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Exponential Base Splitting Expressions-Extensions (Exbaspex)

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Abstract: *This given note is a brief introduction to a new branch of Mathematics which comes under the category Exponentiation. Which deals with the study of base (of an exponential expression) and its split terms, explains the relation between the split terms, gives an extension for exponentiations (like Square, Cube etc.). That we can call as "Exponential Base Splitting Expressions-Extensions" simply abbreviated as 'Exbaspex . The main objective of this branch is to provide a suitable extension for all exponential expressions and thereby increasing the possibilities and futher applications of exponentiations. We hardly find any natural extension for any real base or exponent using the prevailing traditional extension methods. But, we can find so many trends and relations between the exponential results (i.e. square, cube or higher of a number) with the split terms of that number. The first few paragraphs familiarize you, what is an exponentiation and what is a base?, Then it tells how, why and where to split the base?. The general extensions for the first two exponentiations i.e. square and cube of real numbers are added here along with their respective rules governing , including all explanations and proofs with suitable examples. The first ever known application of Exbaspex is from the square extension i.e. the "Square Index Table" is also briefly described below.*
Keywords: *Base, Exponentiation, Extensions, Scindo (Latin), Square Index Table*

I. Introduction

1.1 Some general terminologies used that are prevailing now.

1.1.1 Exponentiation

Exponentiation is a mathematical operation, written as b^n, involving two numbers, the base band the exponent (or index or power) n. When n is a positive integer, exponentiation corresponds to repeated multiplication; in other words, a product of n factors, each of which is equal to b (the product itself can also be called power) :

$$b^n = \underbrace{b \times \cdots \times b}_{n}$$

Fig. 1

Just as multiplication by a positive integer corresponds to repeated addition:

$$b \times n = \underbrace{b + \cdots + b}_{n}$$

Fig. 2

The exponent is usually shown as a superscript to the right of the base. The exponentiation b^n can be read as: b raised to the n-th power, b raised to the power of n, or b raised by the exponent of n, most briefly as b to then. Some exponents have their own pronunciation: for example, b^2 is usually read as b squared and b^3 as b cubed.

The power b^n can be defined also when n is a negative integer, for nonzero b. No natural extension to all real b and n exists, but when the base b is a positive real number, b^n can be defined for all real and even complex exponents n via the exponential function e^z. Trigonometric functions can be expressed in terms of complex exponentiation.

Exponentiation where the exponent is a matrix is used for solving systems of linear differential equations.

Exponentiation is used pervasively in many other fields, including economics, biology, chemistry, physics, and computer science, with applications such as compound interest, population growth, chemical reaction kinetics, wave behavior, and public key cryptography.

1.1.2 Base (Exponentiation)

In exponentiation, the base is the number b in an expression of the form b^n. The number n is called the exponent and the expression is known formally as exponentiation of b by n or the exponential of n with base b. It is more commonly expressed as "the nth power of b", "b to tenth power" or "b to the power n". For example, the fourth power of 10 is 10000 since $10^4 = 10000$. The term power strictly refers to the entire expression, but is sometimes used to refer to the exponent.

When the nth power of b equals a number a, or $a = b^n$, then b is called an "nth root" of a. For example, 10 is a fourth root of 10000.

The inverse function to exponentiation with base b (when it is well-defined) is called the logarithm to base b, denoted $\log_b$.

Even though there is no natural extension for all real bases b and indices n exists. We can find some interesting relations and extensions when we suitably split a base into two.

1.2 Splitting the base: Where? How? Why?

1.2.1 Where?

The Splitting of base term must always been done in between the unit-th place number and tenth place number.

For E.g. If

: b = 09

This can be split into

0_9

" Scindo"

For better convenience, Here we should introduce a new type of symbol ["_" an underscore in between the two numbers] which denotes a "splitting". We have to follow the same notation throughout the paper. So, let us name the new notation [i.e. "_" an underscore in between the two numbers] as "**Scindo**" a Latin word which means "a split".

