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# On Supra Regular Generalized Star b-Closed Sets

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***Abstract:*** *In this paper, the recent developments of topology contributed by various authors are mentioned and definitions cited by them are also presented and $rg^*b$-closed sets, and $rg^*b$-continuous functions are studied in Supra Topological Spaces.*
***Keywords:*** *$rg^*b^\mu$ –closed set, $rg^*b^\mu$ –open set*

## I. Introduction

In 1970, Levine [3], introduced the concept of generalized closed set and discussed the properties of sets, closed and open maps, compactness, normal and separation axioms. In 1983, Mashhour et al [4] introduced supra topological spaces and studied S-continuous maps and $S^*$-continuous maps. In 2008, Devi et al [1] introduced and studied a class of sets called supra α-open and a class of maps called sα-continuous between topological spaces, respectively. In 2010, Sayed and Noiri [7] introduced and studied a class of sets called supra b-open and a class of maps called supra β-open and a class of maps called supra β-continuous, respectively. Ravi et al [6] introduced and studied a class of sets called supra g-closed and a class of maps called supra g-continuous respectively.In this paper, we introduce the concepts of $rg^*b$-closed sets and $rg^*b$-continuous functions in Supra Topological Spaces and studied their some properties.

## II. Preliminaries

**Definition 2.1:** Let (X, μ) be a supra topological space. A subset A of X is called
1) $g^\mu$-closed set [6] if $cl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in X.
2) $sg^\mu$-closed set [2] if $scl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra semi open in X.
3) $gs^\mu$-closed set [5] if $scl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in X.
4) $g\alpha^\mu$-closed set if $\alpha cl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra α-open in X.
5) $\alpha g^\mu$-closed set if $\alpha cl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in X.
6) $gp^\mu$-closed set if $pcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in X.
7) $gpr^\mu$-closed set if $pcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra r-open in X.
8) $gsp^\mu$-closed set if $spcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in X.
9) $rg^\mu$-closed set if $cl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra regular open in X.
10) $gr^\mu$-closed set if $rcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in X.
11) $g^{*\mu}$-closed set if $cl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is $g^\mu$-open in X.
12) $g^*s^\mu$-closed set if $scl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is $g^\mu$-open in X.
13) $g^{\#\mu}$-closed set if $cl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is $\alpha g^\mu$-open in X.
14) $g^\#s$-closed set if $scl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is $\alpha g^\mu$-open in X.
15) $gb^\mu$-closed set if $bcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in X.
16) $g^*b^\mu$-closed set if $bcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in X.
17) $rgb^\mu$-closed set if $bcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra regular open in X.

The complements of the above mentioned closed sets are called their respective open sets.

## III. Supra Regular Generalized Star B-Closed Sets

In this section we introduce supra regular generalized star b-closed set and investigate some of their properties.
**Definition 3.1:** A subset A of a supra topological space (X, μ) is called supra regular generalized star b-closed set (briefly $rg^*b^\mu$-closed set) if $bcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is $rg^\mu$-open in X.

**Theorem 3.2:1)** Every supra closed set is $rg^*b^\mu$-closed.
**2)** Every supra α-closed set is a $rg^*b^\mu$-closed set.
**3)** Every supra semi-closed set is a $rg^*b^\mu$-closed set.

**4)** Every supra $g^*$-closed set is a $rg^*b^\mu$-closed set.
**5)** Every supra gα-closed set is a $rg^*b^\mu$-closed set.
**6)** Every supra regular-closed set is a $rg^*b^\mu$-closed set.
**7)** Every supra g-closed set is $rg^*b^\mu$-closed.
**8)** Every supra gs-closed set is $rg^*b^\mu$-closed.
**9)** Every supra sg-closed set is $rg^*b^\mu$-closed.
**10)** Every supra αg-closed set is $rg^*b^\mu$-closed.
**11)** Every supra gr-closed set is $rg^*b^\mu$-closed.
**12)** Every supra $gr^*$-closed set is $rg^*b^\mu$-closed.
**13)** Every supra $g^*s$-closed set is $rg^*b^\mu$-closed.
**14)** Every supra $g^\#$-closed set is $rg^*b^\mu$-closed.
**15)** Every supra $g^\#s$-closed set is $rg^*b^\mu$-closed.
**16)** Every $rg^*b^\mu$-closed set is a $rgb^\mu$-closed set.
**17)** Every $rg^*b^\mu$-closed set is a $rg^\mu$-closed set.
**18)** Every $rg^*b^\mu$-closed set is a $\alpha gr^\mu$-closed set

The converses of the above need not be true as seen from the following examples.

