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Finite Triple Integral Representation For The Polynomial Set $T_n(x_1,x_2,x_3,x_4)$

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Abstract:-*Recently,we introduced "An unification of certain generalized Geometric polynomial Set T_n (x_1,x_2,x_3,x_4) ,with the help of generating function which contains Appell function of four variables in the notation of Burchnall and Choundy [4] associated with Lauricella function. This generated hypergeometric polynomial Set covers as many as thirty four orthogonal and non-orthogonal polynomials.In the present paper an attempt has been made to express a Triple finite integral representation of the polynomial set T_n (x_1,x_2,x_3,x_4).*

Key words:-*Appell function, Lauricella form, Generalized hypergeometric polynomial.*

I. Introduction

The generalized polynomial set T_n $(x_1,x_2,x_3,x_4)$is defined by means of generating relation

$$e^{\lambda_1 t^{m_1}+d_\mu} F\left[\begin{matrix}(A_r);(C_u);(Eg_i) \\ \qquad\qquad \lambda x_1 t,\ \lambda_2 x_2^{m_2} t^{m_2},\ \lambda_3 x_3^{m_3} t^{m_3},\ \lambda_4 x_4^{m_4} t^{m_4} \\ (B_s);(D_v);(Fq_i)\end{matrix}\right]$$

$$=\sum_{n=0}^{\infty} T_{n,m:m_1:m_2:m_3:m_4:(B_s):(D_v):(Fq_i)}^{\lambda:\lambda_1:\lambda_2:\lambda_3:\lambda_4:(A_r):(C_u):(E_{g_i})}\left(x_1,x_2,x_3,x_4\right)t^n \ldots \quad \mathbf{(1.1)}$$

where (i = 1, 2, 3, 4) and $\lambda, \lambda_1, \lambda_2, \lambda_3, \lambda_4$ are real and m, m_1, m_2, m_3, m_4 are positive integer. The left hand side of (1.1) contains the product of two generalized hypergeometric function which contains Appell function of four variables in the notation of Burchanall and Chaundy [4] associated with Lauricella function. The polynomial set contains a number of parameters for simplicity, it is denoted by T_n (x_1,x_2,x_3,x_4), where n is the order of the polynomial set.

After little ,simplification(1.1)gives,

$$T_n(x_1,x_2,x_3,x_4) = \sum_{P_1=0}^{\left[\frac{n}{m_1}\right]} \sum_{P_2=0}^{\left[\frac{n-m_1P_1}{m_2}\right]} \sum_{P_3=0}^{\left[\frac{n-m_1P_1-m_2P_2}{m_3}\right]} \sum_{P_4=0}^{\left[\frac{n-m_1P_1-m_2P_2-m_3P_3}{m_4}\right]}$$

$$\times\frac{[(A_r)]_{n-m_1P_1-(m_2-1)P_2-(m_3-1)P_3-(m_4-1)P_4}}{[(B_s)]_{n-m_1P_1-(m_2-1)P_2-(m_3-1)P_3-(m_4-1)P_4}}$$

$$\times\frac{[(C_u)]n-m_1P_1-m_2P_2-m_3P_3-m_4P_4\left[(E_{g_1})\right]_{P_1}\left[\left(E_{g_2}\right)\right]_{P_2}\left[\left(E_{g_3}\right)\right]_{P_3}}{\left[(D_v)\right]n-m_1P_1-m_2P_2-m_3P_3-m_4P_4\left[\left(F_{q_1}\right)\right]_{P_1}\left[\left(F_{q_2}\right)\right]_{P_2}\left[\left(F_{q_3}\right)\right]_{P_3}}$$

$$\frac{\left[\left(E_{g_4}\right)\right]_{P_4}\left(\lambda x_1^m\right)^{n-m_1P_1-m_2P_2-m_3P_3-m_4P_4}\lambda_1^{P_1}\left(\alpha_\mu\right)^{P_1}\lambda_2^{P_2}\left(\lambda_3 x_3^{m_3}\right)^{P_3}\left(\lambda_4 x_4^{m_4}\right)^{P_4}}{\left[\left(F_{q_4}\right)\right]_{P_4}\quad P_1!\quad P_2!\quad x_2^{m_2P_2}\quad P_3!\quad P_4!} \quad \ldots\ldots \quad \mathbf{(1.2)}$$

The polynomial Set T_n (x_1,x_2,x_3,x_4) happens to the generalization of as many thirty four orthogonal and non-orthogonal polynomials.