More Examples:

b = 13, 1_3

b = 234, 23_4

b =1765, 176_5

1.2.2 How?

For the sake of using this operation in numerical problems and in deriving general expressions , [that we will see later] let us denote the two split terms using the letters "**n**" and "**m**". Let the number on the right side of the Scindo [underline] be "**n**" and the number on the left be "**m**". So, If:

b=09, n=0, m=9

b=13, n=1, m=3

b=234, n=23, m=4

b=1765, n=176, m=5

Generally we can write

: {b} = { n_m } ….(1)

"This is called ScindoExpression of a base"

The Braces { } shows that the whole expressions inside it denotes or results in a single Scindo term or a base.

1.2.3 Why?

For any base b we used to split in between the unit-th and tenth place numbers. Why because we can find some relations between "n" and "m" [that we will discuss later] only when the Scindo is in between the unit-th and tenth place numbers. These two fragment of numbers separated using a Scindo is known as "Scindo Terms" or "Scindo Fragments" (while mentioning them use italics to denote their Latin origin.)

1.3 Addition Property of Scindo terms on joining

This Property is governed by the **"Scindo Rules" Or "Cut & paste Rules"**

1.3.1 **Rule1: "Any number in its Scindo Expression should only possess two Scindo Terms, whose unit-th place number becomes the Right -Scindo Term (RST) "m" and all the remaining numbers above constitute theLeft–ScindoTerm(LST)"n"."**

Example:

b = 13 , 1_3 , n=1 , m=3

b =234, 23_4 , n=23 , m=4

b =1765, 176_5 , n=176 , m=5

1.3.2 **Rule2: "The joining any two numbers should result in a number who's Scindo Expression should possess an RST which must be the same as that of the unit-th place number of one of the joining numbers. "**

Example:

236_34 as
236 + (6 + 3 = 9)
34
= 2394

As the extension for the exponentiations were derived by "trial and error method", a direct derivation is not possible. So,

In this paper, we approach the extensions in their ultimate form with the help of tabular data and simple calculations as proof.

The general extensions for the first two exponentiations i.e. square and cube of real numbers are added here along with respective rules governing them including all explanations and proofs with suitable examples.

II. Universal Square Extension (U.S.E)

: $\{n\_m\}^2 = \{ ((n\_1 \times n) + n) + 2n(m-1) \}\_m^2$ …(2)

N.B. This is equation represents the general extension for square of all real numbers when and where it is represented in terms of its Scindo Fragments.

The brackets and braces here do not mean any tuples or sets. The brackets () says that the algebraic operations should be done from inner most to outer brackets.

For example, In

: ((n_1 x n) + n)

First operation done must be "n_1 x n"

and then the resulting product is added to "n".

2.1 What is U.S.E?

It is a universal empirical extension for square of any base denoted in terms of its Scindo terms i.e. as ScindoExpression.

2.2.Rules and Explanations

2.2.1 **Rule3 (First Member Relation Rule) : "For any real number [in its Scindo Expression] whose Right Scindo Term [m] is 1, the Left Scindo Term of its square is given as the sum of Left Scindo Term of the number [n] and the product of , Left Scindo Term [n] of the number and the actual number [n_1]."**

: ((n_1 x n) + n) … (3)

Where, "n_1" is the actual. No. (As per the **Scindo Expression** [n_m] here, m=1)

Example:

$11^2 = 121$

n_1 = 1_1 , m=1

n_1= 11 , n=1

(n_1 x1) + n = 12

Explanation:

For every first member, when we consider 10 consecutive natural numbers as a group, [such as 0 to 9, 10 to 19, 20 to 29 etc.] we can find a particular pattern in their squares. This is simply represented by the expression.

: ((n_1 x n) + n) … (3)

Proof:

Table. 1

Number	n	(n_1 x n) + n	Square of the number
11	1	11 x 1 + 1= 12	121
21	2	21 x2 + 2= 44	441
31	3	31 x 3 + 3= 96	961
41	4	41 x 4 + 4 =168	1681
51	5	51 x 5 + 5 =260	2601
61	6	61 x 6 + 6 =372	3721
71	7	71 x 7 + 7 =504	5041
81	8	81 x 8 + 8 =656	6561
91	9	91 x 9 + 9 =828	8281

2.2.2 **Rule4 (Universal Square Ending Rule): “Square of any number ends with the square of its right-end number [unitth place].”**

Example:

12^2 = 144

93^2 = 8649

15^2 = 225

Therefore we can write the general extension for a square as,

:{(n_1 x n) + n}_m^2 …(4)

But, when we take the case of other numbers down the group, this extension cannot give the exact square.

