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**Gα-continuous and **Gα-irresolute maps in Topological Spaces

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Abstract : *In this paper we introduce a new type of function called the **gα-continuous maps and **gα-irresolute maps and discuss their properties.*
Key words: ***gα-continuous maps, **gα-irresolute maps.*

I. Introduction

Levine [15] introduced g-closed sets and studied their most fundamental properties. P.Bhattacharya and B.K.Lahiri [6], S.P.Arya and T.Nour [4], H.Maki et al [17,18] introduced semi generalized-closed sets, generalized semi-closed, α-generalized closed sets and generalized α-closed sets respectively. R.Devi, et al [10] introduced semi generalized-homeomorphism and generalized semi-homeomorphism in topological spaces. R.Devi, et al [9] introduced semi generalized-closed maps and generalized semi-closed maps. M.K.R.S Veera Kumar [23] introduced g^*-closed sets and M.Vigneshwaran, et al [24] introduced *gα-closed sets in topological spaces.

We introduce **gα-continuous maps and **gα-irresolute maps and establish the relationship with the exitssting continuous maps.

II. Preliminaries

Throughout this dissertation (X, τ), (Y, σ) and (Z, η) represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure and the interior of A in X respectively. The power set of X is denoted by P(X).

Let us recall the following definitions, which are useful in the sequel.

Definition 2.1 : A subset A of a topological space (X,τ) is called
(1) a pre-open set [**20**] if A ⊆int(cl(A)) and a pre-closed set if cl(int(A)) ⊆ A.
(2) a semi-open set [**16**] if A ⊆ cl(int(A)) and a semi-closed set if int(cl(A))⊆A.
(3) an α-open set [**22**] if A ⊆int(cl(int(A))) and an α-closed set [**22**] if cl(int(cl(A)))⊆ A.
(4) a semi pre-open set [**2**] (= β-open[**1**]) if A ⊆ cl(int(cl(A))) and a semi pre-closed set [**2**] (= β-closed[**1**]) if int(cl(int(A))) ⊆ A.

The class of all closed (respectively semi pre-closed, α-open) subsets of a space (X,τ) is denoted by C(X,τ) (respectively SPC(X, τ), τ^{α}). The intersection of all semi-closed (respectively pre-closed, semi pre-closed and α-closed) sets containing a subset A of (X, τ) is called the semi-closure (respectively pre-closure, semi pre-closure and α-closure) of A and is denoted by scl(A) (respectively pcl(A), spcl(A) and αcl(A)).

Definition 2.2: A subset A of a topological space (X, τ) is called
(1) a generalized closed set (briefly g-closed) [**15**] if cl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
(2) a semi-generalized closed set (briefly sg-closed) [**6**] if scl(A) ⊆ U whenever A ⊆ U and U is semi-open in (X, τ).
(3) a generalized semi-closed set (briefly gs-closed) [**4**] if scl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
(4) a generalized α-closed set (briefly gα-closed) [**18**] if αcl(A) ⊆ U whenever A ⊆ U and U is α-open in (X, τ).
(5) an α-generalized closed set (briefly αg-closed) [**17**] if αcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
(6) a generalized semi pre-closed set (briefly gsp-closed)[**12**] if spcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
(7) a generalized pre-closed set (briefly gp-closed)[**19**] if pcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
(8) a g^*-closed set [**23**] if cl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
(9) a *gα-closed set [**24**] if cl(A) ⊆ U whenever A ⊆ U and U is gα-open in (X, τ).
(10) a **gα-closed set [**25**] if cl(A) ⊆ U whenever A ⊆ U and U is *gα-open in (X, τ).

The class of all g-closed sets (gsp-closed sets) of a space (X, τ)is denoted by GC(X, τ)(GSPC(X,τ)).