Notations

a)

I. (m)=1.2.3…….m

II. $(A_p)=A_1.A_2.A_3\ldots\ldots.A_p$

III. $[(A_p)]=A_1,A_2,A_3\ldots\ldots.A_p$

IV. $[(A_p)]_n=(A_1)_n,(A_2)_n,(A_3)_n\ldots\ldots.(A_p)_n$

V. $\Delta(a,b)=\frac{b}{a},\ \frac{b+1}{a},\ \ldots\ldots\frac{b+a-1}{a}$

b)

I. $M=\frac{[(A_r)]_n\,[(C_u)]_n\,\left(\lambda x^m\right)^n}{[(B_s)]_n\,[(D_v)]_n\,n!}$

2 Theorem : $m_2>1,\ m_3>1$ and $m_4>1$, we have

$$T_n\left(x_1,x_2,x_3,x_4\right)=\frac{\Gamma\left(a+b+c+1\right)M_1}{\Gamma(a)\,\Gamma(b)\,\Gamma(c)}\int_0^1\int_0^{1-\mu}\int_0^{1-\mu-\nu}\mu^{a-1}\,\nu^{b-1}\,\xi^{c-1}$$

$$F^{1+s+v\,:\,g_1\,:\,g_2\,:\,g_3\,:\,g_4}_{r+u\,:q_1\,:\,q_2\,:\,q_3\,:\,q_4}\left[\begin{matrix}[-n:m_1,m_2,m_3,m_4],[(a+b+c+1):1],\\ \underline{\qquad\qquad\qquad},[(a):1],[(b):1],[(c):1]\end{matrix}\right.$$

$$\left[(1-(B_s)-n):m_1,m_2-1,m_3-1,m_4-1\right],\left[(1-(D_v)-n):m_1,m_2,m_3,m_4\right],$$

$$\left[(1-(A_r)-n):m_1,m_2-1,m_3-1,m_4-1\right],\left[(1-(C_u)-n):m_1,m_2,m_3,m_4\right],$$

$$\frac{\left[(E_{g_1}):1\right],\left[(E_{g_2}):1\right],\left[(E_{g_3}):1\right],\left[(E_{g_4}):1\right]}{\left[(F_{q_1}):1\right],\left[(F_{q_2}):1\right],\left[(F_{q_3}):1\right],\left[(F_{q_4}):1\right]};\ \frac{(-1)^{m_1(r+s+u+v+1)}\left(\lambda_1\alpha_\mu\right)}{\left(\lambda x_1^m\right)^{m_1}},$$

$$\frac{\lambda_2(-1)^{m_2(r+s+u+v+1)+r_1+r_2}}{\left(\lambda x_1^m x_2\right)^{m_2}},\ \frac{\lambda_3 x_3^{m_3}(-1)^{m_3(r+s+u+v+1)+r+s}}{\left(\lambda x_1^m\right)^{m_3}},$$

$$\left.\frac{\lambda_4 x_4^{m_4}(-1)^{m_4(r+s+u+v+1)+r+s}}{\left(\lambda x_1^m\right)^{m_4}}\right]\times d\mu\ \ d\nu\ \ d\xi\ldots\ldots\ \mathbf{(2.1)}$$