Table. 2: Tabular Data of the observations down the group

No.	**n**	**m**	**{(n_1 x n) + n}_m^2**	**Square**	**Deviation**
12	1	2	{(1_1 x 1) + 1}_2^2=12_4	144	144-124=20
13	1	3	{(1_1 x 1) + 1}_3^2=12_9	169	169-129=40
14	1	4	{(1_1 x 1) + 1}_4^2=13_6	196	196-136=60
22	2	2	{(2_1 x 2) + 2}_2^2=44_4	484	484-444=40
23	2	3	{(2_1 x 2) + 2}_3^2=44_9	529	529-449=80
24	2	4	{(2_1 x 2) + 2}_4^2 =45_6	576	576-456=120
32	3	2	{(3_1 x 3) + 3}_2^2=96_4	1024	1024-964=60
33	3	3	{(3_1 x 3) + 3}_3^2=96_9	1089	1089-969=120
34	3	4	{(3_1 x 3) + 3}_4^2=97_6	1156	1156-976=180
42	4	2	{(4_1 x 4) + 4}_2^2=168_4	1764	1764-1684=80
43	4	3	{(4_1 x 4) + 4}_3^2=168_9	1849	1849-1689=160
44	4	4	{(4_1 x 4) + 4}_4^2=169_6	1936	1936-1696=240

Table. 3 : Simple Deviation

G:1	2	4	6	8	10	12	14
G:2	4	8	12	16	20	24	28
G:3	6	12	16	20	24	30	36
G:4	8	16	24	32	40	48	56

From the above two tables we can find that,

Table. 4

Deviation	**m**
0	**1**
2	**2**
4	**3**
6	**4**
8	**5**

“The deviation is an integral multiple of the double of “n”.”

Comparing deviation with “m”,
Thus we got, a correction term for all numbers i.e.
: 2n (m-1) … (5)
(N.B: Since all the first terms has m=1, Correction Term = 2n (1-1) = 0.)

2.2.3 **Rule5 (Deviation-Correction Term Rule for square-extensions): “The deviation of lower members in the group is equal to the product of double of “n” [2n] and consecutive natural number just below the “m” [m-1]”.**

III. Universal Cubic Extension (U.C.E)

: $\{n\_m\}^3 = \{ (n\_1 \times n (n\_1 + 1)) + n) + 3n (m + n\_1) (m-1) \}\_m^3$ …(6)

N.B. This is equation represents the general extension for cube of all real numbers when and where it is represented in terms of its Scindo Fragments.

The brackets and braces here do not mean any tuples or sets. The brackets ( ) says that the algebraic operations should be done from inner most to outer brackets.
For Example,
In (n_1 x n (n_1 + 1)) + n)
First operation done must be “(n_1 + 1)”
and the resulting sum is multiplied to “n_1 x n”
then resulting product is added to “n”.

3.1 What is U.C.E?

It is a universal empirical extension for cube of any base denoted in terms of its Scindo terms i.e. as Scindo Expression.

3.2 Rules and Explanations

3.2.1**Rule3 (First Member-Relation Rule): “For any real number [in its Scindo Expression] whose Right Scindo Term [m] is 1, the Left Scindo Term of its cube is given by the sum of Left Scindo Term of number [n] and the product of, Left Scindo Term [n] , actual number [n_1] and the next natural number consecutive to the actual number [n_1 + 1].”**

: (n_1 x n (n_1 + 1)) + n) …(7)
Example:
$11^3 = 1331$
n_1 = 1_1, m=1
n_1= 11 , n=1
1_1 x1 (1_1 + 1)) + 1 = 133.

