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A mathematical model of two phase, (One phase is Newtonian and other is non-Newtonian) layered renal blood flows in capillaries remote from the heart and proximate to the kidney with special reference to Diabetes

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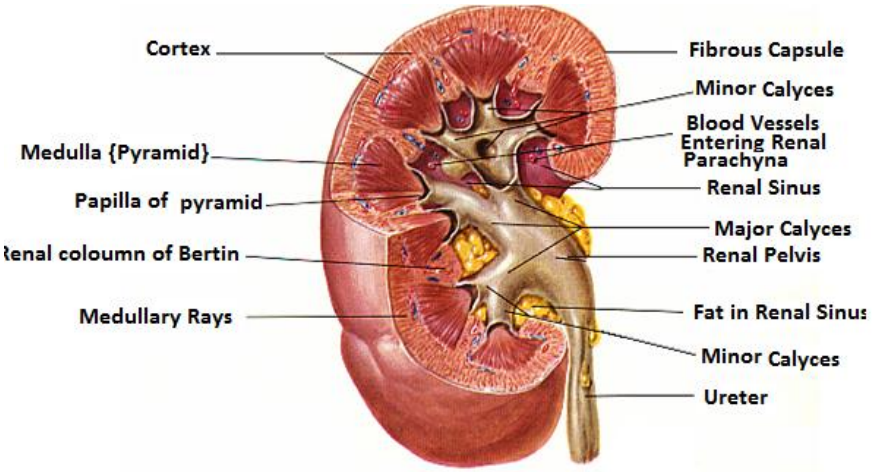
Abstract: *In the present paper we have formulated the renal blood flow along the capillaries in case of renal disease Diabetes . keeping in the view the nature of renal circulatory system in human body. P.N.Pandey and V.Upadhyay have considered the blood flow has two phased one of which is that of red blood cells and other is plasma. According to Fahreaus-Lindqvist effect the blood flow in two separated layers while passing through capillaries. The plasma layer which flows along the surface of the capillaries contains almost no blood cells. The second layer the core layer containing blood cells which flows in plasma along the axis of capillary. We have collected a clinical data in case of Diabetes for hematocrit v/s blood pressure. The graphical presentation for particular parametric value is much closed to the clinical observation. The overall presentation is in tensorial form and solution technique adapted is analytical as well as numerical. The role of hematocrit is explicit in the determination of blood pressure drop in case of renal disease Diabetes*

Keywords*: Pressure drop, hematocrit, renal circulation, Glomerular capillary, Diabetes etc.*

I. Bio-physical problem(Kidney)

The kidney has bean shaped structure that serve the several essential regulatory roles in vertebrates , each kidney has a convex and concave surfaces. The concave surface , the renal hilum , is the point at which the renal artery enter the organs and renal veins & ureter leave . The kidney is surrounded by tough fibrous tissue , renal capsule, which is itself surrounded by perinephric fat , renal fascia(Gerota) and paranephric fat .The anterior part of this tissue is peritoneum , while the posterior (rear) border is the transversalis fascia.[1][2][3] The normal adult kidney is about 10-12 cm long , 5-7 cm wide and 2-3 cm thick and its weighs 125-170g Each kidney composed of parenchyama and collecting system . The parenchyama cosists of an outer cortex and inner medulla .The medulla is divided into an outer (towards the cortex) and inner medulla (toward pelvis) . The collecting system includes the calyces, renal pelvis and the ureter . The major calyces unite to form the renal pelvis . The renal pelvis drain into ureter which connect the kidney to the bladder . [4][5][6] .



[7]

Fig.1

1.1 Function:- Kidney perform three major type of of functions (i) Maintenance of fluid and acid –base balance (ii) Removal of Nitrogenous waste products (iii) synthesis of hormones ; such as renin , erythropoietin and active vitamin D_3 (calcitriol) [8] Each human kidney contains about one million nephrons (Fig. 22.2), each of which consists of a renal corpuscle and a renal tubule. The renal corpuscle consists of a tuft of capillaries, the glomerulus, surrounded by Bowman's capsule. The renal tubule is divided into several segments. The part of the tubule nearest the glomerulus is the proximal tubule. This is subdivided into a proximal convoluted tubule and proximal straight tubule. The straight portion heads toward the medulla, away from the surface of the kidney. The loop of Henle includes the proximal straight tubule, thin limb, and thick ascending limb. Connecting tubules connect the next segment, the short distal convoluted tubule, to the collecting duct system. Several nephrons drain into a cortical collecting duct, which passes into an outer medullary collecting duct. In the inner medulla, inner medullary collecting ducts unite to form large papillary ducts.

1.2 Blood Supply :- Each kidney is typically supplied by a single renal artery, which branches into anterior and posterior divisions, which give rise to a total of five segmental arteries. The segmental arteries branch into interlobar arteries, which pass toward the cortex between the kidney lobes. At the junction of the cortex and medulla, the interlobar arteries branch to form arcuate arteries. These, in turn, give rise to smaller cortical radial arteries, which pass through the cortex toward the surface of the kidney. Several short, wide, muscular afferent arterioles arise from the cortical radial arteries. Each afferent arteriole gives rise to a glomerulus. The glomerular capillaries are followed by an efferent arteriole. The efferent arteriole then divides into a second capillary network, the peritubular capillaries, which surround the kidney tubules. Venous vessels, in general, lie parallel to the arterial vessels and have similar names.[9] In resting adult kidney receive 1.2 to 1.3 l blood per minut or 25% of cardiac output . Renal Blood flow canbe measured with electromagnetic or other type of flow meter or it canbe determined by applying the Fick principle [12]

