ is one so, $h(U)$ will have only one positive root say U^*, by put this value in (5) and (6) we get positive value of E, V say E^*, V^* That is system has only one non negative equilibrium say (U^*, E^*, V^*) .

IV. Stability Analysis

We can check the stability of equilibrium point (U^*, E^*, V^*) by calculate the variational matrix of system (1)

$$M = \begin{bmatrix} -a_2(P+V^*-E^*)-a_3-a_5 & a_2U^*+a_4 & -a_2U^* \\ a_2(P+V^*-E^*)+a_5 & -a_4-a_6-a_2U^* & a_2U^* \\ \alpha & 0 & -\beta \end{bmatrix}$$

The characteristic equation of the matrix is

$$\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0 \qquad \text{\_\_\_\_\_\_\_\_(7)}$$

Where,

$$b_1 = [a_2(P+V^*-E^*)+a_3+a_4+a_5+a_6+a_2U^*+\beta]$$

$$b_2 = \beta[a_4+a_6+a_2U^*+a_2(P+V^*-E^*)+a_3+a_5]+a_2\alpha U^*+a_3a_4+a_2a_6(P+V^*-E^*)$$
$$+a_3a_6+a_5a_6+a_2a_3U^*$$

$$b_3 = a_2a_6\alpha U^*+\beta[a_2a_6(P+V^*-E^*)+a_3a_4+a_3a_6+a_5a_6+a_2a_3U^*]$$

Since a_i, α, β , $i = 1,2,3,4,5,6$ all are positive so, b_1, b_2, b_3 are also positive.

By calculating $b_1b_2 - b_3$ we get,

$$b_1b_2 - b_3 = [a_2(P+V^*-E^*)+a_4+a_5+\beta+a_2U^*][\beta\{a_4+a_2U^*+a_2(P+V^*-E^*)+a_3+a_5\}$$
$$+a_2\alpha U^*+a_3a_4+a_2a_6(P+V^*-E^*)+a_3a_6+a_5a_6]+a_2a_3U^*[a_2(P+V^*-E^*)$$
$$+a_4+a_5+a_2U^*+a_3+a_6+\alpha]+\beta a_6[2a_4+a_2U^*+\beta+a_6]+(a_3+a_6)[\beta\{a_2U^*$$
$$+a_2(P+V^*-E^*)+a_3+a_5\}+a_3a_4+a_2a_6(P+V^*-E^*)+a_3a_6+a_5a_6]$$

Which contains all positive terms with positive elements

$$\therefore\ b_1b_2 - b_3 > 0$$

By Routh Hurwitz criterion all roots of (7) have negative real part and therefore equilibrium point (U^*, E^*, V^*) is locally asymptotically stable.

V. Characteristics of equilibrium values.

5.1 Characteristic of the equilibrium values of unemployed persons, employed persons and newly created vacancies with respect to a_2

$$f(U^*, E^*, a_2) = a_1 - a_2U^*(P + \frac{\alpha U^*}{\beta} - E^*) - a_3U^* + a_4E^* - a_5U^*$$

$$g(U^*, E^*, a_2) = a_2U^*(P + \frac{\alpha U^*}{\beta} - E^*) - a_4E^* + a_5U^* - a_6E^*$$

$$\frac{dU^*}{da_2} = \frac{\begin{vmatrix} \frac{\partial f}{\partial E^*} & \frac{\partial f}{\partial a_2} \\ \frac{\partial g}{\partial E^*} & \frac{\partial g}{\partial a_2} \end{vmatrix}}{\begin{vmatrix} \frac{\partial f}{\partial U^*} & \frac{\partial f}{\partial E^*} \\ \frac{\partial g}{\partial U^*} & \frac{\partial g}{\partial E^*} \end{vmatrix}} = \frac{\frac{\partial f}{\partial E^*}\frac{\partial g}{\partial a_2} - \frac{\partial f}{\partial a_2}\frac{\partial g}{\partial E^*}}{\frac{\partial f}{\partial U^*}\frac{\partial g}{\partial E^*} - \frac{\partial f}{\partial E^*}\frac{\partial g}{\partial U^*}} \qquad \text{\_\_\_\_\_\_\_\_(8)}$$