Explanation:

For every first member, when we consider 10 consecutive natural numbers as a group, [such as 0 to 9, 10 to 19, 20 to 29 etc.] we can find a particular pattern
in their cubes. This is simply represented by an expression.
: (n_1 x n (n_1 + 1)) + n) …(7)
(Where “n” denotes the number above unit-th place.)
Proof:

Table. 5

Number	n	(n_1 x n(n_1 + 1)) + n	Cube of the number
11	1	11 x 1 (11+1) + 1= 133	1331
21	2	21 x2 (21+1) + 2= 926	9261
31	3	31 x3 (31+1) + 3= 2979	29791
41	4	41 x4 (41+1) + 4=6892	68921
51	5	51 x5 (51+1) + 5=13265	132651
61	6	61 x6 (61+1) + 6=22698	226981
71	7	71 x7 (71+1) + 7=35791	357911
81	8	81 x8 (81+1) + 8=53144	531441
91	9	91 x9 (91+1) + 9=75357	753571

3.2.2 **Rule4 (Universal Cubic Ending Rule): “Cube of any number ends with the cube of its right-end number [unitth place].”**

Example:

$12^3 = 1728$
$93^3 = 804357 = 80433\_27$
$15^3 = 3375 = 325\_125$

Therefore we can write the general extension for a cube as,

: {n_1 x n (n_1 + 1)) + n }_m**³** ...(8)

But, In the case of other numbers down the group this extension cannot give the exact cube. So we should introduce a correction term into the expression so that the extension would be applicable to lower members of the group also, which must be zero for the first member. The exercise and analysis of the above expression over the lower members of each group will give you some common deviations and its trends just as that of in U.S.E. (Since those data are unable to tabulate let us omit the calculations here.)

After the comparisons and calculations and there by finding the deviations
We could find the below mentioned observations
Important observations:

- Since, the correction term must be zero for the first member (whose m=1) , here also [just as that of in U.S.E] the correction term should enclose a "m-1" term.
- Deviation is an integral multiple of triple of "n" i.e. "3n"
- Dividing the Deviation with 3n(m-1) we get certain numbers for all the groups which can be generally represented in terms of "m" and the first member [n_1] .i.e. the remaining part of the correction term is the sum of "m" and "n_1" (the first member) .

From the above observations we can formulate the deviation correction term for all real numbers as

: 3n (m + n_1) (m-1) …(9)
(N.B: Since all the first terms has m=1, Correction Term = 2n (1-1) = 0.)
Thus,
3.2.3 **Rule5 (Deviation-Correction Term Rule for cubic-extensions): "The deviation of lower members in the group is equal to the product of Triple of "n" [3n], consecutive natural number just below the "m" [m-1] and sum of "m" first member [n_1]] i.e. " n_1 + m "."**

IV. Conclusion

4.1 We can represent extension of any exponential expressions as the combination of three rules (First Member-Relation Rule, Ending Rules and Deviation-Correction Rule) which are similar for all exponentials.

4.2 First Member-Relation Term and Deviation-Correction Term of a higher exponentiations share more than one term with the lower exponentiations.

E.g. n_1 x n and n are common for First Member Relation Term of both U.S.E and U.C.E.

n(m-1) is common term of Deviation Correction Term for all exponential extensions.

Thus, we can say too that the First Member Relation Term and Deviation Correction Term of any higher exponentiation is a 'modified form' of lower.

4.3 Since, First Member Relation Term is the same for all the members of a group of ten consecutive natural numbers, And Deviation Correction Term shows regular ascending trends over groups, Exbaspex can be used <u>to classify natural numbers based their </u>First Member Relation Term and Deviation Correction Term. This would be useful for the study of trends in Exponential Extensions.