Definition 2.3: A function f: (X,τ) ⟶ (Y, σ) is called
(1) an αg-continuous[**14**] if $f^{-1}(V)$ is an αg-closed set of (X,τ) for every closed set V of (Y, σ).
(2) a gs-continuous [**9**] if $f^{-1}(V)$ is a gs-closed set of (X, τ) for every closed set V of (Y, σ).
(3) a gsp-continuous[**11**] if $f^{-1}(V)$ is a gsp-closed set of (X, τ) for every closed set V of (Y,σ).
(4) a gp-continuous [**24**] if $f^{-1}(V)$ is a gp-closed set of (X, τ) for every closed set V of (Y, σ).
(5) a g^*-continuous[**3**] if $f^{-1}(V)$ is a g^*-closed set of (X, τ) for every closed set V of (Y, σ).
(6) a $^*g\alpha$-continuous [**15**] if $f^{-1}(V)$ is a $^*g\alpha$-closed set of (X, τ) for every closed set V of (Y, σ).
(7) a g^*-irresolute [**5**] if $f^{-1}(V)$ is a g^*-closed set of (X, τ) for every g^*-closed set V of (Y,σ).
(8) a gs-irresolute[**9**] if $f^{-1}(V)$ is a gs-closed set of (X, τ) for every gs-closed set V of (Y, σ).
(9) a $^*g\alpha$-irresolute [**24**] if $f^{-1}(V)$ is a $^*g\alpha$-closed set of (X, τ) for every $^*g\alpha$-closed set V of (Y,σ).

Definition 2.4: A topological space (X, τ) is said to be
(1) ${}_{\alpha}T_{1/2}^{***}$ space if every $^{**}g\alpha$-closed set is closed.
(2) ${}_{\alpha}T_c^{***}$ space if every αg-closed set is $^{**}g\alpha$-closed.
(3) ${}^{***}{}_{\alpha}T_{1/2}$ space if every g-closed set is $^{**}g\alpha$-closed.

III. **Gα-Continuous And **Gα-Irresolute Maps In Topological Spaces

We introduce the following definition

Definition 3.1: A function f: (X, τ) → (Y, σ) is called $^{**}g\alpha$-continuous if $f^{-1}(V)$ is a $^{**}g\alpha$-closed set of (X, τ) for every closed set of V of (Y, σ).

Theorem 3.2: Every continuous map is $^{**}g\alpha$-continuous.

Proof: Let V be a closed set of (Y, σ). Since f is continuous, $f^{-1}(V)$ is closed in (X, τ). But every closed set is $^{**}g\alpha$-closed set. Hence $f^{-1}(V)$ is $^{**}g\alpha$-closed set in (X, τ). Thus f is $^{**}g\alpha$-continuous.

The converse of the above theorem need not be true. It can be seen by the following example

Example 3.3: Let X = {a, b, c} =Y with τ = {X, ϕ, {b, c}} and σ = {Y, ϕ, {a}}
Define f: (X, τ) → (Y, σ) by f(a) = b, f(b) = a, f(c) = c.
$^{**}g\alpha$-closed sets are X, ϕ, {a}, {a, b}, {a, c}
Here f^{-1}({b, c}) = {a, c} is not a closed set in (X, τ). Therefore f is not continuous. However f is $^{**}g\alpha$-continuous.

Theorem 3.4: Every $^*g\alpha$-continuous map is $^{**}g\alpha$-continuous.

Proof: Let V be a closed set of (Y, σ). Since f is $^*g\alpha$-continuous, $f^{-1}(V)$ is $^*g\alpha$-closed in (X, τ). But every $^*g\alpha$-closed set is $^{**}g\alpha$-closed set. Hence $f^{-1}(V)$ is $^{**}g\alpha$-closed set in (X, τ). Thus f is $^{**}g\alpha$-continuous.

The converse of the above theorem need not be true. It can be seen by the following example

Example 3.5: Let X = {a, b, c} =Y with τ = {X, ϕ, {c}} and σ = {Y, ϕ, {b}, {a, b}}
Define f: (X, τ) → (Y, σ) by f(a) = b , f(b) = a, f(c) = c.
$^*g\alpha$-closed sets are X, ϕ, {a, b}
$^{**}g\alpha$-closed sets are X, ϕ, {a}, {b}, {a, b}, {b, c}, {a, c}
Here f^{-1}({a, c}) = {b, c} is not a $^*g\alpha$-closed set in (X, τ)

Therefore f is not $^*g\alpha$-continuous. However f is $^{**}g\alpha$-continuous.

Theorem 3.6: Every g^*-continuous map is $^{**}g\alpha$-continuous.

Proof: Let V be a closed set of (Y, σ). Since f is g^*-continuous, $f^{-1}(V)$ is g^*-closed in (X, τ). But every g^*-closed set is **gα-closed set. Hence $f^{-1}(V)$ is **gα-closed set in (X, τ). Thus f is **gα-continuous.

