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Some New kinds of Connected Domination in Fuzzy Graphs

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Abstract: *In this paper, we introduce the concept of some new kinds of connected domination number of a fuzzy graphs. We determine the domination numbers* γ_{cs}, γ_{ds} ,γ_{lsc} , γ_{rsc} *and the total domination number of* γ_{tcs}, γ_{tds} *for several classes of fuzzy graphs and obtain bounds for the same. We also obtain the Nordhaus – Gaddum type result for these parameters.*

Keywords: *Fuzzy graphs, fuzzy domination , connected strong domination, disconnected strong domination, total connected strong domination, total disconnected strong domination, left semi connected domination, right semi connected domination.*

I. Preliminaries

Definition:1.1[10]

Let V be a finite non empty set. Let E be the collection of all two element subsets of V. A fuzzy graph G=(σ,μ) is a set with two functions $\sigma : V \to [0,1]$ and $\mu: E \to [0,1]$ such that $\mu(\{u ,v\}) \le \sigma(u) \wedge \sigma(v)$ for all u ,v∈ V.

Definition:1.2[10]

Let G=(σ,μ) be a fuzzy graph on V and $V_1 \subseteq V$. Define σ_1 on V_1 by $\sigma_1(u)=\sigma(u)$for all u∈ V_1 and μ_1 on the collection E_1 of two element subsets of V_1 by $\mu_1(\{u ,v\}) = \mu(\{u ,v\})$ for all u ,v∈ V_1,then (σ_1,μ_1) is called the fuzzy subgraph of G induced by V_1 and is denoted by $<V_1>$.

Definition:1.3[10]

The order p and size q of a fuzzy graph G=(σ,μ) are defined to be $p=\sum_{u\in V} \sigma(u)$ and $q=\sum_{(u ,v)\in E} \mu(\{u ,v\})$.

Definition:1.4[10]

Let G=(σ,μ) be a fuzzy graph on V and D⊆V then the fuzzy cardinality of D is defined to be $\sum_{u\in D} \sigma(u)$.

Definition:1.5[10]

An edge e={u ,v} of a fuzzy graph is called an effective edge if $\mu(\{u ,v\}) = \sigma(u) \wedge \sigma(v)$.

$N(u) = \{ v\in V/ \mu(\{u ,v\}) = \sigma(u) \wedge \sigma(v)\}$ is called the neighborhood of u and $N[u]=N(u)\cup \{u\}$ is the closed neighborhood of u.

The effective degree of a vertex u is defined to be the sum of the weights of the effective edges incident at u and is denoted by dE(u). $\sum_{v\in N(u)} \sigma(v)$is called the neighborhood degree of u and is denoted by dN(u). The minimum effective degree $\delta_E(G)=\min\{dE(u)|u\in V(G)\}$ and the
maximum effective degree
$\Delta_E(G)=\max\{dE(u)|u\in V(G)\}$.

Definition :1.6[10]

The complement of a fuzzy graph G denoted by $\bar{G}$ is defined to be $\bar{G} = (\sigma,\overline{\mu})$ where $\overline{\mu}(\{u,v\}) = \sigma(u)\wedge\sigma(v) - \mu(\{u,v\})$.

Definition :1.7[1]

A set of fuzzy vertex which cover all the fuzzy edges is called a fuzzy vertex cover of G and the minimum cardinality of a fuzzy vertex cover is called a vertex covering number of G and denoted by β(G).

Definition :1.8[10]

Let $\sigma:V\to[0,1]$ be a fuzzy subset of V. Then the complete fuzzy graph on σ is defined to be (σ,μ) where $\mu(\{u ,v\})=\sigma(u)\wedge\sigma(v)$ for all {u ,v}∈E and is denoted by K_σ.

Definition :1.9[3]

Let G=(V,E) be a graph. A subset D of V is called a dominating set in G if every vertex in V-D is adjacent to some vertex in D.