$$\frac{\partial f}{\partial U^*} = -a_2(P + \frac{\alpha U^*}{\beta} - E^*) - \frac{a_2U^*\alpha}{\beta} - a_3 - a_5$$

$$\frac{\partial f}{\partial E^*} = a_2U^* + a_4$$

$$\frac{\partial f}{\partial a_2} = -U^*(P + \frac{\alpha U^*}{\beta} - E^*)$$

$$\frac{\partial g}{\partial U^*} = a_2(P + \frac{\alpha U^*}{\beta} - E^*) + \frac{a_2\alpha U^*}{\beta} + a_5$$

$$\frac{\partial g}{\partial E^*} = -a_2U^* - a_4 - a_6$$

$$\frac{\partial g}{\partial a_2} = U^*(P + \frac{\alpha U^*}{\beta} - E^*)$$

$$\frac{\partial f}{\partial E^*}\frac{\partial g}{\partial a_2} - \frac{\partial g}{\partial E^*}\frac{\partial f}{\partial a_2} = -a_6U^*(P + \frac{\alpha U^*}{\beta} - E^*) < 0$$

$$\frac{\partial f}{\partial U^*}\frac{\partial g}{\partial E^*} - \frac{\partial f}{\partial E^*}\frac{\partial g}{\partial U^*} = a_2a_6(P + \frac{\alpha U^*}{\beta} - E^*) + a_3a_4 + a_3a_6 + a_5a_6 + a_2a_3U^* + \frac{a_2a_6\alpha U^*}{\beta} > 0$$

In $\frac{dU^*}{da_2}$, $\frac{\partial f}{\partial E^*}\frac{\partial g}{\partial a_2} - \frac{\partial g}{\partial E^*}\frac{\partial f}{\partial a_2} < 0$ and $\frac{\partial f}{\partial U^*}\frac{\partial g}{\partial E^*} - \frac{\partial f}{\partial E^*}\frac{\partial g}{\partial U^*} > 0$

i.e. in $\frac{dU^*}{da_2}$ numerator is negative and denominator is positive.

$\therefore$ $\frac{dU^*}{da_2} < 0$, which shows that a_2 increases then U^* decreases.

Now,

$$\frac{dE^*}{da_2} = \frac{\begin{vmatrix} \frac{\partial f}{\partial a_2} & \frac{\partial f}{\partial U^*} \\ \frac{\partial g}{\partial a_2} & \frac{\partial g}{\partial U^*} \end{vmatrix}}{\begin{vmatrix} \frac{\partial f}{\partial U^*} & \frac{\partial f}{\partial E^*} \\ \frac{\partial g}{\partial U^*} & \frac{\partial g}{\partial E^*} \end{vmatrix}} = \frac{\frac{\partial f}{\partial a_2}\frac{\partial g}{\partial U^*} - \frac{\partial f}{\partial U^*}\frac{\partial g}{\partial a_2}}{\frac{\partial f}{\partial U^*}\frac{\partial g}{\partial E^*} - \frac{\partial f}{\partial E^*}\frac{\partial g}{\partial U^*}}$$

$$\frac{\partial f}{\partial a_2}\frac{\partial g}{\partial U^*} - \frac{\partial f}{\partial U^*}\frac{\partial g}{\partial a_2} = a_3U^*(P + \frac{\alpha U^*}{\beta} - E^*) > 0$$

In $\frac{dE^*}{da_2}$, $\frac{\partial f}{\partial a_2}\frac{\partial g}{\partial U^*} - \frac{\partial f}{\partial U^*}\frac{\partial g}{\partial a_2} > 0$ and $\frac{\partial f}{\partial U^*}\frac{\partial g}{\partial E^*} - \frac{\partial f}{\partial E^*}\frac{\partial g}{\partial U^*} > 0$

i.e. In $\frac{dE^*}{da_2}$ numerator and denominator both are positive.

So, a_2 increases then E^* increases

From $V = \frac{\alpha U^*}{\beta}$ we get $\frac{dV^*}{da_2} = \frac{\alpha}{\beta}\frac{dU^*}{da_2} < o$

$\therefore$ a_2 increases then V^* decreases.

5.2 Characteristic of the equilibrium values of unemployed persons, employed persons and newly created vacancies with respect to α .