Example : Square Index Table.- New classification of numbers based on Exbaspex

This is a new classification of natural numbers based on First Member-Relation Term and Deviation-Correction Term (DCT) of the Universal Square Extension (U.S.E.) of Exbaspex. [Fig. 3]

Application of Exponential Base Splitting Expressions-Extension(Exbaspex) - Universal Square Extension (U.S.E)

SQUARE INDEX TABLE

		Member 0	Member 1	Member 2	Member 3	Member 4	Member 5	Member 6	Member 7	Member 8	Member 9
Group 1	FMRT	0	0	0	0	0	0	0	0	0	0
	Square	0	1	4	9	16	25	36	49	64	81
	Number	0	1	2	3	4	5	6	7	8	9
	Scindo	0 0	0 1	0 2	0 3	0 4	0 5	0 6	0 7	0 8	0 9
	DCT	0	0	0	0	0	0	0	0	0	0
Group 2	FMRT	12	12	12	12	12	12	12	12	12	12
	Square	100	121	144	169	196	225	256	289	324	361
	Number	10	11	12	13	14	15	16	17	18	19
	Scindo	1 0	1 1	1 2	1 3	1 4	1 5	1 6	1 7	1 8	1 9
	DCT	-2	0	2	4	6	8	10	12	14	16
Group 3	FMRT	44	44	44	44	44	44	44	44	44	44
	Square	400	441	484	529	576	625	676	729	784	841
	Number	20	21	22	23	24	25	26	27	28	29
	Scindo	2 0	2 1	2 2	2 3	2 4	2 5	2 6	2 7	2 8	2 9
	DCT	-4	0	4	8	12	16	20	24	28	32
Group 4	FMRT	96	96	96	96	96	96	96	96	96	96
	Square	900	961	1024	1089	1156	1225	1296	1369	1444	1521
	Number	30	31	32	33	34	35	36	37	38	39
	Scindo	3 0	3 1	3 2	3 3	3 4	3 5	3 6	3 7	3 8	3 9
	DCT	-6	0	6	12	18	24	30	36	42	48
Group 5	FMRT	168	168	168	168	168	168	168	168	168	168
	Square	1600	1681	1764	1849	1936	2025	2116	2209	2304	2401
	Number	40	41	42	43	44	45	46	47	48	49
	Scindo	4 0	4 1	4 2	4 3	4 4	4 5	4 6	4 7	4 8	4 9
	DCT	-8	0	8	16	24	32	40	48	56	64
Group 6	FMRT	260	260	260	260	260	260	260	260	260	260
	Square	2500	2601	2704	2809	2916	3025	3136	3249	3364	3481
	Number	50	51	52	53	54	55	56	57	58	59
	Scindo	5 0	5 1	5 2	5 3	5 4	5 5	5 6	5 7	5 8	5 9
	DCT	-10	0	10	20	30	40	50	60	70	80
Group 7	FMRT	372	372	372	372	372	372	372	372	372	372
	Square	3600	3721	3844	3969	4096	4225	4356	4489	4624	4761
	Number	60	61	62	63	64	65	66	67	68	69
	Scindo	6 0	6 1	6 2	6 3	6 4	6 5	6 6	6 7	6 8	6 9
	DCT	-12	0	12	24	36	48	60	72	84	96
Group 8	FMRT	504	504	504	504	504	504	504	504	504	504
	Square	4900	5041	5184	5329	5476	5625	5776	5929	6084	6241
	Number	70	71	72	73	74	75	76	77	78	79
	Scindo	7 0	7 1	7 2	7 3	7 4	7 5	7 6	7 7	7 8	7 9
	DCT	-14	0	14	28	42	56	70	84	98	112

Fig. 3

Key features:

- In this table Natural numbers are arranged in groups based on their First Member-Relation Term (FMRT).
- Each group is represented in rows and its members in columns.
- Each Member is represented in definite boxes along with their square, FMRT, Deviation-Correction Term (DCT) and its Scindo terms. [Fig. 4]

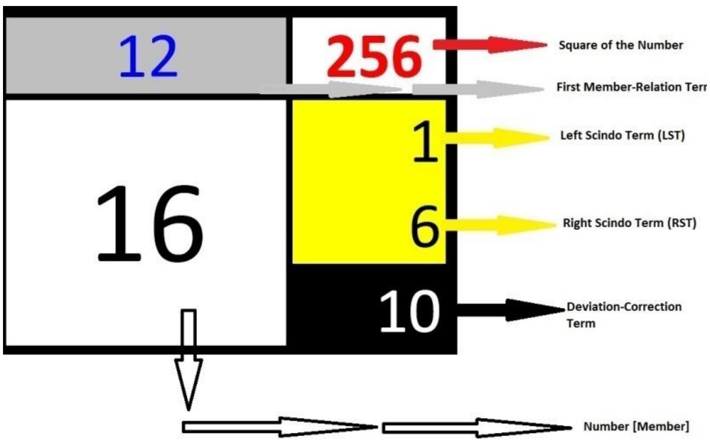

Fig. 4

- Every member of each group possesses the **same** First Member-Relation Term.