The converse of the above theorem need not be true. It can be seen by the following example

Example 3.7: Let X = {a, b, c} =Y with τ = {X, ϕ, { c}} and σ = {Y, ϕ, {a}, { b, c}}
Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c.
g^*-closed sets are X, ϕ, {a, b}
**gα-closed sets are X, ϕ, {a}, {b}, {a, b}, {b, c}, {a, c}
Here f^{-1}({b, c}) = {b, c} is not a g^*-closed set in (X, τ).
Therefore f is not g^*-continuous. However f is **gα-continuous.

Theorem 3.8: Every **gα-continuous map is gs-continuous.

Proof: Let V be a closed set of (Y, σ). Since f is **gα-continuous, $f^{-1}(V)$ is **gα-closed in (X, τ). But every **gα-closed set is gs-closed set. Hence $f^{-1}(V)$ is gs-closed set in (X, τ). Thus f is gs-continuous.

The converse of the above theorem need not be true. It can be seen by the following example

Example 3.9: Let X = {a, b, c} =Y with τ = {X, ϕ, {a}, {b, c}} and σ = {Y, ϕ, {a}, {a, c}}
Define f: (X, τ) → (Y, σ) by f(a) = b, f(b) = c, f(c) = a.
gs-closed sets are X, ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}
**gα-closed sets are X, ϕ, {a}, {b, c}
Here f^{-1}({b, c}) = {a, b} is not a **gα-closed set in (X, τ).
Therefore f is not **gα-continuous. However f is gs-continuous.

Theorem 3.10: Every **gα-continuous map is gsp-continuous.

Proof: Let V be a closed set of (Y, σ). Since f is **gα-continuous, $f^{-1}(V)$ is **gα-closed in (X, τ). But every **gα-closed set is gs-closed set. Hence $f^{-1}(V)$ is gsp-closed set in (X, τ). Thus f is gsp-continuous.

The converse of the above theorem need not be true. It can be seen by the following example

Example 3.11: Let X = {a, b, c} =Y with τ = {X, ϕ, {a}, {b, c}} and σ = {Y, ϕ, {a}, {a, c}}
Define f: (X, τ) → (Y, σ) by f(a) = b, f(b) = c, f(c) = a.
gsp-closed sets are X, ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}
**gα-closed sets are X, ϕ, {a}, {b, c}
Here f^{-1}({b, c}) = {a, b} is not a **gα-closed set in (X, τ).
Therefore f is not **gα-continuous. However f is gsp-continuous.

Theorem 3.12: Every **gα-continuous map is gp-continuous.

Proof: Let V be a closed set of (Y, σ). Since f is **gα-continuous, $f^{-1}(V)$ is **gα-closed in (X, τ). But every **gα-closed set is gp-closed set. Hence $f^{-1}(V)$ is gp-closed set in (X, τ). Thus f is gp-continuous.

The converse of the above theorem need not be true. It can be seen by the following example

Example 3.13: Let X = {a, b, c} =Y with τ = {X, ϕ, {a, b}} and σ = {Y, ϕ, {a}, {a, c}}
Define f: (X, τ) → (Y, σ) by f(a) = b, f(b) = a, f(c) = c.
gp-closed sets are X, ϕ, {a}, {b}, {c}, {a, c}, {b, c}

**gα-closed sets are X, ϕ, {c}, {a, c}, {b, c}
Here f^{-1}({b}) = {a} is not a **gα-closed set in (X, τ). Therefore f is not **gα-continuous. However f is gp-continuous.

Theorem 3.14: Every $^{**}g\alpha$-continuous map is an αg-continuous.

Proof: Let V be a closed set of (Y, σ). Since f is $^{**}g\alpha$-continuous, $f^{-1}(V)$ is $^{**}g\alpha$-closed in (X, τ). But every $^{**}g\alpha$-closed set is αg-closed set. Hence $f^{-1}(V)$ is αg-closed set in (X, τ). Thus f is αg-continuous.

The converse of the above theorem need not be true. It can be seen by the following example

Example 3.15: Let X = {a, b, c} =Y with τ = {X, ϕ, {a},{b, c}} and σ = {Y, ϕ, {b}, {b, c}}
Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = c, f(c) = b.
αg-closed sets are X, ϕ, {a}, {b}, {c},{a, b}, {a, c}, {b, c}
$^{**}g\alpha$-closed sets are X, ϕ, {a}, {b, c}
Here f^{-1}({a, c}) = {a, b} is not a $^{**}g\alpha$-closed set in (X, τ).
Therefore f is not $^{**}g\alpha$-continuous. However f is αg-continuous.