Now let,

$$f(U^*, E^*, \alpha) = a_1 - a_2U^*(P + \frac{\alpha U^*}{\beta} - E^*) - a_3U^* + a_4E^* - a_5U^*$$

$$g(U^*, E^*, \alpha) = a_2U^*(P + \frac{\alpha U^*}{\beta} - E^*) - a_4E^* + a_5U^* - a_6E^*$$

$$\frac{dU^*}{d\alpha} = \frac{\frac{\partial f}{\partial E^*}\frac{\partial g}{\partial \alpha} - \frac{\partial f}{\partial \alpha}\frac{\partial g}{\partial E^*}}{\frac{\partial f}{\partial U^*}\frac{\partial g}{\partial E^*} - \frac{\partial f}{\partial E^*}\frac{\partial g}{\partial U^*}}$$

In $\frac{dU^*}{d\alpha}$, $\frac{\partial f}{\partial E^*}\frac{\partial g}{\partial \alpha} - \frac{\partial g}{\partial E^*}\frac{\partial f}{\partial \alpha} < 0$ and $\frac{\partial f}{\partial U^*}\frac{\partial g}{\partial E^*} - \frac{\partial f}{\partial E^*}\frac{\partial g}{\partial U^*} > 0$

i.e. in $\frac{dU^*}{d\alpha}$ numerator is negative and denominator is positive.

$\therefore$ $\frac{dU^*}{d\alpha} < 0$, which shows that α increases then U^* decreases.

Similarly,

$$\frac{dE^*}{d\alpha} = \frac{\frac{\partial f}{\partial \alpha}\frac{\partial g}{\partial U^*} - \frac{\partial f}{\partial U^*}\frac{\partial g}{\partial \alpha}}{\frac{\partial f}{\partial U^*}\frac{\partial g}{\partial E^*} - \frac{\partial f}{\partial E^*}\frac{\partial g}{\partial U^*}}$$

In $\frac{dE^*}{d\alpha}$, $\frac{\partial f}{\partial \alpha}\frac{\partial g}{\partial U^*} - \frac{\partial f}{\partial U^*}\frac{\partial g}{\partial \alpha} > 0$ and $\frac{\partial f}{\partial U^*}\frac{\partial g}{\partial E^*} - \frac{\partial f}{\partial E^*}\frac{\partial g}{\partial U^*} > 0$

i.e. In $\frac{dE^*}{d\alpha}$ numerator and denominator both are positive.

So, α increases then E^* increases

From $V = \frac{\alpha U^*}{\beta}$ we get $\frac{dV^*}{d\alpha} = \frac{U^*}{\beta} > 0$

$\therefore$ α increases then V^* increases.

5.3 Characteristic of the equilibrium values of unemployed persons and employed persons with respect to a_5

Let,

$$f(U^*, E^*, a_5) = a_1 - a_2 U^*(P + \frac{\alpha U^*}{\beta} - E^*) - a_3 U^* + a_4 E^* - a_5 U^*$$

$$g(U^*, E^*, a_5) = a_2 U^*(P + \frac{\alpha U^*}{\beta} - E^*) - a_4 E^* + a_5 U^* - a_6 E^*$$

$$\frac{dU^*}{da_5} = \frac{\frac{\partial f}{\partial E^*}\frac{\partial g}{\partial a_5} - \frac{\partial f}{\partial a_5}\frac{\partial g}{\partial E^*}}{\frac{\partial f}{\partial U^*}\frac{\partial g}{\partial E^*} - \frac{\partial f}{\partial E^*}\frac{\partial g}{\partial U^*}}$$

In $\frac{dU^*}{da_5}$, $\frac{\partial f}{\partial E^*}\frac{\partial g}{\partial a_5} - \frac{\partial g}{\partial E^*}\frac{\partial f}{\partial a_5} < 0$ and $\frac{\partial f}{\partial U^*}\frac{\partial g}{\partial E^*} - \frac{\partial f}{\partial E^*}\frac{\partial g}{\partial U^*} > 0$

i.e. in $\frac{dU^*}{da_5}$ numerator is negative and denominator is positive.

$\therefore$ $\frac{dU^*}{da_5} < 0$, which shows that a_5 increases then U^* decreases.