Proof :-We have

$$I=\int_0^1\int_0^{1-\mu}\int_0^{1-\mu-\nu}\mu^{a-1}\,\nu^{b-1}\,\xi^{c-1}\sum_{P_1=0}^{\left[\frac{n}{m_1}\right]}\ \sum_{P_2=0}^{\left[\frac{n-m_1P_1}{m_2}\right]}\ \sum_{P_3=0}^{\left[\frac{n-m_1P_1-m_2P_2}{m_3}\right]}\ \sum_{P_4=0}^{\left[\frac{n-m_1P_1-m_2P_2-m_3P_3}{m_4}\right]}$$

$$\times\frac{\left[(A_r)\right]_{n-m_1P_1-(m_2-1)P_2-(m_3-1)P_3-(m_4-1)P_4}\left[(C_u)\right]_{n-m_1P_1-m_2P_2-m_3P_3-m_4P_4}}{\left[(B_s)\right]_{n-m_1P_1-(m_2-1)P_2-(m_3-1)P_3-(m_4-1)P_4}\left[(D_v)\right]_{n-m_1P_1-m_2P_2-m_3P_3-m_4P_4}}$$

$$\times\frac{\left[\left(E_{g_1}\right)\right]_{P_1}\left[\left(E_{g_2}\right)\right]_{P_2}\left[\left(E_{g_3}\right)\right]_{P_3}\left[\left(E_{g_4}\right)\right]_{P_4}\left(\lambda x_1 m\right)^{n-m_1P_1-m_2P_2-m_3P_3-m_4P_4}}{\left[\left(F_{q_1}\right)\right]_{P_1}\left[\left(F_{q_2}\right)\right]_{P_2}\left[\left(F_{q_3}\right)\right]_{P_3}\left[\left(F_{q_4}\right)\right]_{P_4}\left(n-m_1P_1-m_2P_2-m_3P_3-m_4P_4\right)!}$$

$$\times \frac{\lambda_1^{P_1}\ \alpha_\mu^{P_1}\ \lambda_2^{P_2}\left(\lambda_3 x_3^{m_3}\right)^{P_3}\ \left(\lambda_4 x_4^{m_4}\right)^{P_4}\ (a+b+c+1)_{3P_1}\ d\mu\ d\nu\ d\xi}{P_1!\quad x_2^{m_2P_2}P_2!\quad P_3!\quad P_4!\ (a)_{P_1}\quad (b)_{P_1}\quad (c)_{P_1}}$$

$$=\int_0^1 \int_0^{1-\mu} \int_0^{1-\mu-\nu} \mu^{a-1+P_1}\ \nu^{b-1+P_1}\ \xi^{c-1+P_1} \times \sum_{P_1=0}^{\left[\frac{n}{m_1}\right]} \sum_{P_2=0}^{\left[\frac{n-m_1P_1}{m_2}\right]} \sum_{P_3=0}^{\left[\frac{n-m_1P_1-m_2P_2}{m_3}\right]}$$

$$\times \sum_{P_4=0}^{\left[\frac{n-m_1P_1-m_2P_2-m_3P_3}{m_4}\right]} \frac{[(A_r)]_{n-m_1P_1-(m_2-1)P_2-(m_3-1)P_3-(m_4-1)P_4}}{[(B_s)]_{n-m_1P_1-(m_2-1)P_2-(m_3-1)P_3-(m_4-1)P_4}}$$

$$\times \frac{[[(C_u)]]_{n-m_1P_1-m_2P_2-m_3P_3-m_4P_4}\ \left[\left(E_{g_1}\right)\right]_{P_1} \left[\left(E_{g_2}\right)\right]_{P_2} \left[\left(E_{g_3}\right)\right]_{P_3} \left[\left(E_{g_4}\right)\right]_{P_4}}{[[(D_v)]]_{n-m_1P_1-m_2P_2-m_3P_3-m_4P_4}\ \left[\left(F_{q_1}\right)\right]_{P_1} \left[\left(F_{q_2}\right)\right]_{P_2} \left[\left(F_{q_3}\right)\right]_{P_3} \left[\left(F_{q_4}\right)\right]_{P_4}}$$

$$\times \frac{\lambda_1^{P_1}\left(\alpha_\mu\right)^{P_1}\ \lambda_2^{P_2}\ \left(\lambda_3 x_3^{m_3}\right)^{P_3}\ \left(\lambda_4 x^{m_4}\right)^{P_4}\ (a+b+c+1)_{3P_1}}{P_1!\ \left(x_2^{m_2}\right)^{P_2}\ P_2!\quad P_3!\quad P_4!\quad (a)_{P_1}\quad (b)_{P_1}\quad (c)_{P_1}}$$