Theorem 3.16: Every $^{**}g\alpha$-continuous map is an gpr-continuous.

Proof: Let V be a closed set of (Y, σ). Since f is $^{**}g\alpha$-continuous, $f^{-1}(V)$ is $^{**}g\alpha$-closed in (X, τ). But every $^{**}g\alpha$-closed set is gpr-closed set. Hence f^{-1} (V) is gpr-closed set in (X, τ). Thus f is gpr-continuous.

The converse of the above theorem need not be true. It can be seen by the following example

Example 3.17: Let X = {a, b, c} =Y with τ = {X, ϕ, {a}} and σ = {Y, ϕ, {a}, {a, b}}
Define f: (X, τ) → (Y, σ) by f(a) = c, f(b) = b, f(c) = a.
gpr-closed sets are X, ϕ, {a}, {b}, {c},{a, b}, {a, c}, {b, c}
$^{**}g\alpha$-closed sets are X, ϕ,{b}, {c},{a, b}, {a, c} , {b, c}
Here f^{-1}({c}) = {a} is not a $^{**}g\alpha$-closed set in (X, τ). Therefore f is not $^{**}g\alpha$-continuous. However f is gpr-continuous.

Theorem 3.18: Every $^{**}g\alpha$-continuous map is an g-continuous.

Proof: Let V be a closed set of (Y, σ). Since f is $^{**}g\alpha$-continuous, $f^{-1}(V)$ is $^{**}g\alpha$-closed in (X, τ). But every $^{**}g\alpha$-closed set is g-closed set. Hence $f^{-1}(V)$ is g-closed set in (X, τ). Thus f is g-continuous.
The converse of the above theorem need not be true. It can be seen by the following example

Example 3.19: Let X = {a, b, c} =Y with τ = {X, ϕ, {a},{b, c}} and σ = {Y, ϕ, {b}, {a, b}}
Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = c, f(c) = b.
g-closed sets are X, ϕ, {a}, {b}, {c},{a, b}, {a, c}, {b, c}
$^{**}g\alpha$-closed sets are X, ϕ,{a}, {b, c}
Here f^{-1}({a, c}) = {a, b} is not a $^{**}g\alpha$-closed set in (X, τ).
Therefore f is not $^{**}g\alpha$-continuous. However f is g-continuous.

Remark 3.20: $^{**}g\alpha$-continuity is independent of semi-continuity and α-continuity.
It can be seen by the following example.

Example 3.21: Let X = {a, b, c} =Y with τ = {X, ϕ, {b},{a, b}} and σ = {Y, ϕ, {a}, {a, c}}
Define f: (X, τ) → (Y, σ) by f(a) = b, f(b) = c, f(c) = a.
$^{**}g\alpha$-closed sets are X, ϕ, {c}, {a, c}, {b, c}
semi-closed sets are X, ϕ,{a},{c}, {a, c}
α-closed sets are X, ϕ,{a},{c}, {a, c}
Here f^{-1}({b}) = {a} is not a $^{**}g\alpha$-closed set in (X, τ). Therefore f is not $^{**}g\alpha$-continuous. However f is semi-continuous and α-continuous .

Example 3.22: Let X = {a, b, c} =Y with τ = {X, ϕ, {b}} and σ = {Y, ϕ, {a}, {b, c}}
Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c.
$^{**}g\alpha$-closed sets are X, ϕ, {c}, {a, c}, {b, c}

semi-closed sets are X, ϕ,{a},{c}, {a, c}
α-closed sets are X, ϕ,{a},{c}, {a, c}
Here f^{-1}({b, c}) = {b, c} is not a **gα-closed set in (X, τ). Therefore f is not semi-continuous and α-continuous. However f is **gα-continuous.

Remark 3.23: **gα-continuity is independent of pre-continuity and semi- pre-continuity.
It can be seen by the following example.

Example 3.24: Let X = {a, b, c} =Y with τ = {X, ϕ, {b},{a, b}} and σ = {Y, ϕ, {a}, {a, b}}
Define f: (X, τ) → (Y, σ) by f(a) = b, f(b) = c, f(c) = a.
**gα-closed sets are X, ϕ, {c}, {a, c}, {b, c}
semi-pre-closed sets are X, ϕ,{a},{c}, {a, c}
pre-closed sets are X, ϕ,{a},{c}, {a, c}
Here f^{-1}({b}) = {a} is not a **gα-closed set in (X, τ). Therefore f is not **gα-continuous. However f is semi-pre-continuous and pre-continuous.