From renal plasma flow , the renal blood flow can be calculated by dividing by one minus the hematocrit : Hematocrit (HCT) - 45% The renal Blood flow = RPF ×1/(1-HCT) → 700×1/(1-0.45) = 1273 ml/ Minut [45]

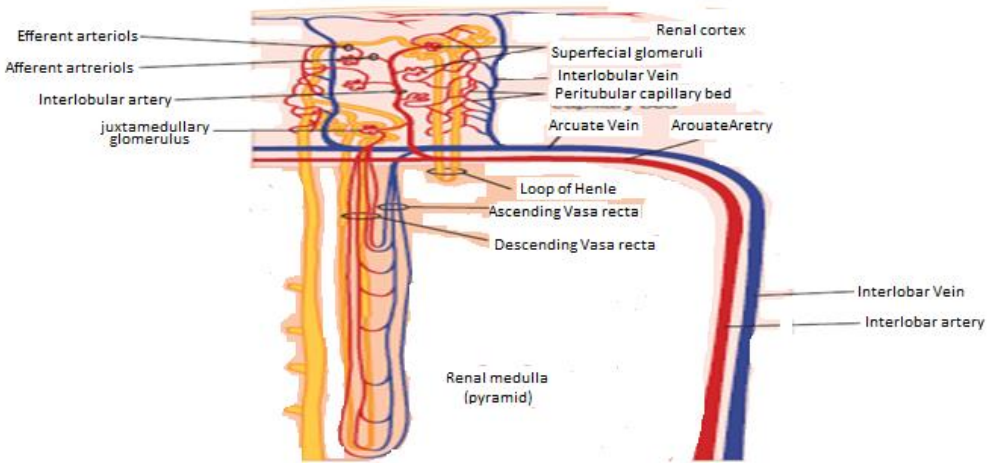


Fig.-2

1.3 Nephron is the functional unit of the kidney :- Each Human kidney contains about one million nephrons , each capable of forming urine .The kidney cannot regenerate new nephrons , therefore with renal injury , disease or normal aging , there is a gradual decrease in nephron numbers .After the the age of 40 years number of functioning nephrons usually decrease about 10 percent every 10 years thus the age of 80 many people have 40 percent fewer functioning nephrons than they did at age of 40

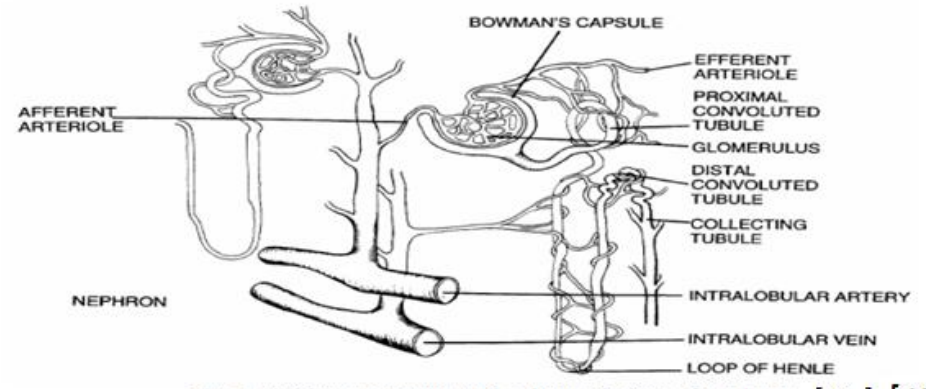


Fig 1.: Kidney anatomy and enhanced view of nephron. [13] [14]

Fig-3

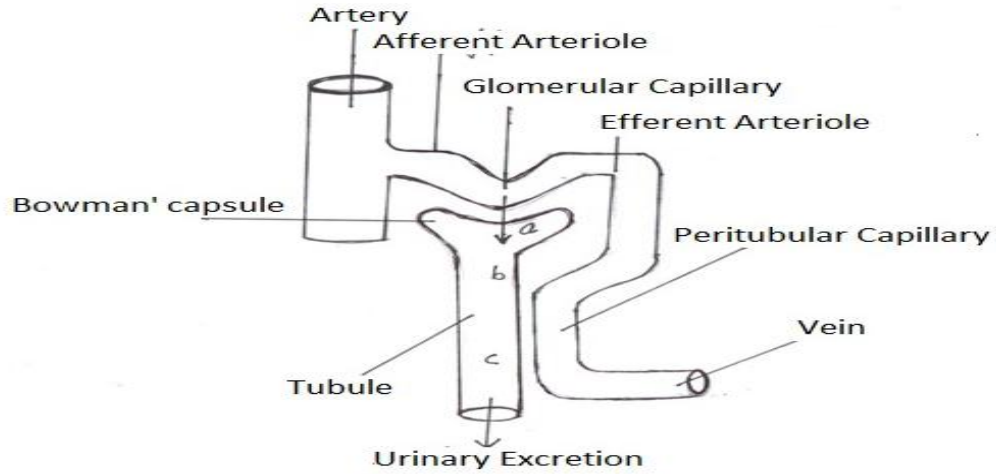


Fig-4

1.4 Pressure in renal Vessels :- The pressure in glomerular capillary has been measured directly in the rat and has been found to be considerably lower than the predicted on thye basis of indirect measurement . When the mean systolic arterial pressure is 100 mmhg , then glomerular capillary pressure is about 45 mmhg . The pressure drop across the glomerulas is only 1 to 3 mmhg , but further drop occurs in the efferent arteriole sothat the pressure in the peritubular capillary is about 8 mmhg . The pressure in renal vein is about 4 mmhg .The pressure gradient are similar to squirrel monkey end presumably in Human with glomerular capillary pressure that is about 40% of systolic arterial pressure [12]