**Example 3.3: 1)** Let X = {a, b, c} with μ = {ϕ, {a, b}, {a, c}, X}. Then the subset {a} is $rg^*b^\mu$-closed set but not a supra closed set.
**2)** Let X = {a, b, c, d} with μ = {ϕ, {a, c, d}, {b, c, d}, X}. Then the subset {b, c} is a $rg^*b^\mu$-closed set but not a supra α-closed set.
**3)** Let X = {a, b, c, d} with μ = {ϕ, {b, c, d}, {a, b, d}, X}. Then the subset {c, d} is a $rg^*b^\mu$-closed set but not a supra semi-closed set.
**4)** Let X = {a, b, c} with μ = {ϕ, {a, c}, {b, c}, X}. Then the subset {c} is $rg^*b^\mu$-closed set but not a supra $g^*$-closed set.
**5)** Let X = {a, b, c, d} with μ = {ϕ, {b, c, d}, {a, b, d}, X}. Then the subset {b, d} is a $rg^*b^\mu$-closed set but not a supra gα-closed set.
**6)** Let X = {a, b, c} with μ = {ϕ, {b, c}, X}. Then the subset {a, b} is $rg^*b^\mu$-closed set but not a supra regular-closed set.
**7)** Let X = {a, b, c} with μ = {ϕ, {a, b}, X}. Then the subset {a} is a $rg^*b^\mu$-closed set but not a supra g-closed set.
**8)** Let X = {a, b, c} with μ = {ϕ, {b, c}, X}. Then the subset {b} is a $rg^*b^\mu$-closed set but not a supra gs-closed set.
**9)** Let X = {a, b, c} with μ = {ϕ, {a, b, c}, {b, c, d}, X}. Then the subset {b, c} is $rg^*b^\mu$-closed set but not a supra sg-closed set.
**10)** Let X = {a, b, c} with μ = {ϕ, {a, b}, {b, c}, X}. Then the subset {b} a is $rg^*b^\mu$-closed set but not a supra αg-closed set.
**11)** Let X = {a, b, c} with μ = {ϕ, {a, b}, X}. Then the subset {a} is a $rg^*b^\mu$-closed set but not a supra gr-closed set.
**12)** Let X = {a, b, c, d} with μ = {ϕ, {b, c, d}, {a, b, d} X}. Then the subset {b, d} is a $rg^*b^\mu$-closed set but not a supra $gr^*$-closed set.
**13)** Let X = {a, b, c, d} with μ = {ϕ, {b, c, d}, {a, c, d} X}. Then the subset {d} is a $rg^*b^\mu$-closed set but not a supra $g^*s$-closed set.
**14)** Let X = {a, b, c} with μ = {ϕ, {b, c}, X}. Then the subset {c} is a $rg^*b^\mu$-closed set but not a supra $g^\#$-closed set.
**15)** Let X = {a, b, c} with μ = {ϕ, {a, c}, {b, c}, X}. Then the subset {c} is a $rg^*b^\mu$-closed set but not a supra $g^\#s$-closed set.
**16)** Let X = {a, b, c, d} with μ = {ϕ, {a, b, c}, {a, c, d}, X}. Then the subset {a, c, d} is a $rgb^\mu$ -closed set but not a $rg^*b^\mu$-closed set.
**17)** Let X = {a, b, c} with μ = {ϕ, {a, c}, {b, c}, X}. Then the subset {b, c,} is a $rg^\mu$-closed set but not a $rg^*b^\mu$-closed set.
**18)** Let X = {a, b, c, d} with μ = {ϕ, {a, b, c}, {a, b, d}, X}. Then the subset {a, b, c,} is a $\alpha gr^\mu$-closed set but not a $rg^*b^\mu$-closed set.