$$\times \frac{\left(\lambda x_1^m\right)^{n-m_1P_1-m_2P_2-m_3P_3-m_4P_4}}{(n-m_1P_1-m_2P_2-m_3P_3-m_4P_4)!}\, d\mu\ d\nu\ d\xi$$

$$=\sum_{P_1=0}^{\left[\frac{n}{m_1}\right]} \sum_{P_2=0}^{\left[\frac{n-m_1P_1}{m_2}\right]} \sum_{P_3=0}^{\left[\frac{n-m_1P_1-m_2P_2}{m_3}\right]} \sum_{P_4=0}^{\left[\frac{n-m_1P_1-m_2P_2-m_3P_3}{m_4}\right]}$$

$$\times \frac{[(A_r)]_{n-m_1P_1-(m_2-1)P_2-(m_3-1)P_3-(m_4-1)P_4}\ [(C_u)]_{n-m_1P_1-m_2P_2-m_3P_3-m_4P_4}}{[(B_s)]_{n-m_1P_1-(m_2-1)P_2-(m_3-1)P_3-(m_4-1)P_4}\ [(D_v)]_{n-m_1P_1-m_2P_2-m_3P_3-m_4P_4}}$$

$$\times \frac{\left[\left(E_{g_1}\right)\right]_{P_1} \left[\left(E_{g_2}\right)\right]_{P_2} \left[\left(E_{g_3}\right)\right]_{P_3} \left[\left(E_{g_4}\right)\right]_{P_4}\ \left(\lambda x_1^m\right)^{m_1P_1-m_2P_2-m_3P_3-m_4P_4}}{\left[\left(F_{q_1}\right)\right]_{P_1} \left[\left(F_{q_2}\right)\right]_{P_2} \left[\left(F_{q_3}\right)\right]_{P_3} \left[\left(F_{q_4}\right)\right]_{P_4}\ (n-m_1P_1-m_2P_2-m_3P_3-m_4P_4)!}$$

$$\times \frac{\left(\lambda_1\alpha_\mu\right)^{P_1}\ \lambda_2^{P_2}\ \left(\lambda_3 x_3^{m_3}\right)^{P_3}\ \left(\lambda_4 x_4^{m_4}\right)^{P_4}\ (a+b+c+1)_{3P_1}}{P_1!\ x_2^{m_2P_2}\ P_2!\quad P_3!\quad P_4!\ (a)_{P_1}\quad (b)_{P_1}\quad (c)_{P_1}}$$

$$\times \frac{\Gamma(a+P_1)\quad \Gamma(b+P_1)\quad \Gamma(c+P_1)}{\Gamma(a+b+c+1+3P_1)}$$

$$= \frac{\Gamma(a)\ \Gamma(b)\ \Gamma(c)}{\Gamma(a+b+c+1)} \sum_{P_1,\ P_2,\ P_3,\ P_4=0}^{\infty} \frac{[1-(B_s)-n]_{m_1P_1+(m_2-1)P_2+(m_3-1)P_3+(m_4-1)P_4}}{[1-(A_r)-n]_{m_1P_1+(m_2-1)P_2+(m_3-1)P_3+(m_4-1)P_4}}$$

$$\times \frac{[1-(D_v)-n]_{m_1P_1+m_2P_2+m_3P_3+m_4P_4} \left[(E_{g_1})\right]_{P_1} \left[(E_{g_2})\right]_{P_2} \left[(E_{g_3})\right]_{P_3} \left[(E_{g_4})\right]_{P_4}}{[1-(C_u)-n]_{m_1P_1+m_2P_2+m_3P_3+m_4P_4} \left[(F_{q_1})\right]_{P_1} \left[(F_{q_2})\right]_{P_2} \left[(F_{q_3})\right]_{P_3} \left[(F_{q_4})\right]_{P_4}}$$