1.5 Blood :- Blood is a characterized fluid of Body that delivers required substances that are the needs of body's cells, such as the oxygen nutrients as well as it transport the waste products away from these cells [15] .Blood complete so many important functions when it is circulated through the body . It transports the oxygen from the lungs to other body tissue and carried away carbon dioxide and excrete the waste products by the kidney . Blood help our body fight off infectious agent and inactivates toxins and it regulate our body temperature .[16] Human blood Contain so many important properties and there is no other substitute which has all properties . In 15^{th} ,And 16^{th} century the idea of blood transfusion was appeared .[17][18] The Blood has the potential components like packed red cells , Platelet rich plasma , fresh frozen plasma , cryoprecipitate and leucocytes [19][20][21][22][23] . Blood and its components are most important part of patient management treatment protocols.[19/36]

Blood cantained 7% of body weight [24][25]. With an average density of appr. 1060kg/m^3 and it is very closed to the desity of water [26] The behavior of the blood two phase non-Newtonian . Viscosity of blood depends on acting Shear force and it is determined by Hematocrit value . From Biological point of view Blood can be considered as tissue to be composed of various cells like as RBC, WBC,and PLETLETS, and liquid cellular material (Plasma) ,but from rheological point of view Blood is considered as two phased liquid . Approx 40% to 45% of the blood volume is occupied by blood cellular elements are RBC . There are 5 millions RBC in 1mm^3 of blood but only 5000 white Blood cells and 300000 Platelets exit in the same volume .[27] [28]

1.6 Disease (diabetes):- Dibetes is the chronic disease that occurs when the pancreas does not producing enough insuline or when the body cannot effectively use the insulin it produces. Diabetes is the life threating condition affecting the millions of people .[29] [30]

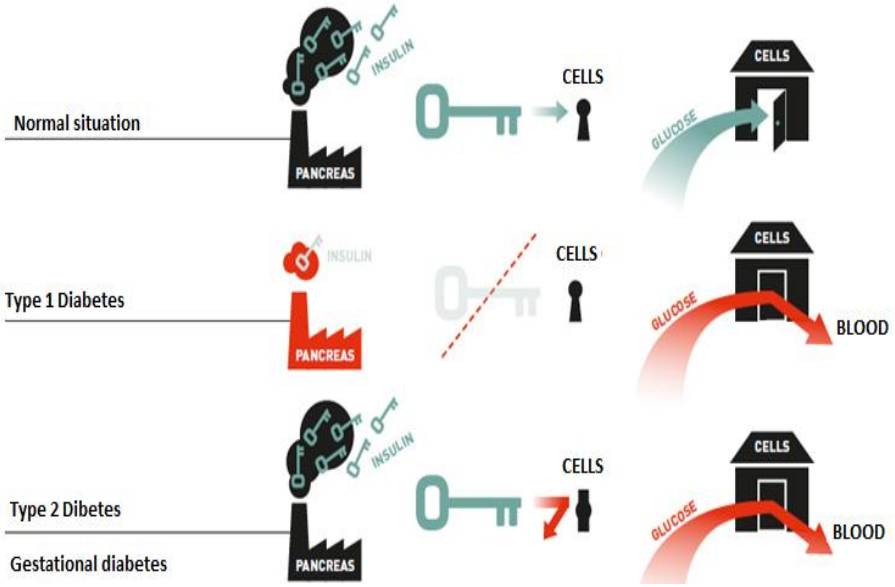
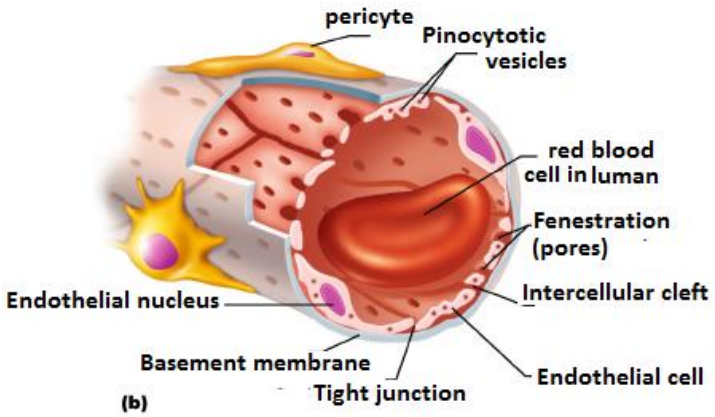


Fig.-5

II. Structure & functions of Renal Capillaries-

The renal circulation is unique in that it has two capillary beds, the glomerular and pertibular capillaries, which are arranged in series and separated by the efferent arterioles that help, regulate the hydrostatic pressures in both sets of capillaries. High hydrostatic pressure in the glomerular capillaries (about 60 mm Hg) causes rapid fluid filtration, whereas much lower hydrostatic pressure in the pertibular capillaries (about 13 mm Hg) permits rapid fluid reabsorption. [32] The pertibular capillaries empty into the vessels of the venous system, which run parallel to the arteriolar vessels and progressively forms the interlobular vein arcuate vein, interlobar vein and renal vein, which leaves the kidney beside the renal artery and ureter. The glomerulus is a tuft of small blood vessels called capillaries located within Bowman's capsule within the kidney [33]



[34] Fig.-6

1.8 Both layers are Newtonian-

1.8.1 Description of the problem-

How the blood flow in capillaries is possible as we know that these vessels are far enough from the heart as well as thin. It's a natural question because the blood flows very slowly in arterioles where there is high viscosity. The satisfactory answer of this problem is given by Fahreaus-Lindqvist effect. According to this effect the blood flows in two separated layers while passing through capillaries. The plasma layer containing almost no blood cells. The second layer is that of blood cells. The second layer is that of blood cells which float in plasma on the axis of the capillary. In this process the effective blood viscosity depends upon radius of the capillary. That's why the effective viscosity decreases, as the radius and thus the blood flow becomes possible.