**Theorem 3.4:** A set A is $rg^*b^\mu$-closed set iff $bcl^\mu(A)$-A contains no non empty $rg^\mu$-closed set.
**Proof: Necessity:** Let A be an $rg^*b^\mu$-closed set in (X, μ). Let F be a $rg^\mu$-closed set in X such that $F \subseteq bcl^\mu(A)$-A and X – F is $rg^\mu$-open, Since A is and $bcl^\mu(A) \subseteq X – F$. (i.e) $F \subseteq (X - bcl^\mu(A)) \cap (bcl^\mu(A)\text{-}A)$. Therefore F = ϕ.

**Sufficiency:** Let us assume that $bcl^{\mu}(A)$-A contains no non empty $rg^{\mu}$-closed set. Let A⊆U, U is $rg^{\mu}$-open. Suppose that $bcl^{\mu}(A)$ is not contained in U, $bcl^{\mu}(A) \cap U^{C}$ is non-empty $rg^{\mu}$-closed set of $bcl^{\mu}(A)$-A which is a contradiction. Therefore $bcl^{\mu}(A)$⊆U. Hence A is $rg^{*}b^{\mu}$-closed.

**Remark 3.5:** The intersection of any two subsets of $rg^{*}b^{\mu}$-closed sets in X is $rg^{*}b^{\mu}$-closed in X.

**Remark 3.6:** The union of any two subsets of $rg^{*}b^{\mu}$-closed sets in X need not to be $rg^{*}b^{\mu}$-closed in X.

**Example 3.7:** Let X = {a, b, c, d} with μ={ ϕ, {b, c, d}, {a, b ,d} }. The sets {ϕ, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, c, d}} are $rg^{*}b^{\mu}$-closed. Then the union of the sets {b, c} and {b, d} is {b, c, d}, which is not $rg^{*}b^{\mu}$-closed and the intersection of the sets {a, b} and {a, c} is {a}, which is $rg^{*}b^{\mu}$-closed.

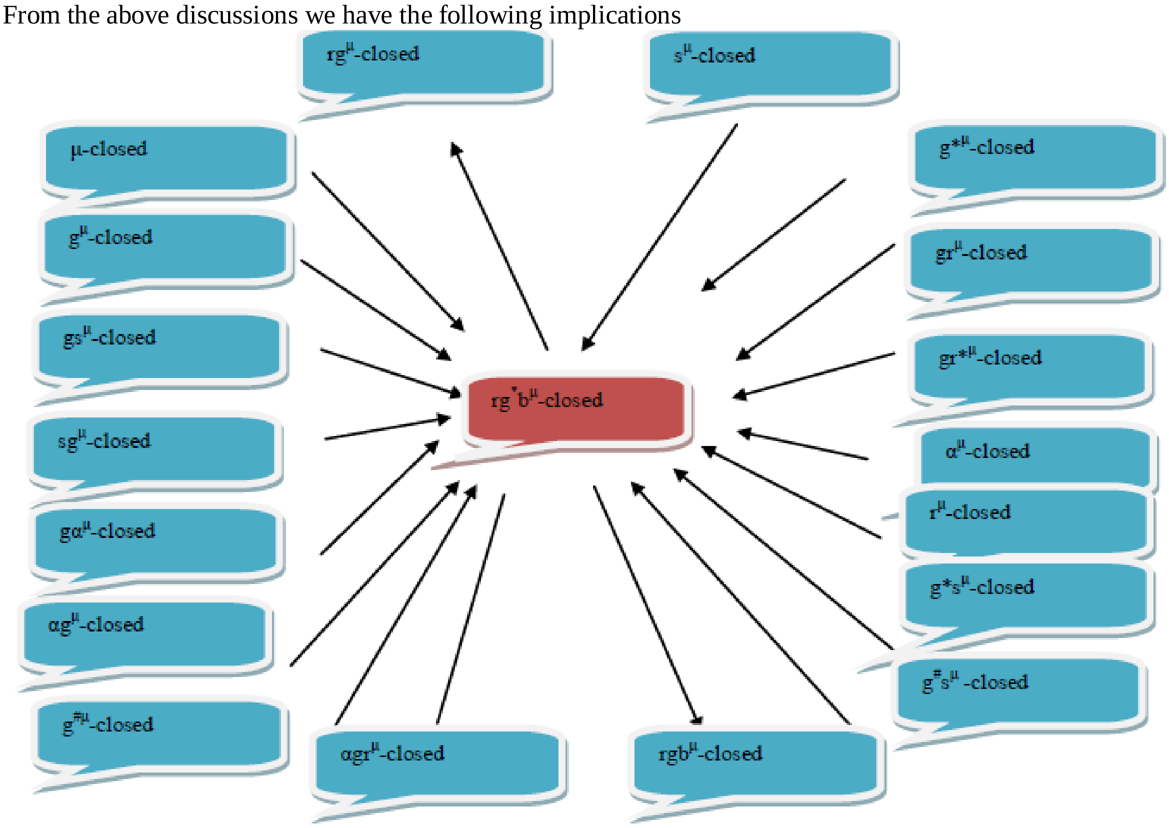
**Theorem 3.8:** If A is $rg^{*}b^{\mu}$-closed set in X and A⊆B⊆$bcl^{\mu}(A)$, then B is a $rg^{*}b^{\mu}$-closed set in X.
**Proof:** Since B⊆$bcl^{\mu}(A)$, we have $bcl^{\mu}(B)$⊆$bcl^{\mu}(A)$ then $bcl^{\mu}(B)$-B ⊆ $bcl^{\mu}(A)$-A. By Theorem 3.38, $bcl^{\mu}(A)$-A contains no non empty supra rg-closed set. Hence $bcl^{\mu}(B)$-B contains no empty supra rg-closed set. Therefore B is a $rg^{*}b^{\mu}$-closed set in X.