$$\times \frac{(\lambda_1\alpha_\mu)^{P_1}\ (-1)^{m_1(r+s+u+v+1)P_1}\ \ \lambda_2(-1)^{\{m_2(r+s+u+v+1)+r+s\}P_2}}{(\lambda x_1^m)^{m_1P_1}\ \ P_1!\ \ (\lambda x_1^m x_2)^{m_2P_2}\ \ P_2!}$$

$$\times \frac{\lambda_3 x_3^{m_3}\ (-1)^{\{m_3(r+s+u+v+1)+r+s\}P_3}\ \ \lambda_4 x_4^{m_4}\ \ (-1)^{\{m_4(r+s+u+v+1)+r+s+\}P_4}\ (-n)_{m_1P_1+m_2P_2+m_3P_3+m_4P_4}}{P_3!\ \ (\lambda x_1^m)^{m_3P_3}\ \ (\lambda x_1^m)^{m_4P_4}\ \ P_4!} \quad \text{... (2.2)}$$

The single terminating factor $(-n)_{m_1P_1+m_2P_2+m_3P_3+m_4P_4}$ makes all summation in (2.2) runs upto ∞.

$$= \frac{\Gamma(a)\ \Gamma(b)\ \Gamma(c)}{\Gamma(a+b+c+1)}\ T_n(x_1,\ x_2,\ x_3,\ x_4)$$

On using [16], hence the theorem.

$$\int_0^1 \int_0^{1-n} \int_0^{1-x-y} x^{l-1}\ y^{m-1}\ z^{m-1}\ z^{n-1}\ dx\ dy\ dz = \frac{\Gamma(l)\ \Gamma(m)\ \Gamma(n)}{\Gamma(l+m+n+1)}$$

Where $x+y+z<1$

3 Particular Cases

I. On taking $r=0=s=u=v=q_1, g_1=1=\lambda=m=m_1=\alpha_\mu;\ E_1=1+\lambda_2,\ \ \lambda_1=-1$ and y for x_1 in (2.1), we achieve

$$A_n^{\lambda_2}(y) = \frac{\Gamma(a+b+c+1)\ y^n}{\Gamma(a)\ \Gamma(b)\ \Gamma(c)\ \ n!} \int_0^1 \int_0^{1-\mu} \int_0^{1-\mu-\nu} \mu^{a-1}\ \nu^{b-1}\ \xi^{c-1}$$

$$\times\ F\left[\begin{matrix}-n, 1+\lambda_2,\ a+b+c+1; & \frac{1}{y}\\ a,\ b,\ c; & \end{matrix}\right] d\mu\ d\nu\ d\xi$$

(ii)If we set $r=0=s=u=v;\ g_1=1=q_1=m=m_1=\lambda=\lambda_1=\mu_\alpha;\ E_1=(a_p),\ F_1=(b_q)$ and $x_1=-\frac{1}{x}$ in (2.1) we have

$${}_1F_1(-n, b; x) = \frac{\Gamma(a+b+c+1)}{\Gamma(a)\ \Gamma(b)\ \Gamma(c)} \int_0^1 \int_0^{1-\mu} \int_0^{1-\mu-\nu} \mu^{a-1}\ \nu^{b-1}\ \xi^{c-1}$$

$$\times F\left[\begin{matrix}-n,\left(a_p\right),\ a+b+c+1; \\ \left(b_q\right),\ a,b,c;\end{matrix}\ x\right] d\mu\ d\nu\ d\xi$$

(iii) On setting $r=0=s=v=g_1=q_1; u=1=m=m_1=\lambda_1=\alpha_\mu,\ \lambda_1=-1, C_1=1+\lambda$ and $x=\frac{1}{y}$ in (2.1) we have

$$A_n^{(\lambda)}(y)=\frac{\Gamma(a+b+c+1)\ (1+\lambda)_n\ (-1)^n}{\Gamma(a)\ \Gamma(b)\ \Gamma(c)\ n!}\int_0^1\int_0^{1-\mu}\int_0^{1-\mu-\nu}\mu^{a-1}\ \nu^{b-1}\ \xi^{c-1}$$
$$\times\ F\left[\begin{matrix}-n,\ a+b+c+1; \\ -\lambda-n,a,b,c;\end{matrix}\ -y\right] d\mu\ d\nu\ d\xi$$