1.9 Real Model

Blood is a complex fluid consisting of particulate corpuscles suspended in a non-Newtonian fluid. The particulate solids are red blood cells (RBCs), white blood cells (WBCs) and platelets. 55% of the plasma and 45% of the blood cells in a whole blood and approximately 98% of RBCs in 45% of blood cells and there are a few parts (approximately 2%) of the other cells. Which are ignorable, so one phase of the bloods plasma and 2nd phase of blood is RBCs.[35]

The first and foremost reason is that the blood is not an ideal fluid but it is a mixture of the two phases one is of plasma and other one is of blood cells. These blood cells, semi permeable packages of liquid of a density greater than that of plasma, are capable of changing their shape and size while flowing through different blood vessels [36]. Plasma is a liquid containing semi permeable packages of RBCs.

The behavior of blood is almost Newtonian at high shear rate, while at low shear rate the blood exhibits yield stress and non-Newtonian behavior [37]. We have selected generalized three dimensional orthogonal curvilinear co-ordinate system, briefly prescribed as E3 called as 3-dim Euclidean space. Here we have some quantities related to moving blood in cylindrical vessels: blood velocity $V^k = V^k(x^i, t)$, k=1,2,3 blood pressure $P = p(x^i,t)$ and density $\rho = \rho(x^i, t)$ where x^i be the co-ordinates of any point in space and i-1,2,3

If let us consider that the both phases- plasma and blood cells are equally distributed in whole blood. Then blood treated as homogeneous mixture.

Equation of Continuity-

When there is absence of source and sink in any region of flowing fluid, the fluid mass is conserved in that region. As we observed that there is no source or sink in the whole circuit of the human blood circulatory system, the heart behaves merely like a pumping station, so the law of conservation of mass can well be applied to hemodynamic [38]. Since, whole blood flow circuit of the kidney is called a Renal Circulatory System. Hence renal circulatory system is a sub system of human circulatory system. Blood enter in kidney by arteries and out by veins and in a kidney no source or sink.

Mass of enter the blood = mass of outer the blood

Therefore law of conservation of mass can also be applied for renal circulatory system.

The flow of blood is affected by the presence of blood cells. This effect is directly proportional to the volume occupied by blood cells.

Let X is the volume portion covered by the blood cells in unit volume. And X can be replaced by H/100, where H is the hematocrit the volume percentage of blood cells. Then the volume portion covered by plasma will be 1-X. if the mass ratio of blood cells to plasma is r, then clearly

$$r = \frac{x\rho_c}{(1-x)\rho_p}$$

Where ρ_c and ρ_p are densities of blood cells and blood plasma respectively. Usually this mass ratio is not constant; even then this may be supposed to be constant in present context [39].

The both phase of blood, i.e., blood cells and plasma move with a common velocity. Campbell and Pitcher have presented a model for this situation. According to this model we consider the two phases of blood separately [40]. Hence according to principle of conservation of mass, the equations of continuity for the two phases are as follows [41].

$$\frac{\partial(X\rho_c)}{\partial t} + \left(X\rho_c V^i\right)_{,i} = 0$$

$$\frac{\partial(1-X)\rho_p}{\partial t} + \left((1-X)\rho_p V^i\right)_{,i} = 0$$

Where v is the common velocity of the two phases blood cells and plasma and $\left(X\rho_c V^i\right)_{,i}$ is co-variant derivative of $\left(X\rho_c V^i\right)$ with respect to X^i, in the same way $\left((1-X)\rho_p V^i\right)$ with respect to X^i.

If we define the uniform density ρ_m as follows:

$$\frac{1+r}{\rho_m} = \frac{r}{p_c} + \frac{1}{\rho_p} \qquad (1.91)$$

Then the equations can be combined together as follows;

$$\frac{\partial \rho_m}{\partial t}+\left(\rho_m V^i\right)_{,i}=0$$

As we know that blood is incompressible fluid, hence ρ_m will be a constant quantity. Thus the equation of continuity for blood flow takes the following form:

$$V_{,i}^{\ i}=0$$

i.e.

$$\frac{\partial V^i}{\partial X^i}+\frac{V^i\partial\sqrt{g}}{\sqrt{g}\partial X^i}=\frac{1}{\sqrt{g}}\left(\sqrt{g}V^i\right)_{,i}=0$$

Equation of Motion- According to this principle, the total momentum of any fluid system is conserved in absence of external force. So the law of conservation of momentum can well apply to renal circulatory system. In other words, the rate of change of momentum of a fluid particle with respect to time equals to external force exerted on it. This is also called Newton's 2nd law of motion.