Definition :1.10[10]

Let G=(σ,μ) be a fuzzy graph on V. Let $u,v \in V$. We say that u dominates v in G if $\mu(\{u,v\})=\sigma(u)\wedge\sigma(v)$. A subset D of V is called a dominating set in G if for every $v \notin D$, there exists $u \in D$ such that u dominates v. The minimum fuzzy cardinality of a dominating set in G is called the domination number of G and is denoted by γ(G)or γ.

Definition :1.11[10]

A fuzzy graph G=(σ,μ) is said to be a bipartite if the vertex V can be partitioned into two nonempty sets V_1 and V_2 such that $\mu(v_1,v_2)=0$ if $v_1,v_2 \in V_1$ or $v_1,v_2 \in V_2$. Further, if $\mu(u,v)=\sigma(u) \wedge \sigma(v)$ for all $u \in V_1$ and $v \in V_2$ then G is called a complete bipartite graph and is denoted by K_{σ_1,σ_2} where σ_1 and σ_2 are, respectively, the restrictions of σ to V_1 and V_2.

Definition :1.12[10]

A dominating set D of a fuzzy graph G=(σ,μ) is connected dominating set, if the induced fuzzy sub graph $H=(<D>,\sigma',\mu')$ is connected. The minimum fuzzy cardinality of a connected dominating set of G is called the connected dominating number of G and is denoted by $\gamma_c(G)$ (or) γ_c.

Definition :1.13[10]

A dominating set D of a fuzzy graph G is said to be a minimal dominating if no proper subset D′ of D is dominating set of G such that |D′|<|D|.

II. Fuzzy Connected Strong Domination Number

Definition : 2.1

Let G=(σ,μ) be a fuzzy graph without isolated vertices. A subset D_{cs} of V is said to be a fuzzy connected strong domination set if both induced subgraphs $<D_{cs}>$ and $<V-D_{cs}>$ are connected. The fuzzy connected strong domination number $\gamma_{cs}(G)$ is the minimum fuzzy cardinality taken over all connected strong dominating sets of G.

Example:2.1

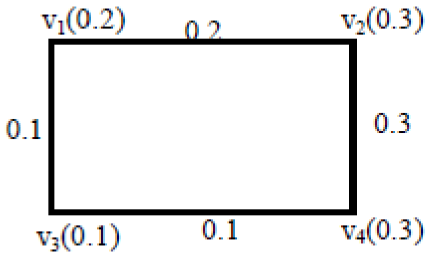

Fig.(1)

$D_{cs} = \{v_1,v_3\}$,
$V - D_{cs} = \{v_2,v_4\}$
$\gamma_{cs}(G) = 0.3$
$<D_{cs}>$, $<V - D_{cs}>$ are connected.

One possible application of the concept of connected strong domination is that to consider D_{cs} is the Indian embassy and V- D_{cs} is a foreign embassy to maintain better political relationship among each one has its own connected team for effective management.

Definition : 2.2

Let G=(σ,μ) be a fuzzy graph, A subset D_{cs} of V is said to be a fuzzy total connected strong domination set if

(i) D_{cs} is connected strong dominating set
(ii) $N[D_{cs}] = V$

The fuzzy total connected strong domination number $\gamma_{tcs}(G)$ is the minimum cardinality taken over all total connected strong dominating sets in G.

Proposition 2.1

$\gamma_{cs}(p_n) = \min \{p-\sigma(v_1), p-\sigma(v_n)\}$,
when v_1 and v_n are the pendent vertices.

Proposition 2.2

$\gamma_{cs}(C_n) = \min \left\{\sum_{i=1}^{n-2} \sigma(v_i), \sum_{i=2}^{n-1} \sigma(v_i), \ldots \sum_{i=n-2}^{1} \sigma(v_i)\right\}$

Proposition 2.3

$\gamma_{cs}(W_n) = \sigma(v)$, v is the centre vertex

Proposition 2.4

$\gamma_{cs}(K_\sigma) = \sigma(v)$, v is the vertex of minimum cardinality.

Proposition 2.5

$\gamma_{cs}(\text{star}) = \sigma(v_i) + \sigma(v_j)$, v_i is a vertices adjacent with all other vertices and v_j is the all pendent vertices of minimum cardinality, except one pendent vertex has maximum cardinality.