**Theorem 3.9:** If A⊆Y⊆X and suppose that A is $rg^{*}b^{\mu}$-closed in X, then A is $rg^{*}b^{\mu}$-closed relative to Y.
**Proof:** Given that A⊆Y⊆X and A is a $rg^{*}b^{\mu}$-closed set in X. To prove that A is a $rg^{*}b^{\mu}$-closed set relative to Y. Let us assume that A⊆Y∩U, where U is supra rg-open in X. Since A is a $rg^{*}b^{\mu}$-closed set, A⊆U implies $bcl^{\mu}(A)$⊆U. It follows that Y∩ $bcl^{\mu}(A)$⊆Y∩U. That is A is a $rg^{*}b^{\mu}$-closed set relative to Y.

**Definition 3.10:** A subset A of a supra topological space (X, μ) is called supra regular generalized star b-open set (briefly $rg^{*}b^{\mu}$-open set) if $A^{C}$ is $rg^{*}b^{\mu}$-closed in X. The family of all $rg^{*}b^{\mu}$-open sets in X is denoted by $RG^{*}B^{\mu}$-O(X).

**Theorem 3.11:** If $int^{\mu}(A)$⊆B⊆A and if A is $rg^{*}b^{\mu}$-open in X, then B is $rg^{*}b^{\mu}$-open in X.
**Proof:** Suppose that $int^{\mu}(A)$⊆B⊆A and A is $rg^{*}b^{\mu}$-open in X, then $A^{c}$⊆$B^{c}$⊆$bcl^{\mu}(A^{c})$. Since $A^{c}$ is $rg^{*}b^{\mu}$-closed in X, by Theorem 3.8 is $rg^{*}b^{\mu}$-open in X.

From the above discussions we have the following implications



## IV. $rg^*b^\mu$-Continuous Functions

This chapter is devoted to introduce and study the concepts of $rg^*b$-continuous functions in supra topological spaces.

**Definition 4.1:** A function $f : X \to Y$ is called $rg^*b^\mu$-continuous if $f^{-1}(V)$ is $rg^*b^\mu$-closed in X for every supra closed set V in Y.

**Remark 4.2:** Since every supra closed set is $rg^*b^\mu$-closed, every supra continuous function is $rg^*b^\mu$-continuous. But the converse need not true, which is verified by the following example.

**Example 4.3:** Let X = Y = {a, b, c}, $\mu_1 = \{\phi, \{a\}, X\}$ and $\mu_2 = \{\phi, \{a, b\}, Y\}$. Let $f : (X, \mu_1) \to (Y, \mu_2)$ be an identity map. Then $rg^*b^\mu C(X, \mu_1) = \{\phi, \{b\}, \{c\}, \{b, c\}, X\}$. Hence f is $rg^*b^\mu$-continuous. But f is not supra continuous, since for the supra closed set {c} in Y, $f^{-1}(\{c\}) = \{c\}$ is not supra closed in X.