So, the rate of change of momentum is equal to sum of about two mentioned forces, which may be symbolically presented as follows.

$$\frac{dp}{dt}=-P+F$$

Where,

$\frac{dp}{dt}$ = Rate of change of momentum

P= internal pressures

F= viscous force

The hydro dynamical pressure p between the two phases of blood can be supposed to be uniform because the both phases i.e. blood cells and plasma are always in equilibrium state in blood [42]. Taking viscosity coefficient of blood cells to be η_c and applying the principle of conservation of momentum, we get the equation of motion for the phase of blood cells as follows:

$$X\rho_c\frac{\partial v^i}{\partial t}+\left(X\rho_c v^i\right)v_{,j}^{\ i}=-Xp_{,j}g^{ij}+X\eta_c\left(g^{jk}v_{,k}^{\ i}\right)_{,j} \qquad (1.92)$$

Similarly, taking the viscosity coefficient of plasma to be η_p the equation of motion for plasma will be as follows:

$$(1-X)\rho_p\frac{\partial v^i}{\partial t}+\left\{(1-X)\rho_p v^j\right\}v_{,j}^{\ i}=-(1-X)p_{,j}g^{ij}+(1-X)\eta_p\left(g^{jk}v_{,k}^{\ i}\right)_{,j} \qquad (1.93)$$

Now adding equations (1.92) and (1.93) and using relation (1.91) , the equation of motion for blood flow with the both phases will be as follows:

$$\rho_m\frac{\partial v^i}{\partial t}+\left(\rho_m v^j\right)v_{,j}^{\ i}=-p_{,j}g^{ij}+\eta_m\left(g^{jk}v_{,k}^{\ i}\right)_{,j}$$

Where

$\eta_m=X\eta_c+(1-X)\eta_p$ Is the viscosity coefficient of blood as a mixture of two phases.

Different constitutive equations for blood

Generally blood is non-homogeneous mixture of plasma and blood cells. Though for practical purposes it may be considered to be homogeneous two-phase mixture of plasma and blood cells. The constitutive equations proposed for whole blood mixture are as follows:

(1) Newtonian equation

$$\tau=\eta e$$

Where η is the viscosity coefficient.

This is found to hold good in the broad blood vessels where there is low hematocrit [43].

(2) The non-Newtonian power law equation

$$\tau = \eta e^{n}$$

This is found to be conformable for strain rate between 5 and 200 sec-1,

$$0.68 \le n \le 0.80 \quad [44]$$

The non-Newtonian Herschel – Bulkley equation [45]

$$\tau = \eta e^{n} + \tau_0 \left(\tau \ge \tau_0\right)$$

$$e = 0 \left(\tau < \tau_0\right)$$

It holds good when blood shows yield stress τ_0. We notice that the yield stress arise because blood cells form aggregates in the form of rouleaux at low strain rate.

If $\tau < \tau_0$, no blood flow-takes place. It is found that yield stress is given by the following formula:

$$\tau_0^{\frac{1}{3}} = \frac{A\left(H\text{-}H_m\right)}{100}$$

Where, $A = \left(0.008 \pm 0.002\,\text{dyne/cm}^2\right)^{\frac{1}{3}}$

H is normal hematocrit and H_m is the hematocrit below which there is no yield stress.

Hematocrit-

Hematocrit is the volume percentage (%) of red blood cells in blood. It is normally 45% for men and 40% for women. [46] Hematocrit is the most important determinant of whole blood viscosity. [47] Blood viscosity and vascular resistance affect total peripheral resistance to blood flow,[48] According to Berkow, Robert The hematocrit (expressed as percentage points) is normally about three times the hemoglobin concentration (reported as grams per deciliter).[49]

Boundary Conditions are as follows:

1. The velocity of blood flow on the axis of capillaries at r=0 will be maximum and finite, say V_0 = maximum velocity
2. The velocity of blood flow on the wall of blood vessels at r=R, where, R is the radius of capillary, will be zero. This condition is well known as no-slip condition.

1.8.2 Mathematical Modeling-

We consider the two layer blood flow to be Newtonian. The first layer is that of plasma while second one is core layer. Let the viscosity of plasma layer be η_p and that of core layer η_m where $\eta_m = X\eta_c + \left(1\text{-}X\right)\eta_p$ where η_c is viscosity of blood cells and X is portion of blood cells in unit

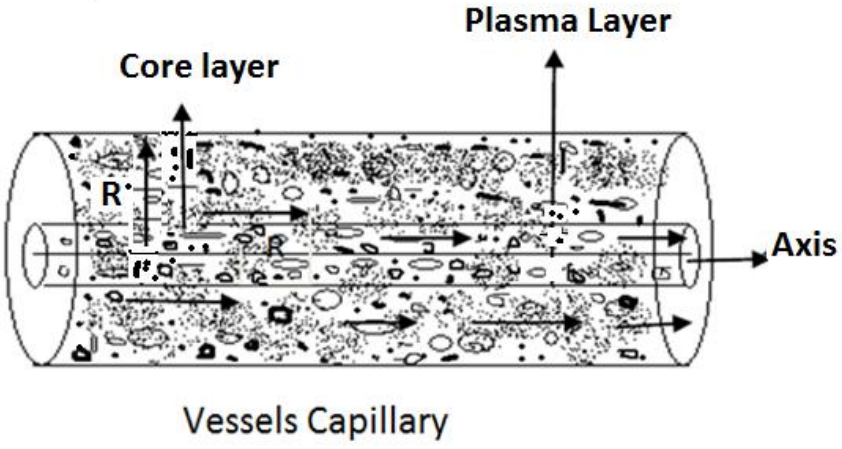


Fig-7

1.9 Solution-

Now we describe the basic equations for Power law blood flow as follows:

1.91 Equation of Continuity

in tensorial form as follows:

$$\frac{1}{\sqrt{g}}\left(\sqrt{g}v^{i}\right)_{,i} = 0$$ ……..(1.911)

1.9.2 Equation of motion:

$$\rho_m \frac{\partial v^{i}}{\partial t} + \rho_m v^{j} v^{i}_{,j} = -\rho_{,j} g^{ij} + \eta_m \left(g^{jk} v^{i}_{,k}\right)_{,j}$$ …….(1.921)

Where,

η_p = Viscosity of Plasma layer

η_m = Viscosity of core layer

η_c = Viscosity of blood cells

$\eta_m = X\eta_c + (1\text{-}X)\eta_p$

X = portion of blood cells in unit volume

X = H/100

ρ_m = density of mixture blood

ρ_p = density of plasma

ρ_c = density of blood cells

$$\rho_m = X\rho_c + (1\text{-}X)\rho_p$$

We have transformed in cylindrical form eq.(1.911) &(1.921)

The blood flow in capillary is symmetric w.r.t. axis.

Hence,

v_θ =0, v_z , v_r and p do not depend upon θ.

Since only one Component of velocity which is along axis is effective.

We have,

$$v_r = 0, v_\theta = 0, v_z = V$$

Since, flow is steady,

$$\frac{\partial P}{\partial t} = \frac{\partial v_r}{\partial t} = \frac{\partial v_\theta}{\partial t} = \frac{\partial v_z}{\partial t} = 0$$

$$\frac{\partial v_z}{\partial z} = 0$$

$$v_z = V(r)$$

r-component

$$\rho_m(0) = -\frac{\partial p}{\partial r} + \eta_m(0)$$

$$\frac{\partial p}{\partial r} = 0$$

P = p(z)

θ - component

$$\rho_m(0) = 0 + \eta_m(0)$$

0 = 0

z-component

$$\rho_m v_z \frac{\partial v_z}{\partial t} = -\frac{\partial p}{\partial z} + \eta_m \left[\frac{1}{r}\frac{\partial}{\partial r}\left\{ r \frac{\partial v_z}{\partial r} \right\} + \frac{\partial^2 v_z}{\partial z^2} \right]$$

$$\rho_m V_r \frac{\partial V(r)}{\partial t} = -\frac{\partial p}{\partial z} + \eta_m \left[\frac{1}{r}\frac{\partial}{\partial r}\left\{ r \frac{\partial V(r)}{\partial r} \right\} + \frac{\partial^2 V(r)}{\partial Z^2} \right]$$

And pressure p depends on Z.

i.e. $p = -\frac{\partial p}{\partial z}$

By using first & second boundary condition, we get

$$V = \frac{P}{4\eta_m}\left(R^2 - r^2 \right)$$

The velocity of plasma layer is obtained by replacing η_m by η_p in formula …… of Newtonian model, which is as follows:

$$v_p = \frac{P}{4\eta_p}\left(R^2 - r^2 \right); \quad R-\delta \le r \le R$$

The velocity of core layer can also be obtained in a similar way as follows:

$$v_m = \frac{P}{4\eta_m}\left(R^2 - r^2 \right) + \frac{P}{4\eta_m}\left[R^2 - \left(R-\delta \right)^2 \right]\left(\frac{\eta_m}{\eta_p} - 1 \right); \; 0 \le r \le R-\delta$$

Where R is the radius of the capillary and δ is the thickness of the plasma layer. δ Is supposed to be independent of R. [35

III. Bio-Physical Interpretation

Clinical Data

Sno.	1	2	3	4	5
HB	12.5	13.4	12.1	12.3	12.5
H	37.5	40.2	36.3	36.9	37.5
BP	130/80	140/100	160/100	130/90	130/90

Average systolic Pressure = 138 mmhg

Average Diostolic pressure = 92 mmhg

Pressure at Capillary $= 69\left(\frac{\frac{D+S}{2}+D}{3} \right)$

Pressure on Venules $\frac{2}{3}\text{of Capillary} = \frac{2}{3}\times 69 = 46\,\text{mmhg}$

$p_i = \text{BloodPressure on capillary} = 69\times 133.322 = 9199.2\,p$

$p_f = \text{Blood Pressure on Venules} = 46\times 133.322 = 6132.8\,p$

Now We have ,

η_m, Viscosity of mixture $=3.5\times10^{-3}$ ps

η_p, Viscosity of plasma $=1.2\times10^{-3}$ ps

R, Radius of capillary $=0.0965$ m

δ, Thickness of RBC layer $=\frac{1}{3}R=0.0322$

$R-\delta=0.0643$

Average HB = 12.56 & Hematocrit $3\times12.56=37.68$

Q, flow flux of blood = 0.01833

Length of capillary = 19000 m

$\eta_m=\eta_c X+(1-X)\eta_p$; $X=\frac{H}{100}$,

$3.5\times10^{-3}=\eta_c\times\frac{37.8}{100}+\left(1-\frac{37.8}{100}\right)\times\eta_p$

