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A Study on linear time-varying Electronic Circuit Using Adomian Decomposition Method

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Abstract : *In this paper an interesting and famous realistic electronic circuit problem is discussed using the Adomian Decomposition Method (ADM) is presented. To illustrate the effectiveness of the ADM an example from linear time-varying electrical circuit problem have been considered and compared with the Runge-Kutta Butcher algorithm (RKB)[5] and with exact solutions of the problems, and are found to be very accurate. Error graphs for the linear time-varying electrical circuit problem are presented in a graphical form to show the efficiency of this ADM. This ADM can be easily implemented in a digital computer and the solution can be obtained for any length of time.*

Keywords:*Runge-Kutta method, Runge-Kutta Butcher algorithm, time-varying singular systems, Adomian Decomposition Method.*

I. Introduction

Singular systems play a vital role in various disciplines of engineering. Obtaining accurate numerical solutions of singular systems which govern physical systems has always been a major problem for scientist and engineers. So for we have seen many model of singular systems which have been solved through orthogonal functions, STWS and the classical fourth order RK methods etc [4]. Shih [11] used the block pulse function (BPF) technique to analyse time-varying and non-linear networks. Subbayyan and Zakariah [12] applied the STWS approach to the computer-aided design of electronic circuits. Balachandran and Murugesan [2] introduced the Single Term Walsh Series (STWS) for the singular system of electronic circuits.

In this paper we developed numerical methods for addressing time-varying electronic circuit by an application of the Adomian Decomposition Method which was studied by Sekar and team of his researchers [6-10]. Recently, Murugesan *et al*. [5] discussed the time-varying electronic circuit using RK-Butcher algorithm. In this paper, the same time-varying electronic circuit problem was considered (discussed by Murugesan *et al.* [5]) but present a different approach using the Adomian Decomposition Method with more accuracy for time-varying electronic circuit. In this paper we show the simulation results in graphical form to highlight the effectives of ADM compare to RK-Butcher algorithm.

II. Adomian Decomposition Method

Suppose k is a positive integer and $f_1, f_2, \dots, f_k$ are k real continuous functions defined on some domain G. To obtain k differentiable functions $y_1, y_2, \dots, y_k$ defined on the interval I such that $(t, y_1(t), y_2(t), \dots, y_k(t)) \in G$ for $t \in I$.

Let us consider the problems in the following system of ordinary differential equations:

$$\frac{dy_i(t)}{dt} = f_i(t, y_1(t), y_2(t), \dots, y_k(t)) \quad , \quad y_i(t)|_{t=0} = \beta_i \tag{1}$$

where β_i is a specified constant vector, $y_i(t)$ is the solution vector for $i = 1, 2, \dots, k$. In the decomposition method, (1) is approximated by the operators in the form: $Ly_i(t) = f_i(t, y_1(t), y_2(t), \dots, y_k(t))$ where L is the first order operator defined by $L = d/dt$ and $i = 1, 2, \dots, k$.

Assuming the inverse operator of L is L^{-1} which is invertible and denoted by $L^{-1}(.) = \int_{t_0}^{t} (.)\, dt$, then applying L^{-1} to $L\, y_i(t)$ yields

$$L^{-1} L y_i(t) = L^{-1} f_i(t, y_1(t), y_2(t), \dots, y_k(t))$$

where $i = 1, 2, \dots, k$. Thus

$$y_i(t) = y_i(t_0) + L^{-1} f_i(t, y_1(t), y_2(t), \ldots, y_k(t)) .$$

Hence the decomposition method consists of representing $y_i(t)$ in the decomposition series form given by

$$y_i(t) = \sum_{n=0}^{\infty} f_{i,n}(t, y_1(t), y_2(t), \ldots, y_k(t))$$

where the components $y_{i,n}$, $n \geq 1$ and $i = 1, 2, \ldots, k$ can be computed readily in a recursive manner. Then the series solution is obtained as

$$y_i(t) = y_{i,0}(t) + \sum_{n=1}^{\infty} \{L^{-1} f_{i,n}(t, y_1(t), y_2(t), \ldots, y_k(t))\} .$$

For a detailed explanation of decomposition method and a general formula of Adomian polynomials, we refer reader to [Adomian 1].