**Theorem 4.4:** Let $f : X \to Y$ be a function where X and Y are supra topological spaces. Then the following are equivalent:
1) f is $rg^*b^\mu$-continuous.
2) For each point $x \in X$ and each supra open set V in Y with $f(x) \in V$, there is a $rg^*b^\mu$-open set U in X such that $x \in U$ and $f(U) \subseteq V$.

**Proof: (1) → (2):** Let V be a supra open set in Y and let $f(x) \in V$, where $x \in X$. Since f is $rg^*b^\mu$-continuous, $f^{-1}(V)$ is a $rg^*b^\mu$-open set in X. Also $x \in f^{-1}(V)$. Take $U = f^{-1}(V)$. Then $x \in U$ and $f(U) \subseteq V$.
**(2) → (1):** Let V be a supra open set in Y and let $x \in f^{-1}(V)$. Then $f(x) \in V$ and there exists a $rg^*b^\mu$-open set U in X such that $x \in U$ and $f(U) \subseteq V$. Then $x \in U \subseteq f^{-1}(V)$. Hence $f^{-1}(V)$ is a $rg^*b^\mu$-nbhd of x and it is $rg^*b^\mu$-open. Then $f^{-1}(V) = U$. Hence f is $rg^*b^\mu$-continuous.

**Theorem 4.5:** Let $f : X \to Y$ be a function where X and Y are supra topological spaces. Then the following are equivalent:
1) f is $rg^*b^\mu$-continuous.
2) The inverse of each supra open set in Y is $rg^*b^\mu$-open in X.
3) For each supra subset A of X, $f(rg^*b^\mu\text{-}cl(A)) \subseteq cl^\mu(f(A))$.

**Proof: (1) → (2):** Let B be a supra open subset of Y. Then Y-B is supra closed in Y. Since f is $rg^*b^\mu$-continuous, $f^{-1}(Y\text{-}B)$ is $rg^*b^\mu$-closed in X. That is, $X\text{-}f^{-1}(B)$ is $rg^*b^\mu$-closed in X. Hence $f^{-1}(B)$ is $rg^*b^\mu$-open in X.
**(2) → (1):** Let G be a supra closed subset of Y. Then Y-G is supra open in Y. Then $f^{-1}(Y\text{-}B)$ is $rg^*b^\mu$-open in X. That is, $X\text{-}f^{-1}(G)$ is $rg^*b^\mu$-open in X. Hence $f^{-1}(G)$ is $rg^*b^\mu$-closed in X, which implies that f is $rg^*b^\mu$-continuous.
**(1) → (3):** Let A be a supra subset of X. Since $A \subset f^{-1}(f(A))$, $A \subset f^{-1}(cl^\mu(f(A)))$. Now $cl^\mu(f(A))$ is a supra closed in Y. Then by (1), $f^{-1}(cl^\mu(f(A)))$ is $rg^*b^\mu$-closed in X containing A. But $rg^*b^\mu\text{-}cl(A)$ is the smallest $rg^*b^\mu$-closed in X containing A. Therefore $rg^*b^\mu\text{-}cl(A) \subseteq f^{-1}(cl^\mu(f(A)))$. Hence $f(rg^*b^\mu\text{-}cl(A)) \subseteq cl^\mu(f(A))$.
**(3) → (1):** Let B be a closed subset of Y. Then $f^{-1}(B)$ is a subset of X. By (3) $f(rg^*b^\mu\text{-}cl(f^{-1}(B))) \subseteq cl^\mu(f(f^{-1}(B)) \subset cl^\mu(B) = B$. This implies, $rg^*b^\mu\text{-}cl(f^{-1}(B)) \subseteq f^{-1}(B)$. But $f^{-1}(B) \subseteq rg^*b^\mu\text{-}cl(f^{-1}(B))$. Hence $f^{-1}(B) = rg^*b^\mu\text{-}cl(f^{-1}(B))$ and $f^{-1}(B)$ is $rg^*b^\mu$-closed in X. This implies that f is $rg^*b^\mu$-continuous.