Example 3.25: Let X = {a, b, c} =Y with τ = {X, ϕ, {b}} and σ = {Y, ϕ, {a}, {b, c}}
Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c.
**gα-closed sets are X, ϕ, {c}, {a, c}, {b, c}
semi-pre-closed sets are X, ϕ,{a},{c}, {a, c}
pre-closed sets are X, ϕ,{a},{c}, {a, c}
Here f^{-1}({b, c}) = {b, c} is not a **gα-closed set in (X, τ). Therefore f is not semi-pre-continuous and pre-continuous. However f is **gα-continuous .

Remark 3.26: The composition of two **gα-continuous map need not be a **gα-continuous.
It can be seen by the following example.

Example 3.27: Let X = {a, b, c} =Y=Z with τ = {X, ϕ, {b}, {a, b}}, σ = {Y, ϕ, {a}, {c}, {a, c}} and η = {Z, ϕ, {c}, {a, b}}
Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c.
Define g: (Y, σ) → (Z, η) by f(a) = b, f(b) = a, f(c) = c
**GαC(X, τ) = { X, ϕ, {c}, {a, c}, {b, c}}
**GαC(Y, σ) = { Y, ϕ, {b}, {a, b}, {b, c}}
Hear {a, b} is a closed set in (Z, η).But $(gof)^{-1}$({a, b}) = {a, b} is not a **gα-closed set in (X, τ).Therefore gof is not **gα-continuous.

We introduce the following definition

Definition 3.28: A function f : (X, τ) → (Y, σ) is called **gα-irresolute if f^{-1}(V) is a **gα-closed set of (X, τ) for every **gα-closed set of V of (Y, σ).

Theorem 3.29: Every **gα-irresolute map is **gα-continuous.

Proof: Let V be a closed set of (Y, σ). Since every closed set is **gα-closed set, V is **gα-closed set of (Y, σ). Since f is **gα-irresolute, Hence f^{-1}(V) is **gα-closed set in (X, τ). Thus f is **gα-continuous.

The converse of the above theorem need not be true. It can be seen by the following example

Example 3.30: Let X = {a, b, c} =Y with τ = {X, ϕ, {a, b}} and σ = {Y, ϕ, {a}}.
Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c.
Define g: (Y, σ) → (Z, η) by f(a) = b, f(b) = a, f(c) = c
**gαC(X, τ) = { X, ϕ, {c}, {a, c}, {b, c}}
**gαC(Y, σ) = { Y, ϕ, {b}, {a, b}, {b, c}}

Hear f is **gα-continuous but f is not **gα-irresolute. Since {a, b} is a **gα-closed set in (Y, σ) but f^{-1}({a, b}) = {a, b} is not a **gα-closed set in (X, τ).

Theorem 3.31: Let f: $(X, \tau) \to (Y, \sigma)$ and g: $(Y, \sigma) \to (Z, \eta)$ be any two functions. Then gof: $(Y, \sigma) \to (Z, \eta)$ is $^{**}g\alpha$-continuous if g is continuous and f is $^{**}g\alpha$-continuous.

Proof: Let V be a closed set in (Z, η). Since g is continuous, $g^{-1}(V)$ is closed in (Y, σ). Since f is $^{**}g\alpha$-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is $^{**}g\alpha$-closed in (X, τ). Therefore g o f is $^{**}g\alpha$-continuous.

Theorem 3.32:Let f: $(X, \tau) \to (Y, \sigma)$ and g: $(Y, \sigma) \to (Z, \eta)$ be any two functions. Then gof: $(Y, \sigma) \to (Z, \eta)$ is $^{**}g\alpha$-irresolute if both g and f are $^{**}g\alpha$-irresolute.

Proof: Let V be a closed set in (Z, η). Since g is $^{**}g\alpha$-irresolute, $g^{-1}(V)$ is $^{**}g\alpha$-closed in (Y, σ). Since f is $^{**}g\alpha$-irresolute, $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is $^{**}g\alpha$-closed in (X, τ). Therefore gof is $^{**}g\alpha$-irresolute.