Proposition 2.6

$\gamma_{cs}(K_{\sigma_1,\sigma_2}) = \min \{\sigma(v_i)\} + \min \{\sigma(v_j)\}$ where $v_i \in V_1$ and $v_j \in V_2$.

Proposition 2.7

Let G be the Peterson graph, (i) If all fuzzy vertices having equal cardinality then $\gamma_{cs}(G) = 5\sigma(v_i)$, for i=1to 10.
(ii) If an unequal fuzzy vertex cardinality, then

$$\gamma_{cs}(G) = \min \left\{ \sum_{i=1}^{5} \sigma(v_i),\ \sum_{i=6}^{10} \sigma(v_i) \right\}$$

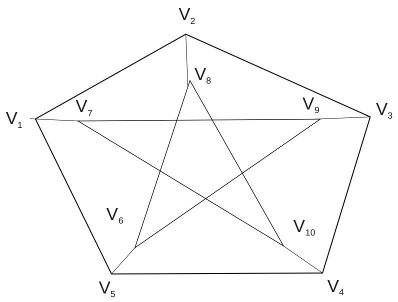

Fig.(2)

Theorem :2.1

A connected strong dominating set D_{cs} of G is a minimal dominating set iff for each vertex $d \in D_{cs}$, one of the following condition holds.

(i) $N(d) \cap D_{cs} = \phi$
(ii) There exist $c \in V-D_{cs}$ such that $N(c) \cap D_{cs} = \{d\}$

Proof :

Suppose that D_{cs} is minimal and there exists a vertex $v \in D_{cs}$ such that v does not satisfy any of the above conditions. Then by condition (i) and (ii), $D' = D_{cs}-\{v\}$ is a dominating set of G, This implies that D' is connected strong dominating set of G which is contradiction.

Theorem : 2.2.

Let G=(σ,μ) be a fuzzy connected strong dominating set iff G contains a path P_n or cycle C_n, depends upon the number of vertices of the graph G.

Proof : Let G be a fuzzy graph

Let D be the γ_{cs} – set of G, then<D> and <V-D> are connected graphs.
For each u ,v∈G, then there exists a path. since<D> and <V-D> are connected.
Suppose G is a path or any tree the result holds good.
Consider G contains any cycle then there is a cycle contains u and v. since<D> and <V-D> are connected.

Theorem :2.3

For any Fuzzy graph G=(σ,μ),
$p\text{-}q \le \gamma_{cs}(G) \le P\text{-}\Delta$

Proof :
Let v be a vertex of a fuzzy graph, such that $d_N(v) = \Delta$ then V/N(v) is a dominating set of G, so that $\gamma_{cs}(G) \le |V \setminus N(v)| \le p\text{-}\Delta$.

Example 2.2

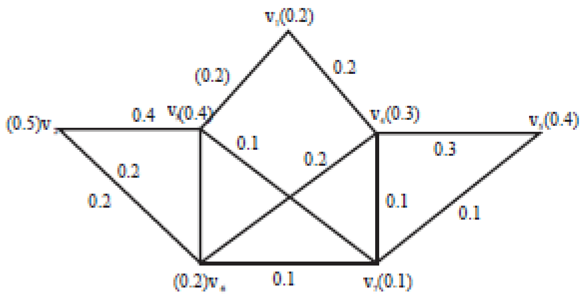

Fig.(3)

D = $\{v_1,v_3,v_4\}$
<V-D> = $\{v_2,v_5,v_6,v_7\}$
p = 2.1, q = 2.1, Δ = 0.9
γ_{cs} (G) = 0.9
$p\text{-}q \le \gamma_{cs}(G) \le p\text{-}\Delta$

Theorem :2.4
For any fuzzy graphs G,
$\gamma_{cs}(G) + \gamma_{cs}(\overline{G}) \le 2p$,

where $\gamma_{cs}(\overline{G})$ is the connected strong domination number of $\overline{G}$ and equality holds iff $0 \le \mu(x,y) < \sigma(x) \wedge \sigma(y)$, for all x , y∈V.