**Corollary 4.6:** Let $f : X \to Y$ be a function where X and Y are supra topological spaces. Then the following are equivalent:
1) f is $rg^*b^\mu$-continuous.
2) For each subset B of Y, $rg^*b^\mu\text{-}cl(f^{-1}(B)) \subseteq f^{-1}(cl^\mu(B))$.

**Proof: (1) → (2):** Let B be a supra subset of Y. Then $f^{-1}(B)$ is a subset of X. Since f is $rg^*b^\mu$-continuous, $f(rg^*b^\mu\text{-}cl(f^{-1}(B))) \subseteq cl^\mu(f(B))$, for each subset A of X, Therefore $f(rg^*b^\mu\text{-}cl(f^{-1}(B))) \subseteq cl^\mu(f(f^{-1}(B)) \subset cl^\mu(B)$. Hence $rg^*b^\mu\text{-}cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$.
**(2) → (1):** Let B be a closed subset of Y. Then by (2), $rg^*b^\mu\text{-}cl(f^{-1}(B)) \subseteq f^{-1}(cl^\mu(B))$. This implies, $f(rg^*b^\mu\text{-}cl(f^{-1}(B))) \subseteq cl^\mu(f(f^{-1}(B)) \subseteq cl^\mu(B)$. Take B = f(A), where A is subset of X. Then $f(rg^*b^{\mu}\text{-}cl(f^{-1}(B))) \subseteq cl^\mu(f(A))$. Hence f is $rg^*b^\mu$-continuous.

**Remark 4.7:** The composition of two $rg^*b^\mu$-continuous functions need not to be a $rg^*b^\mu$-continuous function in general as seen from the following example:

**Example 4.8:** Let X = Y = Z = {a, b, c}, $\mu_1 = \{\phi, \{a, c\}, \{b, c\}, X\}$, $\mu_2 = \{\phi, \{a, b\}, Y\}$, and $\mu_3 = \{\phi, \{a\}, Z\}$. Let $f : (X, \mu_1) \to (Y, \mu_2)$ and $g : (Y, \mu_2) \to (Z, \mu_3)$ be identity maps. Then $rg^*b^\mu C(X, \mu_1) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, X\}$, $rg^*b^\mu C(Y, \mu_2) = \{\phi, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}, Y\}$ and $rg^*b^\mu C(Z, \mu_3) = \{\phi, \{b\}, \{c\}, \{b, c\}, Z\}$. Then f and g are $rg^*b^\mu$-continuous but $g \circ f: (X, \mu_1) \to (Z, \mu_3)$ is not $rg^*b^\mu$-continuous, since the subset {b, c} is supra closed in $(Z, \mu_3)$ but $(g \circ f)^{-1}(\{b, c\}) = \{b, c\}$ is not $rg^*b^\mu$-closed in $(X, \mu_1)$.

**Definition 4.9:** A function $f : X \to Y$, where X and Y are supra topological spaces, is called $rg^*b^\mu$-irresolute if the inverse image of each $rg^*b^\mu$-closed set in Y is a $rg^*b^\mu$-closed set in X.

**Theorem 4.10:** A function $f : X \to Y$ is $rg^*b^\mu$-irresolute if and only if $f^{-1}(V)$ is $rg^*b^\mu$-open in X for every $rg^*b^\mu$-open set V in Y.
**Proof:** Necessity: Let V be a $rg^*b^\mu$-open set in Y. Then $V^C$ is $rg^*b^\mu$-closed in Y. Since f is $rg^*b^\mu$-irresolute, $f^{-1}(V^C)$ is $rg^*b^\mu$-closed in X. But $f^{-1}(V^C) = (f^{-1}(V))^C$. Hence $(f^{-1}(V))^C$ is $rg^*b^\mu$-closed in X and hence $f^{-1}(V)$ $rg^*b^\mu$-open in X.
Sufficiency: Let V be a $rg^*b^\mu$-closed in Y. Then $V^C$ is is $rg^*b^\mu$-open in Y. Since the inverse image of each $rg^*b^\mu$-open set in Y is a $rg^*b^\mu$-open set in X, $f^{-1}(V^C)$ is $rg^*b^\mu$-open in X. Also $f^{-1}(V^C) = (f^{-1}(V))^C$. Hence $(f^{-1}(V))^C$ is $rg^*b^\mu$-open in X and hence $f^{-1}(V)$ is $rg^*b^\mu$-closed in X. Hence f is $rg^*b^\mu$-irresolute.