Theorem 3.33: Let f: $(X, \tau) \to (Y, \sigma)$ and g: $(Y, \sigma) \to (Z, \eta)$ be any two functions. Then gof: $(Y, \sigma) \to (Z, \eta)$ is $^{**}g\alpha$-continuous g is $^{**}g\alpha$-continuous and f is $^{**}g\alpha$-irresolute.

Proof: Let V be a closed set in (Z, η). Since g is $*^{*}g\alpha$-continuous, $g^{-1}(V)$ is $^{**}g\alpha$-closed in (Y,σ). Since f is $^{**}g\alpha$-irresolute, $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is $^{**}g\alpha$-closed in (X, τ). Therefore gof is $^{**}g\alpha$-continuous.

Theorem 3.34: Let f: $(X,\tau) \to (Y,\sigma)$ be a $^{**}g\alpha$-continuous map. If (X,τ) is an ${}_{\alpha}T_{1/2}^{***}$ space, then f is continuous.

Proof: Let V be a closed set in (Y, σ). Since f is $*^{*}g\alpha$-continuous, $f^{-1}(V)$ is $^{**}g\alpha$-closed in (X, τ). Since s(X, τ) is an ${}_{\alpha}T_{1/2}^{***}$, $f^{-1}(V)$ is closed in (X, τ). Therefore f is continuous.

Theorem 3.35: Let f: $(X, \tau) \to (Y, \sigma)$ be a gs-continuous map . If (X, τ) is an T_c^{***} space, then f is $^{**}g\alpha$-continuous.

Proof: Let V be a closed set in (Y, σ). Since f is gs-continuous, $f^{-1}(V)$ is gs-closed in (X, τ). Since (X, τ) is an T_c^{***}, $f^{-1}(V)$ is $^{**}g\alpha$-closed in (X, τ). Therefore f $^{**}g\alpha$-continuous.

Theorem 3.36: Let f: $(X,\tau) \to (Y,\sigma)$ be a αg-continuous map. If (X,τ) is ${}_{\alpha}T_c^{***}$space, then f is $^{**}g\alpha$-continuous.

Proof: Let V be a closed set in (Y, σ). Since f is αg-continuous, $f^{-1}(V)$ is αg-closed in (X,τ). Since (X,τ) is an ${}_{\alpha}T_c^{***}$, $f^{-1}(V)$ is $^{**}g\alpha$-closed in (X, τ). Therefore f $^{**}g\alpha$-continuous.

Theorem 3.37: Let f: $(X, \tau) \to (Y, \sigma)$ be a g-continuous map . If (X, τ) is $^{***}{}_{\alpha}T_{1/2}$ space, then f is $^{**}g\alpha$-continuous.

Proof: Let V be a closed set in (Y, σ). Since f is g-continuous, $f^{-1}(V)$ is g-closed in (X, τ). Since (X, τ) is a $^{***}{}_{\alpha}T_{1/2}$, $f^{-1}(V)$ is $^{**}g\alpha$-closed in (X, τ). Therefore f $^{**}g\alpha$-continuous.

Theorem 3.38: Let f: $(X, \tau) \to (Y, \sigma)$ be a surjective , $^{*}g\alpha$-irresolute and a closed map. Then f (A) is $^{**}g\alpha$-closed set of (Y,σ) for every $^{**}g\alpha$-closed set A of (X,τ).

Proof: Let A be a $^{**}g\alpha$-closed set of (X, τ). Let U be a $^{*}g\alpha$-open set of (Y, σ) such that $f(A) \subseteq U$. Since f is surjective and $^{*}g\alpha$-irresolute, $f^{-1}(U)$ is a $^{*}g\alpha$-open set of (X, τ). Since $A \subseteq f^{-1}(U)$ and A is$^{**}g\alpha$-closed set of (X, τ) $cl(A) \subseteq f^{-1}(U)$. Then $f(cl(A)) \subseteq f(f^{-1}(U)) = U$. since f is closed, $f(cl(A)) \subseteq cl(f(cl(A)))$. This implies $cl(f(A)) \subseteq cl(f(cl(A))) = f(cl(A)) \subseteq U$. Therefore f(A) is a $^{**}g\alpha$-closed set of (Y, σ).

Theorem 3.39: Let f: $(X, \tau) \to (Y, \sigma)$ be a surjective, $^{**}g\alpha$-irresolute and a closed map. If (X, τ) is an ${}_{\alpha}T_{1/2}^{***}$ space, then (Y, σ) is also an ${}_{\alpha}T_{1/2}^{***}$ space.