Similarly,

$$\frac{dE^*}{da_5} = \frac{\frac{\partial f}{\partial a_5}\frac{\partial g}{\partial U^*} - \frac{\partial f}{\partial U^*}\frac{\partial g}{\partial a_5}}{\frac{\partial f}{\partial U^*}\frac{\partial g}{\partial E^*} - \frac{\partial f}{\partial E^*}\frac{\partial g}{\partial U^*}}$$

In $\frac{dE^*}{da_5}$, $\frac{\partial f}{\partial a_5}\frac{\partial g}{\partial U^*} - \frac{\partial f}{\partial U^*}\frac{\partial g}{\partial a_5} > 0$ and $\frac{\partial f}{\partial U^*}\frac{\partial g}{\partial E^*} - \frac{\partial f}{\partial E^*}\frac{\partial g}{\partial U^*} > 0$

i.e. In $\frac{dE^*}{da_5}$ numerator and denominator both are positive.

So, a_5 increases then E^* increases.

From, above discussion we analyze that at the equilibrium the number of unemployed persons decrease as a_2 increases. (i.e. at equilibrium level rate of movement of unemployed person to employed class increases then number of unemployed persons decrease.) , number of employed persons increase and newly created vacancies decrease when a_2 increases. As well as α and a_5 (i.e. rate of newly created vacancies and rate of self employment) increases then number of unemployed persons decrease and number of employed persons increase.

VI. Numerical Simulation

For the Numerical Simulation through Bisection method we consider the following data,

$a_1 = 5000$, $P = 15000$, $a_2 = 0.00001$, $a_3 = 0.05$, $a_4 = 0.0001$,

$a_5 = 0.07$, $a_6 = 0.05$, $\alpha = 0.3$, $\beta = 0.2$

So, the equilibrium values of model are:

$U^* = 34932$, $V^* = 52398$, $E^* = 65066$

The eigenvalues of the matrix corresponding to the equilibrium (U^*, E^*, V^*) of model system (1) are: $-0.3463 + 0.2888\,i$, $-0.3463 - 0.2888\,i$, -0.0501 which are either negative or have a negative real part. So, the equilibrium (U^*, E^*, V^*) is locally asymptotically stable.

Using above data , Fig.1, Fig.2, Fig.3 represent the variations in the number of unemployed persons with respect to time with different values of a_2, α and a_5 respectively. From Fig.1 it is observe that as the rate of movement of unemployed persons in employed class increases then number of unemployed decrease, from Fig.2 it is clear that if rate of newly created vacancies is higher then number of unemployed persons decrease and Fig.3 represent that with high rate of self employment rate of unemployment is law.