Example:2.3

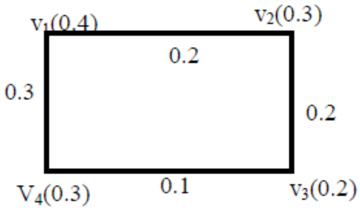

Fig. (4)

$D_{cs} = \{v_3,v_4\}$, $\gamma_{cs}(G) = 0.5$, P = 1.2

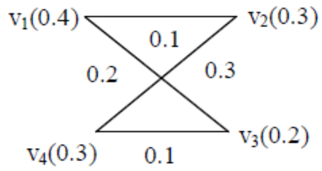

Fig.(5)

$D_{cs} = \{v_3,v_4\}$, $\gamma_{cs}(\overline{G}) = 0.6$

$\gamma_{cs}(G) + \gamma_{cs}(\overline{G}) \leq 2p$

Remark :

A fuzzy graph G=(σ,μ)has an equal cardinality of all vertices then $\gamma_{cs}(\overline{G})$ does not exist.

Theorem 2.5

For any fuzzy graph G=(σ,μ),

$\gamma_{cs}(G) \leq \beta(G) \leq \Gamma_{cs}(G)$

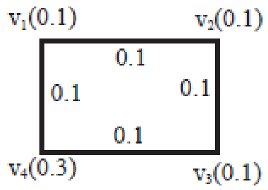

Fig.(6)

$\gamma_{cs}(G) = 0.2$, $\Gamma_{cs}(G) = 0.6$, $\beta(G) = 0.2$

Remark : For equal fuzzy cardinality

$\gamma_{cs}(G) = \beta(G) = \Gamma_{cs}(G)$

Theorem 2.6 :

For any fuzzy graph G=(σ,μ),

$\gamma_{cs}(G) \leq \gamma_{tcs}(G) \leq 2\gamma_{cs}(G)$

Proof : Since every total connected strong dominating set is a connected strong dominating set. Therefore, $\gamma_{cs}(G) \leq \gamma_{tcs}(G)$. Let D_{cs} be a connected strong dominating set with finite vertices say $\{v_1,v_2,\ldots v_n\}$. For each $v_i \in D_{cs}$, choose one vertex $u_i \in V - D_{cs}$ such that v_i and u_i are adjacent.

This is possible since G has no isolated vertices.

Now the set $\{v_1, v_2, \ldots v_n, u_1, u_2, \ldots u_n\}$ is a total connected strong dominating set of G,

$\therefore \gamma_{tcs}(G) \leq 2\gamma_{cs}(G)$

Hence $\gamma_{cs}(G) \leq \gamma_{tcs}(G) \leq 2\gamma_{cs}(G)$

Example 2.4

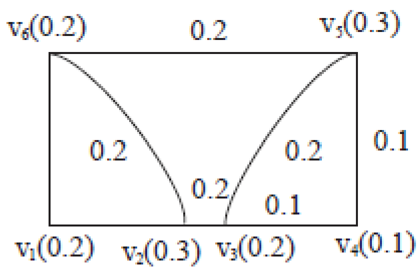

Fig.(7)

$D_{cs} = \{v_2, v_3\}$, $\gamma_{cs} = 0.5$
$D_{tcs} = (v_2, v_3\}$, $\gamma_{tcs} = 0.5$
$\gamma_{cs}(G) \leq \gamma_{tcs}(G) \leq 2\gamma_{cs}(G)$

Theorem :2.7

Let $G=(\sigma,\mu)$ be a connected fuzzy graph and H be a spanning fuzzy subgraph of G. If H has a connected strong dominating set then $\gamma_{tcs}(G) \leq \gamma_{tcs}(H)$

Proof :

Let $G=(\sigma,\mu)$ be a fuzzy graph and $H=(\sigma',\mu')$ be the spanning fuzzy subgraph of G. Let $D_{tcs}(G)$ be the fuzzy minimum total connected strong dominating set of H but not minimum.