Proof: Let A be a $^{**}g\alpha$- closed set of (Y, σ). Since f is $^{**}g\alpha$-irresolute, $f^{-1}(A)$ is a $^{**}g\alpha$-closed set of (X, τ). Since (X, τ) is an ${}_{\alpha}T_{1/2}^{***}$, $f^{-1}(A)$ is closed set of (X, τ). Then $f(f^{-1}(A)) = A$ is closed in (Y, σ). Therefore (Y, σ) is an ${}_{\alpha}T_{1/2}^{***}$ space.

Definition 3.40: A function f: (X, τ) → (Y, σ) is called pre-$^{**}g\alpha$-closed if f(A) is a $^{**}g\alpha$-closed set of (Y, σ) for every $^{**}g\alpha$-closed set of A of (X, τ).

Theorem 3.41: Let f: (X,τ) → (Y, σ) be a surjective, gs-irresolute and pre-$^{**}g\alpha$ -closed map. If (X, τ) is a T_c^{***} space, then (Y, σ) is also T_c^{***}space.

Proof: Let A be a gs-closed set of (Y, σ). Since f is gs-irresolute, $f^{-1}(A)$ is gs-closed set in (X, τ). Since (X, τ) is a T_c^{***} space, $f^{-1}(A)$ is a $^{**}g\alpha$-closed in (X, τ). Since f is pre-$^{**}g\alpha$ -closed map, $f(f^{-1}(A))$ is $^{**}g\alpha$-closed in (Y, σ) for every $^{**}g\alpha$ -closed set $f^{-1}(A)$ of (X, τ). Thus A is a $^{**}g\alpha$-closed in (Y, σ). Therefore (Y, σ) is a T_c^{***}space.

Theorem 3.42: Let f: (X, τ) → (Y, σ) be a surjective, αg-irresolute and pre-$^{**}g\alpha$ -closed map. If (X, τ) is an ${}_{\alpha}T_c^{***}$ space, then (Y, σ) is also ${}_{\alpha}T_c^{***}$ space.

Proof: Let A be a αg-closed set of (Y, σ). Since f is αg-irresolute, $f^{-1}(A)$ is αg-closed set in (X, τ). Since (X, τ) is an ${}_{\alpha}T_c^{***}$ space, $f^{-1}(A)$ is a $^{**}g\alpha$-closed in (X, τ). Since f is pre-$^{**}g\alpha$ -closed map , $f(f^{-1}(A))$ is $^{**}g\alpha$-closed in (Y, σ) for every $^{**}g\alpha$-closed set $f^{-1}(A)$ of (X, τ). Since f is surjection, $A=f(f^{-1}(A))$. Thus A is a $^{**}g\alpha$-closed set of (Y, σ). Therefore (Y, σ) is an ${}_{\alpha}T_c^{***}$ space.

Theorem 3.43: Let f: (X, τ) → (Y, σ) be a surjective, gc-irresolute and pre-$^{**}g\alpha$ -closed map. If (X, τ) is a $^{***}{}_{\alpha}T_{1/2}$ space, then (Y, σ) is also $^{***}{}_{\alpha}T_{1/2}$ space.

Proof: Let A be a g-closed set of (Y, σ). Since f is gc-irresolute, $f^{-1}(A)$ is g-closed set in (X, τ). Since (X, τ) is a $^{***}{}_{\alpha}T_{1/2}$ space, $f^{-1}(A)$ is a $^{**}g\alpha$-closed in (X, τ). Since f is pre-$^{**}g\alpha$-closed map , $f(f^{-1}(A))$ is $^{**}g\alpha$-closed in (Y, σ) for every $^{**}g\alpha$-closed set $f^{-1}(A)$ of (X, τ). Since f is surjection, $A=f(f^{-1}(A))$. Thus A is a $^{**}g\alpha$-closed set of (Y, σ). Therefore (Y, σ) is a $^{***}{}_{\alpha}T_{1/2}$ space.

IV. Conclusion

The class of $^{**}g\alpha$-continuous map and $*^{*}g\alpha$-irresolute map defined using $*^{*}g\alpha$-closed set. The $*^{*}g\alpha$-closed sets can be used to derive a new homeomorphism, connectedness, compactness and new separation axioms. This concept can be extended to soft topological space and nano topological spaces.

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