Figure 1

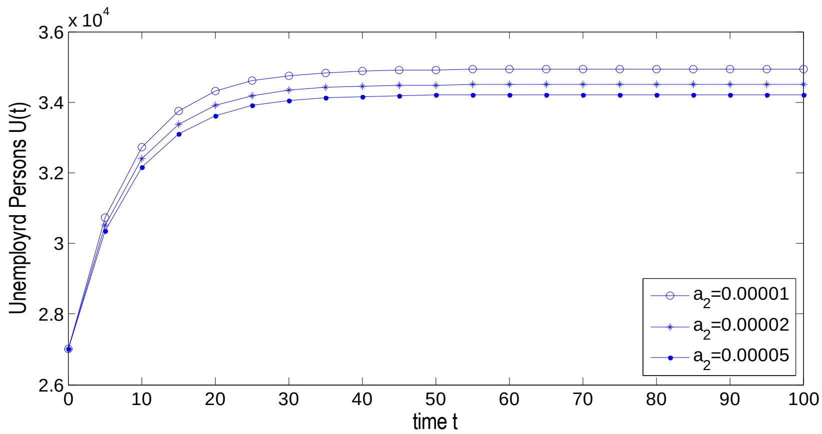


Figure 2

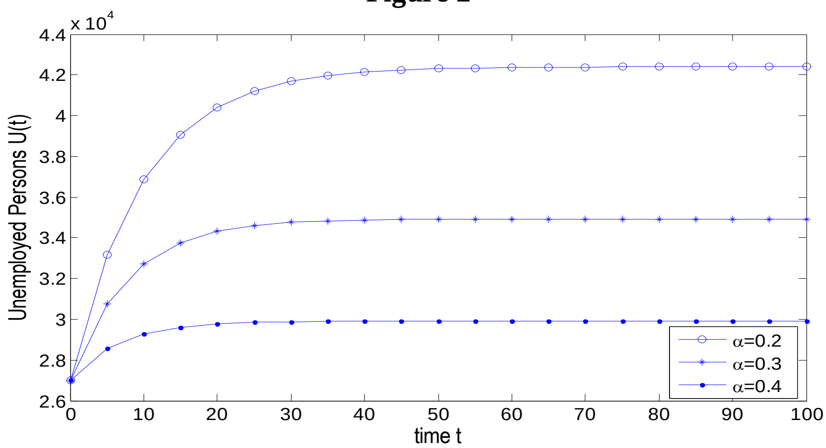
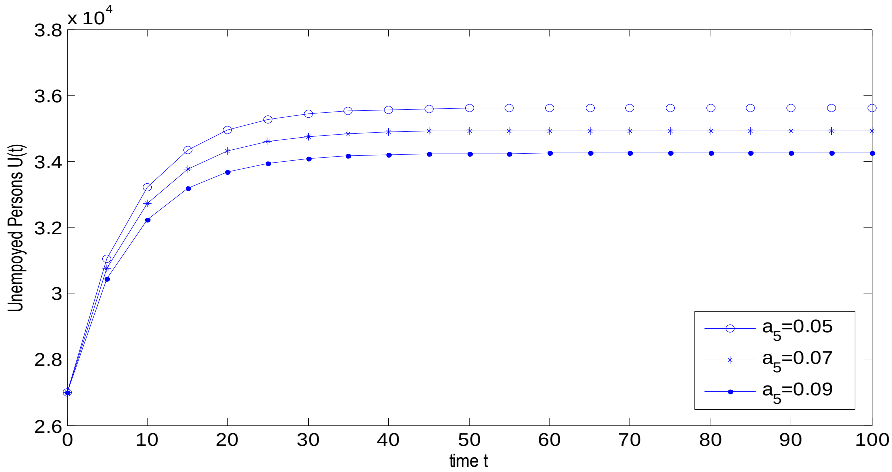


Figure 3



VII. Conclusion

In this paper, we described and analyzed a non linear mathematical model of unemployment for better understanding of unemployment problem and their possible solutions. We check the stability of the equilibrium and verify it with numerical data. We find that if rate of newly created vacancies increases then unemployment decreases. That is if government or private sector create more new vacancies proportional to number of unemployed persons then unemployment can control at some level. Also, if unemployed person try for their independent business (at any level) then it is helpful to survive the person as well as control the unemployment. Unemployment is a serious problem and it can control by the efforts of government and private sector by creating new vacancies and also by individual unemployed person by taking a step of self employment.

References

[1] C.V.Nikolopoulos, DE Tzanetis, A model for housing allocation of a homeless population due to natural disaster, Nonlinear Anal.4,2003,pp.561-579.

[2] A.K.Misra, A.K.Singh, A mathematical model for unemployment, IJTPC 4,2013 pp.2322-3138.

[3] A.K.Misra, A.K.Singh, A Delay mathematical model for the control of unemployment , Differ Equ Dyn Syst 21(3), 2013,pp.291-307.

[4] N. Sirghi, M.Neamtu , Deterministic and stochastic advertising diffusion model With delay, WSEAS Transaction on system and control, 4(8), 2013,pp. 141-150.

[5] M. Neamtu, M.Pirtea, G. Mircea, D. Opris, Stochastic , Fuzzy, Hybrid delayed Dynamics Heterogeneous competitions with Product Differentiation, WSEAS Transaction on mathematics , 7(9), 2010, pp.581-591.

[6] M.Neamtu, A dynamic model for unemployment control with migration and distributed delay, Mathematical Methods in finance and business Administration,Proceeding of the International Business Administration conference, Tenerife, Spain, 1,2014, pp.23-31.

[7] N.Sirghi, M.Neamtu, A dynamic model for unemployment control with distributed delay, Mathematical Methods in finance and business Administration,Proceeding of the International Business Administration conference, Tenerife Spain, 1,2014, pp.42-48