Example 2.5

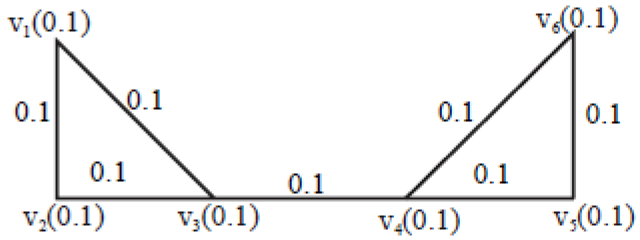

Fig.(8)

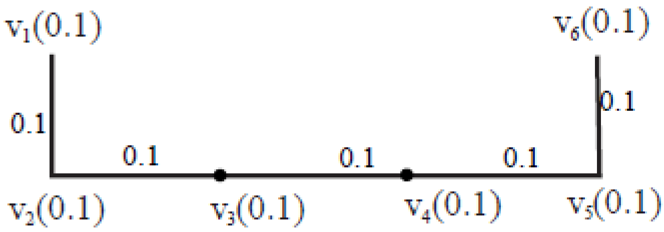

Fig.(9)

$D_{tcs}(G) = \{v_1, v_2, v_3, v_4\}$; $\gamma_{tcs}(G) = 0.4$
$D_{tcs}(H) = \{v_1, v_2, v_3, v_4, v_5\}$; $\gamma_{tcs}(H) = 0.5$

Theorem 2.8

If $G=(\sigma,\mu)$ is complete then$\gamma_{cs}(G) = \sigma(v_i)$, where v_i is the vertex of minimum fuzzy cardinality.

Proof : Let G be a complete fuzzy graph.

Every vertices are adjacent to all other,vertices,therefore each vertexdominates the other and every minimum dominating set of fuzzy complete graph K_σ contains exactly one vertex having minimum cardinality
$\therefore \gamma_{cs}(G)=\sigma(v_i)$ where v_i is a vertex of minimum fuzzy cardinality.

Theorem : 2.9

If $G=(\sigma,\mu)$ is a path, then G has exactly two different connected strong dominating set.

Proof:

In every Path P_n,
$V = \{v_1, v_2, \ldots v_n\}$ be a fuzzy vertex set,
by definition 2.1

Clearly $D_1 = \{v_1, v_2, \ldots v_{n-1}\}$ and
$D_2 = \{v_2, v_3, \ldots v_n\}$ are the two fuzzy connected strong dominating sets.

Theorem 2.10

Every connected strong dominating set is not an independent dominating set.

Proof :

Let G=(σ,μ) be a fuzzy graph assume that D_{cs} is a connected strong dominating set. Therefore D_{cs} is not independent dominating set since D_{cs} is connected.

Theorem 2.11

If G is a path, all connected strong dominating set are minimal dominating set.

Proof : By theorem 2.9,G has exactly two different connected dominating sets.

i.e) $D_1 = \{v_1,v_2,\dots v_{n-1}\}$

$D_2 = \{v_2,v_3,\dots v_n\}$

Obviously $D_1 - \{v_i\}$ is not a connected strong dominating set, for all $v_i \in D_1$.

Hence D_1 is a minimal connected strong dominating set.

Similarly for D_2.

Therefore both D_1 and D_2 are minimal connected strong dominating set.

Theorem 2.12: The fuzzy graph G=(σ,μ) has a connected strong dominating set iff G is connected.

Proof : Assume D is connected strong dominating set.

By definition 2.1

<D> and <V-D> are connected

To prove that G is connected.

Suppose G is not connected, then every dominating set is not a connected dominating set.Which is contradiction to assumption ∴G is connected.

Conversely,assume that G is connected.

To prove that G has a connected strong dominating set. G is connected, then there exists a set D such that <D> and <V-D> are connected.

∴G has a connected strong dominating set.

Theorem 2.13

If G=(σ,μ) is a fuzzy graph then $2\sigma(v_i) \leq \gamma_{cs}(G) + \sigma(v_i) \leq p$

Proof : Let G=(σ,μ) be a fuzzy graph.