**Remark 4.11:** Every $rg^*b^\mu$-irresolute function is $rg^*b^\mu$-continuous but not the converse, which is shown by the following example.

**Example 4.12:** X = {a, b, c, d} and Y = {a, b, c, d}, $\mu_1$ = {ϕ, {a, b, c}, {a, c, d}, X} and $\mu_2$ = {ϕ, {a, b, c}, {b, c, d}, Y}. Then $rg^*b^\mu C(X, \mu_1)$ = {ϕ, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d},{a, b, d}, {b, c, d}, X} and $rg^*b^\mu C(Y, \mu_2)$ = {ϕ, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d},{a, b, d}, {a, c, d}, Y}. Define $f : (X, \mu_1) \to (Y, \mu_2)$ by f({a}) = {a}, f({b}) = {b}, f({c}) = {c} and so on. Then f is $rg^*b^\mu$-continuous. However {a, c, d}, which is $rg^*b^\mu$-closed in $(Y, \mu_2)$ and $f^{-1}(\{a, c, d\}) = \{a, c, d\}$, is not $rg^*b^\mu$-closed in $(X, \mu_1)$. Therefore f is not $rg^*b^\mu$-irresolute.

**Theorem 4.13:** If $f : X \to Y$ and $g : Y \to Z$ are both $rg^*b^\mu$-irresolute, then $g\circ f : X \to Z$ is also $rg^*b^\mu$-irresolute.
**Proof:** Let A be a $rg^*b^\mu$-closed set in Z. Then $g^{-1}(A)$ is $rg^*b^\mu$-closed set in Y and $f^{-1}(g^{-1}(A))$ is also $rg^*b^\mu$-closed in X, since f and g are $rg^*b^\mu$-irresolute. Thus $(g\circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is $rg^*b^\mu$-closed in X and hence $g \circ f$ is also $rg^*b^\mu$-irresolute.

**Theorem 4.14:** Let X, Y and Z be any supra topological spaces. For any $rg^*b^\mu$-irresolute function $f : X \to Y$ and for any $rg^*b^\mu$-continuous function $g : Y \to Z$, the composition $g\circ f : X \to Z$ is is $rg^*b^\mu$-continuous.
**Proof:** Let A be a supra closed set in Z. Then $g^{-1}(A)$ is $rg^*b^\mu$-closed set in Y, since g is $rg^*b^\mu$-continuous and $f^{-1}(g^{-1}(A))$ is also $rg^*b^\mu$-closed in X, since f is $rg^*b^\mu$-irresolute.But $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$, so that $(g \circ f)^{-1}(A)$ is $rg^*b^\mu$-closed set in X. Hence $g \circ f$ is $rg^*b^\mu$-continuous.

**Theorem 4.15:** Let $f : X \to Y$ be a function, where X and Y are supra topological spaces. Then the following are equivalent:
1) f is $rg^*b^\mu$-irresolute.
2) For each point $x\in X$ and each $rg^*b^\mu$-open set V in Y with $f(x)\in V$, there is a $rg^*b^\mu$-open set U in X such that $x \in U$ and $f(U)\subseteq V$.
**Proof: (1) → (2):** Let V be a supra open set in Y and let $f(x)\in V$, where $x \in X$.. Since f is $rg^*b^\mu$-irresolute, $f^{-1}(V)$ is a $rg^*b^\mu$-open set in X. Also $x \in f^{-1}(V)$. Take $U = f^{-1}(V)$. Then $x\in U$ and $f(U)\subseteq V$.
**(2) → (1):** Let V be an open set in Y and let $x\in f^{-1}(V)$. Then $f(x)\in V$ and there exists a $rg^*b^\mu$-open set U in X such that $x\in U$ and $f(U)\subseteq V$. Then $x\in U\subseteq f^{-1}(V)$. Hence $f^{-1}(V)$ is a $rg^*b^\mu$-nbhd of x and let it be $rg^*b^\mu$-open. Then $f^{-1}(V) = U$. Hence f is $rg^*b^\mu$-irresolute.

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