III. Linear Time-Varying Electronic Circuit

Consider the physical model of an electronic circuit discussed on p.347 of Chua and Lin [3] and analyzed by Balachandran and Murugesan [2], as shown in the Fig. 1. This electronic circuit is governed by the following hybrid equations, Yan [13].

$$\begin{bmatrix} i_1 \\ i_4 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_4 \\ i_2 \\ i_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ 1 & 0 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} E_a \\ J_b \end{bmatrix} \tag{3}$$

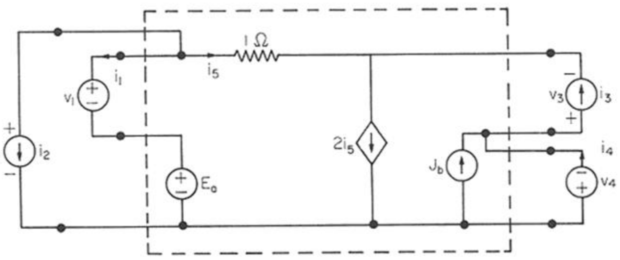

Fig. 1 Electronic Circuit.

Since $i_c = C\dot{v}_c$ and $v_L = L\dot{i}_L$, substituting $i_1 = 2\dot{v}_1$, $i_2 = 2\dot{v}_2$, $v_3 = 2\dot{i}_3$ and $v_4 = 2\dot{i}_4$ into (3) then we obtain:

$$\begin{bmatrix} 2\dot{v}_1 \\ i_4 \\ v_2 \\ 2\dot{i}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ 2\dot{i}_4 \\ 2\dot{v}_2 \\ i_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ 1 & 0 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} E_a \\ J_b \end{bmatrix}$$

After re-arranging the terms, we obtain the singular system of equations as:

$$\begin{bmatrix} 2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ \dot{i}_3 \\ \dot{i}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ i_3 \\ i_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} E_a \\ J_b \end{bmatrix} \tag{4}$$

This is of the form $K\dot{x}(t) = Ax(t) + Bu(t)$

In order to study the effectiveness of the time-varying singular system in electronic circuits, a hypothetical system is formed by transforming the matrices K, A and B which are basically time independent in equation (4) with time-varying components.

Hence, the singular system of time-varying electronic circuit is of the form

$$\begin{bmatrix} 2 & 2 & 0 & 0 \\ 0 & 0 & 2t & 2t \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ \dot{i}_3 \\ \dot{i}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -t & 0 \\ 0 & 0 & t & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & t & -t \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ i_3 \\ i_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -t & 0 \\ t & 0 \\ 0 & -t \end{bmatrix} \begin{bmatrix} t \\ \cos(t) \end{bmatrix} \quad (5)$$

This is of the form $K(t)\dot{x}(t) = A(t)x(t) + B(t)u(t)$

The exact solution of equation (5) is

$$\left.\begin{aligned} v_1 &= \frac{-t^3}{12} - t^2 - \frac{(4-t)\cos(t) + (4t+1)\sin(t)}{34} + \frac{59(t-4)e^{\frac{t}{4}}}{17} + \frac{255}{17} \\ v_2 &= v_1 + t^2 \\ i_3 &= (t+4) + \frac{2}{17}[\sin(t) + 4\cos(t)] - \frac{59e^{\frac{t}{4}}}{17} \\ i_4 &= i_3 - \cos(t) \end{aligned}\right\} \quad (6)$$

with initial conditions $[v_1(0) \quad v_2(0) \quad i_3(0) \quad i_4(0)]^T = [1 \quad 1 \quad 1 \quad 0]^T$

IV. Numerical Results

Time t	$v_1(t)$		$v_2(t)$	
	RK-Butcher Algorithm Error	ADM Error	RK-Butcher Algorithm Error	ADM Error
0.00	1.00E-09	1.00E-11	1.00E-09	1.00E-11
0.50	1.00E-09	1.00E-11	2.00E-09	2.00E-11
1.00	1.00E-09	1.00E-11	3.00E-09	3.00E-11
1.50	1.00E-09	1.00E-11	4.00E-09	4.00E-11
2.00	1.00E-09	1.00E-11	5.00E-09	5.00E-11
2.50	1.00E-09	1.00E-11	6.00E-09	6.00E-11
3.00	1.00E-09	1.00E-11	7.00E-09	7.00E-11
3.50	1.00E-09	1.00E-11	8.00E-09	8.00E-11
4.00	1.00E-09	1.00E-11	8.00E-09	8.00E-11
4.50	1.00E-09	1.00E-11	9.00E-09	9.00E-11
5.00	1.00E-09	1.00E-11	9.00E-09	9.00E-11