By definition of fuzzy dominating set,

$\gamma(G) \leq p$.

Clearly $\gamma(G) \leq \gamma_{cs}(G)$,

Suppose all fuzzy vertices are isolated then $\gamma(G) = p$. Clearly $\gamma_{cs}(G) < p$.

By theorem 2.8,

We have $\sigma(v_i) \leq \gamma_{cs}(G)$ when G is complete or not

$\therefore \sigma(v_i) \leq \gamma_{cs}(G) \leq P-\sigma(v_i)$

Theorem 2.14

If G is connected fuzzy graph, then

$\gamma_{cs}(G) \leq P- \{\sigma(v_i) + \sigma(v_j)\}$, where v_i , v_j are the vertex having first two maximum fuzzy cardinality among the all vertices.

III. Fuzzy Disconnected Strong Domination Number

Definition 3.1

Let G=(σ,μ) be a fuzzy graphs without isolated vertices. A subset D_{ds} of V is said to be a fuzzy disconnected strong domination set if both induced subgraphs<D_{ds}> and <V-D_{ds}>are disconnected. The fuzzy disconnected strong domination number $\gamma_{ds}(G)$ is the minimum fuzzy cardinality taken overall disconnected strong dominating sets of G.

Example:3.1

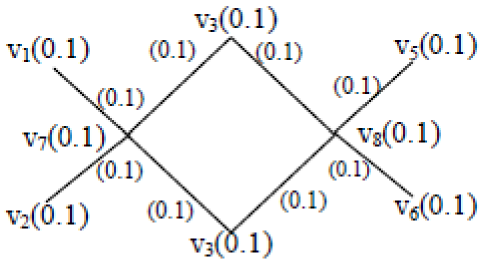

Fig.(10)

$D_{ds} = \{v_7,v_8\}$, V- $D_{ds} = \{v_1,v_2,v_3,v_4,v_5,v_6\}$
$\langle D_{ds}\rangle$ and $\langle V\text{-}D_{ds}\rangle$ are disconnected.
$\gamma_{ds}(G) = 0.2$

Definition 3.2

Let $G=(\sigma,\mu)$ be a fuzzy graph, A subset D_{tds} of V is said to be a fuzzy total disconnected strong dominating set if.

(i) D_{tds} is disconnected strong dominating set.
(ii) $N[D_{tds}] = V$

The fuzzy total disconnected strong dominating number $\gamma_{tds}(G)$ is the minimum cardinality taken over all total disconnected strong dominating set in G.

Proposition : 3.1

$\gamma_{ds}(P_n) = \left\lfloor \frac{n}{2} \right\rfloor \sigma(v_i)$, $\sigma(v_i)$'s are equal.

Proposition : 3.2

$\gamma_{ds}(C_n) = \left\lceil \frac{n}{3} \right\rceil \sigma(v_i), \sigma(v_i)'$s are equal, $n \geq 4$.

Proposition : 3.3

$\gamma_{ds}(K_{\sigma_1,\sigma_2}) = \min \left\{ \sum_{i=1}^{m} \sigma(v_i), \sum_{j=1}^{n} \sigma(v_j) \right\}$

Where $v_i \in V_1$ and $v_j \in V_2$.

Remark:

$\gamma_{ds}(K_n)$, $\gamma_{ds}(W_n)$, γ_{ds}(star) does not exist.

Theorem : 3.1

A disconnected strong dominating set D_{ds} of a fuzzy graph $G=(\sigma,\mu)$ is minimal dominating set iff for each $d \in D_{ds}$ one of the following two conditions holds.

(i) $N(d) \cap D_{ds} = \phi$
(ii) There exist $c \in V\text{-}D_{ds}$ such that $N(c) \cap D_{ds} = \{d\}$

Proof :

Suppose that D_{ds} is minimal and there exists a vertex $v \in D_{ds}$ such that v does not satisfy any of the above conditions, then by condition (i) and (ii), $D' = D_{ds} - \{v\}$ is a dominating set of G. This implies that D′ is disconnected strong dominating set of G, which is contradiction.