$\eta_c=0.0073$

Now $p=-\frac{dp}{dz}\Rightarrow p\int_{z_i}^{z_f}dz=-\int_{p_i}^{p_f}dp \quad\Rightarrow p\left(z_f-z_i\right)=\left(p_i-p_f\right)$

$$\Rightarrow\ p=\frac{p_i-p_f}{z_f-z_i}=\frac{\Delta p}{\text{length of capillary}}$$

$$\text{Flow flux } Q=\int_0^{R-\delta}\left[\begin{array}{l}\left(\frac{p}{2\eta_m}\right)^{\frac{1}{n}}\left(\frac{n}{n+1}\right)\left(R^{\frac{1}{n}+1}-r^{\frac{1}{n}+1}\right)+\\ +\left\{\frac{p}{4\eta_p}\left(R^2-(R-\delta)^2\right)-\left(\frac{p}{2\eta_m}\right)^{\frac{1}{n}}\frac{n}{n+1}\left(R^{\frac{1}{n}+1}-(R-\delta)^{\frac{1}{n}+1}\right)\right\}\end{array}\right]2\pi r dr$$

$$+\int_{R-\delta}^{R}\frac{p}{4\eta_p}\left(R^2-r^2\right)2\pi r dr$$

$$Q=\int_0^{0.0643}\left[\begin{array}{l}\left(\frac{0.1612}{2\times3.5\times10^{-3}}\right)^{\frac{1}{n}}\left(\frac{n}{n+1}\right)\left(R^{\frac{n+1}{n}}2\pi r dr-2\pi r^{\frac{n+1}{n}+1}dr\right)+\\ \left\{\left(\frac{0.1612}{4\times1.2\times10^{-3}}\right)\left(0.0965^2-0.0643^2\right)2\pi r dr-\right.\\ \left.\frac{0.1612}{2\times3.5\times10^{-3}}\times\frac{n}{n+1}\left(0.0965^{\frac{n}{n+1}}-0.0643^{\frac{n}{n+1}}\right)2\pi r dr\right\}\end{array}\right]$$

$$+\int_{0.0643}^{0.0965}\frac{0.1612}{4\times1.2\times10^{-3}}\left(0.0965^2\,2\pi r dr-2\pi r^3 dr\right)$$

$$Q=\int_0^{0.0643}(23.03)^{\frac{1}{n}}\times\frac{n}{n+1}\times 0.0965^{\frac{n+1}{n}}2\pi\left[\frac{r^2}{2}\right]_0^{0.0643}-2\pi\left[\frac{r^{\frac{3n+1}{n}}}{3n+1}\right]_0^{0.0643}$$

$$+1.09\left[\frac{r^2}{2}\right]_0^{0.0643}-(23.03)^{\frac{1}{n}}\frac{n}{n+1}\left(0.0965^{\frac{n+1}{n}}-0.0643^{\frac{n+1}{n}}\right)2\pi\left[\frac{r^2}{2}\right]_0^{0.0643}$$

$$+33.6\left[0.0585\left[\frac{r^2}{2}\right]_{0.0643}^{0.0965}-2\pi\left(\frac{r^4}{4}\right)_{0.0643}^{0.0965}\right]$$

$$Q=\left[\begin{array}{l}(23.03\times 0.0643)^{\frac{1}{n}}\left(\frac{n}{n+1}\right)0.00125\times\left(\frac{0.0965}{0.0643}\right)^{\frac{1}{n}}-\\ -\left(\frac{n}{3n+1}\right)\times 0.0017+(0.0023)\\ -(23.03)^{\frac{1}{n}}\times\frac{n}{3n+1}\times(0.0643)^{\frac{1}{n}}(0.606)\left(\frac{0.0965}{0.0643}\right)^{\frac{1}{n}}-0.404\end{array}\right]\times 0.00207+0.0014$$

$$Q=0.01833=(1.48)^{\frac{1}{n}}\frac{n}{n+1}\left\{0.00125\times(1.5)^{\frac{1}{n}}-(1.5)^{\frac{1}{n}}\times 0.00125\right\}$$

$$-\left\{1.48^{\frac{1}{n}}\frac{n}{n+1}\right\}\left\{\frac{0.0017n}{3n+1}+0.00084\right\}+0.0037$$

$$0.01833-0.0037=(1.48)^{\frac{1}{n}}\left(\frac{n}{n+1}\right)\left\{-\frac{0.0017n}{3n+1}-0.00084\right\}$$

$$0.01463=(1.48)^{\frac{1}{n}}\frac{n}{n+1}\left\{-\frac{0.0017n}{3n+1}-0.00084\right\}.....$$

On solving we et n=-0.335388

Now again,

$$Q=\int_0^{R-\delta}\left[\begin{array}{l}\left(\frac{p}{2\eta_m}\right)^{\frac{1}{n}}\times\frac{n}{n+1}\left(R^{\frac{n+1}{n}}-r^{\frac{n+1}{n}}\right)+\\ \left\{\frac{p}{4\eta_p}\times\left(R^2-(R-\delta)^2\right)-\left(\frac{p}{2\eta_m}\right)^{\frac{1}{n}}\times\frac{n}{n+1}-\left(R^{\frac{n+1}{n}}-(R-\delta)^{\frac{n+1}{n}}\right)\right\}\end{array}\right]2\pi r dr$$

$$+\int_{R-\delta}^{R}\frac{p}{4\eta_p}\left(R^2-r^2\right)2\pi r dr$$

$$Q=\int_0^{0.0643}\left[\begin{array}{l}\left(\frac{p}{2\eta_m}\right)^{\frac{1}{-0.335388}}\times\left(\frac{-0.335388}{-0.335388+1}\right)\left\{(0.0965)^{\frac{-0.335388}{-0.335388+1}}-r^{\frac{-0.335388}{-0.335388+1}}\right\}2\pi rdr\\ +\left\{\left(\frac{p}{2\times1.2\times10^{-3}}\right)\left(0.0965^2-0.0643^2\right)\right\}2\pi rdr\\ -\left\{\left(\frac{p}{2\eta_m}\right)^{\frac{1}{-0.335388}}\left(\frac{-0.335388}{-0.335388+1}\right)\times\right.\\ \left.\left\{\left(0.0965^{\frac{-0.335388}{-0.335388+1}}-0.0643^{\frac{-0.335388}{-0.335388+1}}\right)\right\}\right\}2\pi rdr\end{array}\right]$$