Table 1 Error for time-varying (Electronic circuit) $v_1(t)$ and $v_2(t)$

Time t	$i_3(t)$		$i_4(t)$	
	RK-Butcher Algorithm Error	ADM Error	RK-Butcher Algorithm Error	ADM Error
0.00	1.00E-09	1.00E-11	1.00E-09	1.00E-11
0.50	1.00E-09	1.00E-11	2.00E-09	2.00E-11
1.00	1.00E-09	1.00E-11	3.00E-09	3.00E-11
1.50	1.00E-09	1.00E-11	4.00E-09	4.00E-11
2.00	1.00E-09	1.00E-11	5.00E-09	5.00E-11
2.50	1.00E-09	1.00E-11	6.00E-09	6.00E-11
3.00	1.00E-09	1.00E-11	7.00E-09	7.00E-11
3.50	1.00E-09	1.00E-11	8.00E-09	8.00E-11
4.00	1.00E-09	1.00E-11	8.00E-09	8.00E-11
4.50	1.00E-09	1.00E-11	9.00E-09	9.00E-11
5.00	1.00E-09	1.00E-11	9.00E-09	9.00E-11

Table 2 Error for time-varying (Electronic circuit) $i_3(t)$ and $i_4(t)$

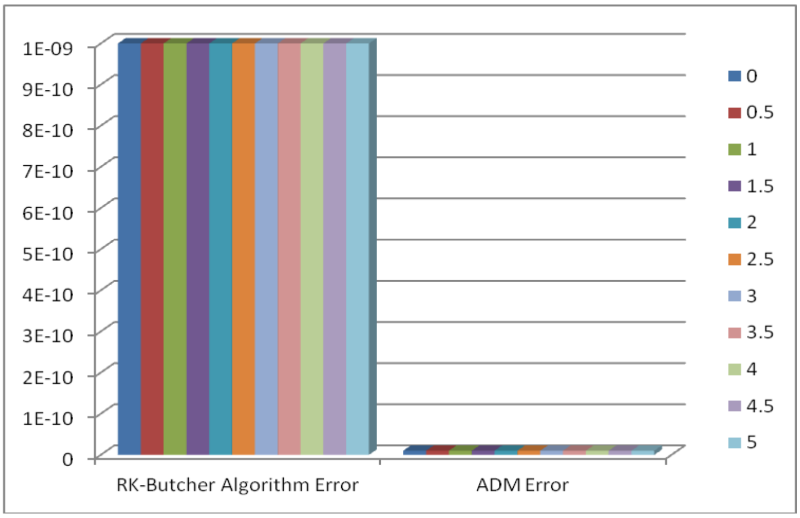

Fig. 2 Error graph for $v_1(t)$

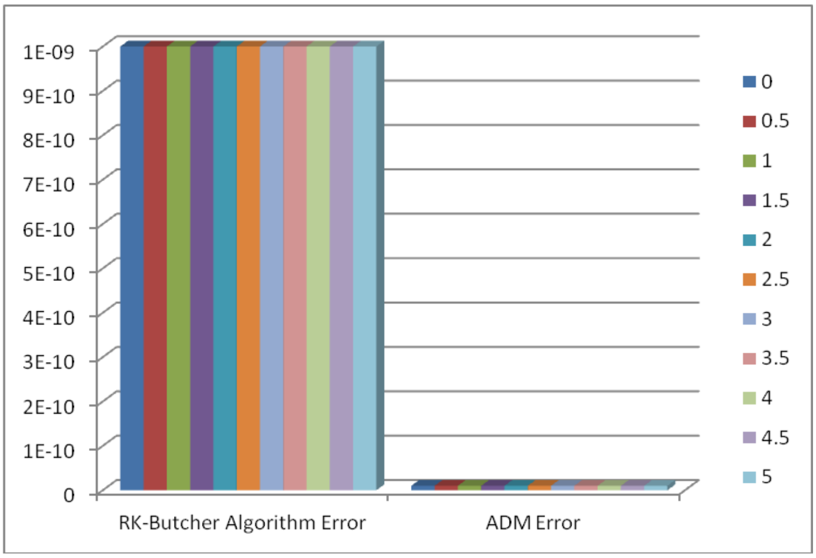

Fig. 3 Error graph for $v_2(t)$

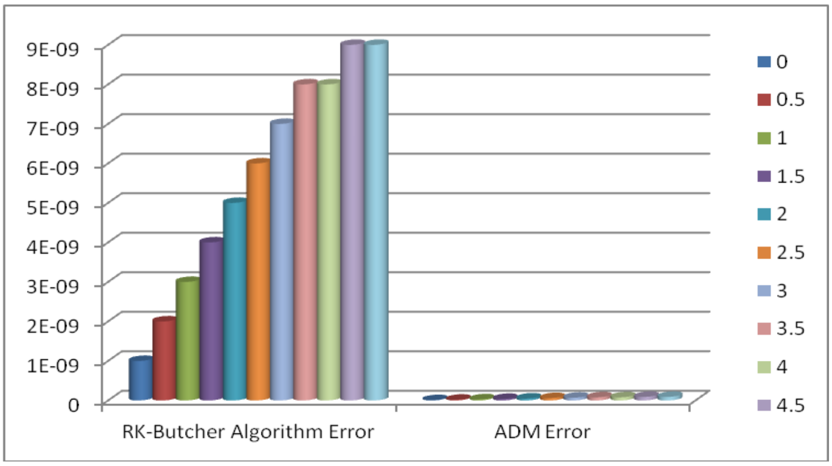

Fig. 4 Error graph for $i_3(t)$

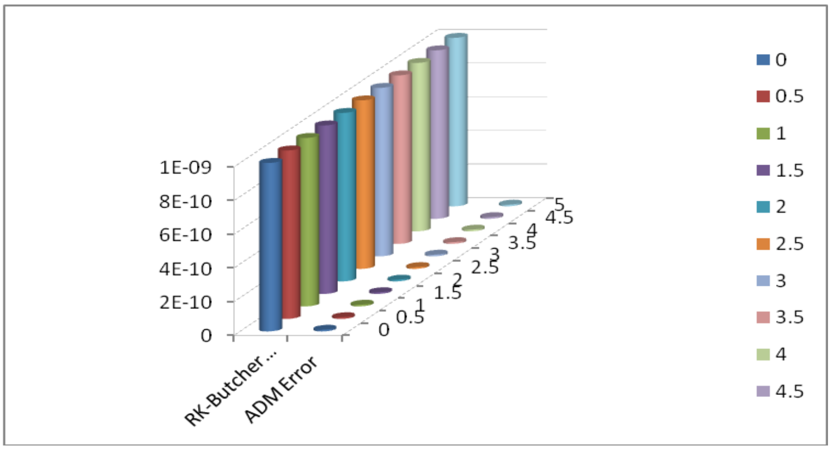

Fig. 5 Error graph for $i_4(t)$

The approximate solutions of the system (5) have been determined using the ADM (with step-size h=0.5) and are compared with the exact solutions of the system (5) presented in equation (6) along with the solutions obtained using RKB method and are presented in Table 1-2. This ADM yields more accurate results when compared to the RKB method. Solution curves for the capacitor voltages for $v_1(t)$ and $v_2(t)$ as well as the inductor currents $i_3(t)$ and $i_4(t)$ are drawn for various time intervals and are presented in Fig. 2-5.

Errors between the exact and approximate solutions are also given in Table 1-2. To exhibit the efficiency of the discussed methods, an error graph is presented for the variable $v_1(t), v_2(t), i_3(t)$ and $i_4(t)$ in Fig. 2-5 at various time intervals and from this it is observed that the ADM gives more accurate results when compared to the RKB method discussed by Murugesan *et al*. [5].

V. Conclusion

The approximate solutions obtained using the ADM give more accurate values when compared to the RKB method discussed by Murugesan *et al*. [5]. From the Table 1-2, it can be observed that the solutions obtained by the ADM match well with the exact solutions of the electronic circuit problem irrespective of time-varying cases, but the RKB method yields a little error. This is revealed from Fig. 2-5.

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