Theorem 3.2

For any fuzzy graph $G=(\sigma,\mu)$,
$p\text{-}q \leq \gamma_{ds}(G) \leq p\text{-}\Delta$

Proof :

Let V be a vertex of a fuzzy graph, such that dN(v) = Δ, then V/N(v) is a dominating set of a fuzzy graph G(σ,μ)

So that $\gamma_{ds}(G) \leq |V \backslash N(v)| = p-\Delta$

From Fig. (9)

$D_{ds}(G) = \{v_7,v_8\}$

$V-D = \{v_1,v_2,v_3,v_4,v_5,v_6\}$

p = 0.8, q = 0.8

$\Delta = 0.4$, $\gamma_{ds}(G) = 0.2$

$p - q \leq \gamma_{ds}(G) \leq p-\Delta$

Theorem 3.3

For any fuzzy graph G=(σ,μ),

$\gamma_{ds}(G) \leq \gamma_{tds}(G) \leq 2\ \gamma_{ds}(G)$.

Proof :

Since every total disconnected strong dominating set is a disconnected strong dominating set, therefore $\gamma_{ds}(G) \leq \gamma_{tds}(G)$. Let D_{ds} be a disconnected strong dominating set with finite vertices, say $\{v_1,v_2,\ldots v_n\}$. For each $v_i \in D_{ds}$, Choose one vertex $u_i \in V-D_{ds}$ such that v_i and u_i are adjacent.

This is possible since G has no isolated vertices. Now the set $\{v_1,v_2,\ldots v_n, u_1,u_2,..u_n\}$ is a total disconnected strong dominating set of G, and hence $\gamma_{tds}(G) \leq 2\gamma_{ds}(G)$.

Hence $\gamma_{ds}(G) \leq \gamma_{tds}(G) \leq 2\gamma_{ds}(G)$.

Theorem 3.4

If G=(σ,μ) has atleast one cut vertex v and having one or more blocks with v is adjacent to all other vertices of the blocks, then v is in every disconnected strong dominating set.

IV. Fuzzy Left Semi Connected Domination Number

Definition 4.1

Let G=(σ,μ) be a fuzzy graph without isolated vertices. A subset D_{lsc} of V is said to be a fuzzy left semi connected dominating set if the induced fuzzy subgraph$\langle D_{lsc}\rangle$ is connected and induced fuzzy subgraph$\langle V-D_{lsc}\rangle$ is disconnected. The fuzzy left semi connected domination number $\gamma_{lsc}(G)$ is the minimum fuzzy cardinality taken over all left semi connected dominating sets of G.

Example:4.1

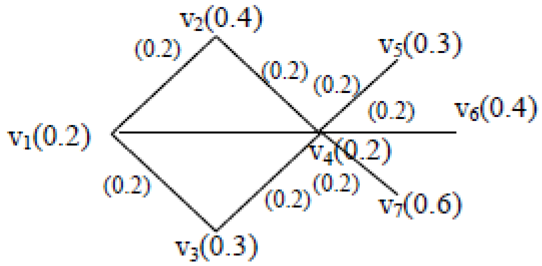

Fig.(11)

$D_{lsc} = \{v_1,v_4\}$, $V- D_{lsc} = \{v_2,v_3,v_5,v_6,v_7\}$

$\langle D_{lsc}\rangle$ is connected and

$\langle V- D_{lsc}\rangle$ is disconnected

$\gamma_{lsc}(G) = 0.4$

Proposition 4.1

$\gamma_{lsc}(P_n) = P - (\sigma(v_1) + \sigma(v_n))$

Proposition 4.2

$\gamma_{lsc}(C_n) = 0$

Proposition 4.3

$\gamma_{lsc}(W_n) = 3\,\sigma(v_i)$, $\sigma(v_i)$'s are equal.

Proposition 4.4

$\gamma_{lsc}(K_\sigma)$ = does not exist.

Proposition 4.5

$\gamma_{lsc}(\text{star}) = \sigma(v_i)$ where v_i is the fuzzy vertex having maximum effective degree.