$$+\int_{0.0643}^{0.0965}\left(\frac{p}{2\times1.2\times10^{-3}}\right)\left(0.0965^2\times2\pi rdr-2\pi r^3dr\right)$$

$$Q=\int_0^{0.0643}\left(\frac{p}{2\eta_m}\right)^{-2.98}\times(-.505)\left(102.87\times2\pi rdr-r^{-1.98}\times2\pi r\right)$$

$$+\left\{\begin{array}{l}208.3p\times0.0052\times2\pi rdr-\left(\frac{p}{2\eta_m}\right)^{-2.98}\\ \times(-0.505)(102.87)\times2\pi rdr\ -\ 229.97\times2\pi rdr\end{array}\right\}$$

$$+\int_{0.0643}^{0.0965}208.3\times p\left(0.0585rdr-2\pi r^3dr\right)$$

$$Q=\left(\frac{p}{2\eta_m}\right)^{-2.98}\times(-0.505)\left\{1.34-2\pi\left(\frac{r^{0.02}}{0.02}\right)_0^{0.0643}\right\}+\left\{0.0141p-\left(\frac{p}{2\eta_m}\right)^{-2.98}\times(-0.505)(1.336-2.987)\right\}$$

$$+208.3p\left(1.51\times10^{-4}-1.09\times10^{-4}\right)$$

$$Q=\left(\frac{p}{2\eta_m}\right)^{-2.98}\times(-0.505)(-296.04)+0.141p-\left(\frac{p}{2\eta_m}\right)^{-2.98}\times0.834+0.0087p$$

$$Q=\left(\frac{p}{2\eta_m}\right)^{-2.98}\times149.5+0.0141p-\left(\frac{p}{2\eta_m}\right)^{-2.98}\times0.834+0.87p$$

$$Q=\left(\frac{p}{2\eta_m}\right)^{-2.98}(149.5-0.834)+0.0087p$$

$$Q=148.65\left(\frac{p}{2\eta_m}\right)^{-2.98}+0.0228p$$

Now ignoring the term 0.0228p (negligible value)

$$0.01833=148.65\left(\frac{\Delta p}{2\times 19000\eta_m}\right)^{-2.98},$$

$$p=-\frac{dp}{dz}=\frac{\Delta p}{\text{length of capillary}}$$

$$\frac{\Delta p}{38000\eta_m}=\left(\frac{0.01833}{2148.65}\right)^{\frac{1}{-2.98}}$$

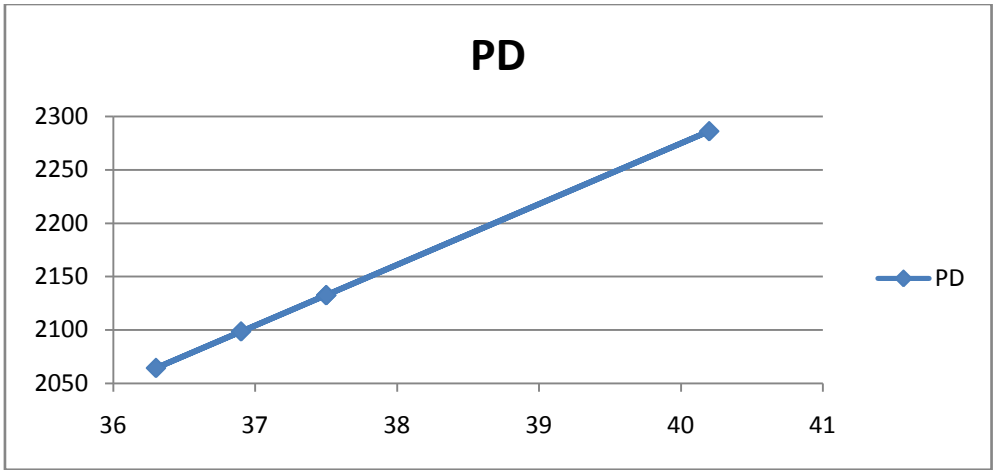
$$\Rightarrow 20.5=\frac{\Delta p}{38000\eta_m}$$

$$\Rightarrow \Delta p=778985.5\left[\eta_c\left(\frac{H}{100}\right)+\left(1-\frac{H}{100}\right)\eta_p\right]$$

$$\Delta p = 778985.5\times 0.0073H+1.2\times 10^{-3}-1.2\times 10^{-5}H$$

$$\Delta p = 56.87H+1.2\times 10^{-3}$$

Sno.	1	2	3	4	5
H	37.5	40.2	36.3	36.9	37.5
Δ P	2132.6	2286.2	2064.4	2098.5	2132.6



Hence Pressure drop is proportional to Hematocrit

Hence the two phase non Newtonian model is verified in clinical data of the Diabetetic patient and pressure drop is proportional to Hematocrit

It is remarkable that velocity of plasma layer is taken as if whole capillary is filled with plasma. Again the velocity of core layer is taken as if the core layer blood is filled in whole capillary.
The relative velocity of the both layers is also added to it.

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