(iv) If we set $r=0=s=u=g_1; v=1=q_1=m=m_1=\alpha_\mu;$

$D_1=1+\beta,\ F_1=1+\alpha,\ \lambda=\frac{1}{2}=\lambda$ and $x_1=\frac{x+1}{x-1}$, in (2.1), we get

$$P_n^{(\alpha,\beta)}(x)=\frac{\Gamma(a+b+c+1)\,(1+\alpha)_n\,(x+1)^n}{\Gamma(a)\ \Gamma(b)\ \Gamma(c)\ n!\ 2^n}\int_0^1\int_0^{1-\mu}\int_0^{1-\mu-\nu}\mu^{a-1}\ \nu^{b-1}\ \xi^{c-1}$$
$$\times F\left[\begin{matrix}-n,-\beta-n,\ a+b+c+1; \\ 1+\alpha,\ a,\ b,\ c;\end{matrix}\ \frac{x-1}{x+1}\right] d\mu\ d\nu\ d\xi$$

(v) On taking $r=0=s=u=g_1; v=1=q_1=m=m_1=\alpha_\mu;$ $\lambda=\frac{1}{2}=\lambda_1;$ $D_1=1+\alpha,\ F_1=1+\beta$ and writing $\frac{x-1}{x+1}$ for x_1, in (2.1), we get

$$P_n^{(\alpha,\,\beta)}(x)=\frac{\Gamma(a+b+c+1)\,(1+\beta)_n\,(x-1)^\mu}{\Gamma(a)\ \Gamma(b)\ \Gamma(c)\ n!\ 2^n}\int_0^1\int_0^{1-\mu}\int_0^{1-\mu-\nu}\mu^{a-1}\ \nu^{b-1}\ \xi^{c-1}$$
$$\times\ F\left[\begin{matrix}-n,-\alpha-n;\ a+b+c+1; \\ 1+\beta,\ a,\ b,\ c;\end{matrix}\ \frac{x+1}{x-1}\right] d\mu\ d\nu\ d\xi$$

(vi) On making the substitution $r=0=s=u=g_1; v=1=q_1=m=m_1=\alpha_\mu;$ $\lambda=\frac{1}{2}=\lambda_1;$ $D_1=1+\alpha,\ F_1=1+\alpha$ and writing $\frac{x+1}{x-1}$ for x_1 in (2.1), we get

$$P_n^{(\alpha,\,\alpha)}(x)=\frac{\Gamma(a+b+c+1)\ (1+\alpha)_n\ (x+1)^n}{\Gamma(a)\ \Gamma(b)\ \Gamma(c)\ n!\ 2^n}\int_0^1\int_0^{1-\mu}\int_0^{1-\mu-\nu}\mu^{a-1}\,\nu^{b-1}\,\xi^{c-1}$$
$$\times\ F\left[\begin{matrix}-n,-\alpha-n,\ a+b+c+1; \\ 1+\alpha,\ a,\ b,\ c;\end{matrix}\ \frac{x-1}{x+1}\right] d\mu\ d\nu\ d\xi$$

(vii) If we take $r=0=s=v=g_1=q_1$; $u=1=m=m_1=\alpha_\mu=\lambda=\lambda_1$; $c_1=-\lambda$ and $\frac{1}{x}$ for x, in (2.1), we get

$$F_n(x)=\frac{\Gamma(a+b+c+1)\ (-\lambda)_n}{\Gamma(a)\ \Gamma(b)\ \Gamma(c)\ \ n!}\int_0^1\int_0^{1-\mu}\int_0^{1-\mu-\nu}\mu^{a-1}\ \nu^{b-1}\ \xi^{c-1}$$
$$\times F\left[\begin{matrix}-n,\ a+b+c+1; \\ 1+\lambda-n,\ a,\ b,\ c;\end{matrix}\ x\right]d\mu\ d\nu\ d\xi$$

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