Proposition 4.6

$$\gamma_{lsc}(K_{\sigma_1,\sigma_2}) = \min\left\{\sum_{i=1}^{m}\sigma(u_i)+\min\{\sigma(v_j)/j=1\,\text{to}\,n\},\ \sum_{i=1}^{n}\sigma(v_j)+\min\{\sigma(u_i)/i=1\,\text{to}\,m\}\right\}$$

Theorem 4.1 :

For any fuzzy graph $G=(\sigma,\mu)$

$\gamma(G) \leq \gamma_{lsc}(G)$

Theorem 4.2.

Let $G=(\sigma,\mu)$ be a connected fuzzy graph and $H=(\sigma',\mu')$ be a spanning fuzzy subgraph of G, If H has a left semi connected dominating set then

$\gamma_{lsc}(G) \leq \gamma_{lsc}(H)$.

Theorem 4.3

For any fuzzy graph $G=(\sigma,\mu)$, $\gamma_{lsc}(G)+\gamma_{lsc}(\overline{G}) \leq 2p$ where $\gamma_{lsc}(\overline{G})$ is the left semi connected domination number of $\overline{G}$ and equality holds iff $0 \leq \mu(x,y) < \sigma(x) \wedge \sigma(y)$ for all $x,y \in V$.

V. Fuzzy Right Semi Connected Domination Number

Definition 5.1.

Let $G=(\sigma,\mu)$ be a fuzzy graph without isolated vertices. A subset D_{rsc} of V is said to be fuzzy right semi connected dominating set if the induced fuzzy subgraph $\langle D_{rsc}\rangle$ is disconnected and induced fuzzy subgraph $\langle V-D_{rsc}\rangle$ is connected. The fuzzy right semi connected domination number $\gamma_{rsc}(G)$ is the minimum fuzzy cardinality taken over all right semi connected dominating sets of G.

Example:4.2

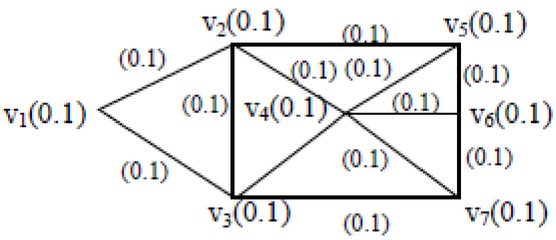

Fig.(12)

$D_{rsc} = \{v_1,v_4\}$, $V-D_{rsc} = \{v_2,v_3,v_5,v_6,v_7\}$

$\langle D_{rsc}\rangle$ is disconnected and

$\langle V-D_{rsc}\rangle$ is connected

$\gamma_{rsc}(G) = 0.2$

Proposition 5.1

$\gamma_{rsc}(P_n) = \min\left\{\sum_{i=1}^{n-3}\sigma(v_i)+\sigma(v_n),\ \sigma(v_1)+\sum_{i=4}^{n}\sigma(v_i)\right\}$

Proposition : 5.2

$\gamma_{rsc}(C_n)$ does not exist.

Proposition : 5.3

$$\gamma_{rsc}(W_n) = \left\lceil \frac{n}{3} \right\rceil \sigma(v_i)$$

for all $\sigma(v_i)$ is are equal.

Theorem 5.1

For any fuzzy graph $G=(\sigma,\mu)$

$\gamma(G) \leq \gamma_{rsc}(G)$

Theorem 5.2 :

Let $G=(\sigma,\mu)$ be a connected fuzzy graph and $H(\sigma',\mu')$ be a spanning fuzzy subgraph of G, If H has a left semi connected dominating set then

$\gamma_{rsc}(G) \leq \gamma_{rsc}(H)$

Theorem 5.3

For any fuzzy graph $G=(\sigma,\mu)$, $\gamma_{rsc}(G) + \gamma_{rsc}(\overline{G}) \leq 2p$ where $\gamma_{rsc}(\overline{G})$ is the right semi connected domination number of $\overline{G}$ and equality holds iff $0 \leq \mu(x,y) < \sigma(x) \wedge \sigma(y)$ for all $x,y \in V$.

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