Group 2	12	12	12	12	12	12	12
	100	121	144	169	196	225	
	10	11	12	13	14	15	16
	1 0	1 1	1 2	1 3	1 4	1 5	
	-2	0	2	4	6	8	

Fig. 5

Example: Group 2 (10 to 19), FMRT = 12 [Fig.5]

- Members [numbers] are arranged in each group in the **ascending order** of their Deviation-Correction Term.

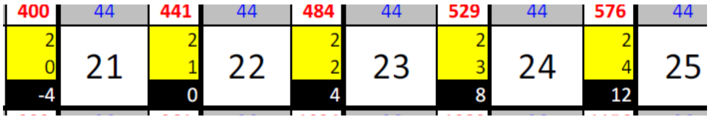

Fig. 6

- The Deviation-Correction Term of every First Member [Member 1] is zero.[since its deviation is zero.]

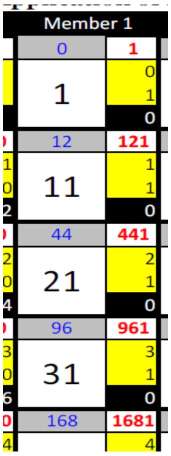

Fig. 7

- Since, all the numbers whose RST is zero possesses a unique property of having a negative DCT [except Group 1]. So, they are included as a separate column called **Zeroth Members** [Member 0].
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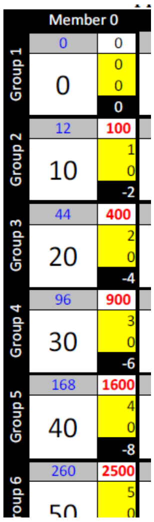

Fig. 8

4.4 To find exponentials of integers
4.4.1 To find the square of any integer.

Example:

$\{0\_9\}^{2} = \{((0\_1 \text{ x } 0) + 0\ ) + 2 \text{ x } 0\ (9\text{-}1)\}\_9^{2}$
= {(01x0) +0} +0_81= 81.

$\{1\_2\}^{2} = \{(\ (1\_1 \text{ x } 1) +1\ ) +2 \text{ x1 } (2\text{-}1)\}\_2^{2}$
= {((11 x 1) + 1) + 2} _4 =144.

4.4.2 To Find the Cube of any integer.
Example:

(a) $\{0\_9\}^3$ = {(0_1 x 0 (0_1+1)) + 0)+ 3x0 (9+ 0_1) (9 -1) }_9^3
= 0_9^3 = 729
(b) $\{5\_6\}^3$ = { (5_1 x 5 (5_1 + 1)) + 5) + 3x5 (6 + 5_1) (6-1) }_6^3
= { (5_1 x 5 (52)) + 5) + 3x5 (6 + 5_1) (5) }_216
= { (255 x 52) + 5) + 15 x (57) (5) }_216
= { 13260 + 5) + 855 x 5 }_216
= {13265 + 4275 }_216
={17540}_216
= 17561_6 = 175616.
(c) $\{32\_4\}^3$ = { (32_1 x 32 (32_1 + 1)) + 32) + 3 x 32 (4 + 32_1) (4-1) }_4^3
= { (32_1 x 32 (322)) + 32) + 3 x 32 (325) (3) }_4^3
= { (10272 x 322)) + 32) + 96 (325) (3) }_64
= { (3307584+ 32) + 96 x 325 x 3) }_64
= { (3307616) + 93600) }_64
= { 3401216 }_64
= 3401222_4 = 34012224.

4.5 Even though it will be more complex to use extensions to find exponentials, study of this method helps to familiarize and memorize squares and cubes of numbers easily, especially for school students.

References

[1]. Wikipedia, the free encyclopedia. http://en.wikipedia.org/wiki