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The Sum Span of a Finite Subset of A Completely Bounded Artex Space over ABi-Monoid

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Abstract: *Completely Bounded Artex Spaces over bi-monoids contain the least and greatest elements namely 0 and 1. These elements play a good role in our study. Sum Combination of elements of a Completely Bounded ArtexSpace over a bi-monoid is defined. The sum span of a finite subset of a completely bounded Artex space over a bi-monoid is defined. Some propositions were found and proved. Examples are provided.*
Keywords: *Bi-monoids, Artex Spaces over bi-monoids, Completely Bounded Artex Spaces over bi-monoids, Sum Combination,Sumspan.*

I. Introduction

The aim of considering semi-groups is to provide an introduction to the theory of rings. A more general concept than that of a group is that of a semi-group. The algebraic system Bi-semi-group is more general to the algebraic system ring or an associative ring. We introduced Artex Spaces over Bi-monoids. As a development of Artex Spaces over Bi-monoids, we introduced SubArtex spaces of Artex spaces over bi-monoids. From the definition of a SubArtex space, it is clear that not every subset of an Artex space over a bi-monoid is a SubArtex space. We found and proved some propositions which qualify subsets to become SubArtex Spaces. Completely Bounded Artex Spaces over bi-monoids were introduced. It contains the least and greatest elements namely 0 and 1. These elements play a good role in our study. In our study Sum Combination of elements of an Artex Space over a bi-monoid is defined. The sum span of a finite subset of a completely bounded artex space over a bi-monoid is defined. Some propositions were found and proved. Examples are provided. As the theory of Artex spaces over bi-monoids is developed from lattice theory, this theory will play a good role in many fields especially in science and engineering and in computer fields. In Discrete Mathematics this theory will create a new dimension.

II. Preliminaries

2.1 Semi-group : A non-empty set S together with a binary operation **.** is called a Semi-group if for all a,b,c ϵ S, a.(b . c) = (a.b).c

2.2 Monoid : A non-empty set N together with a binary operation **.** is called a monoid if
(i) for all a,b,c ϵ N, a.(b . c) = (a.b).c and
(ii) there exists an element denoted by e in N such that a.e = a = e.a, for all a ϵ N.
The element e is called the identity element of the monoid N.

2.3 Relation : Let S be a non-empty set. Any subset of S×S is called a relation in S.
If R is a relation in S, then R is a subset of S×S.
If (a,b) belongs to the relation R, then we can express this by aRb or by a ≤ b.
Note : A relation may be denoted by ≤

2.4 Partial Ordering : A relation ≤ on a set P is called a partial order relation or a partial ordering in P if
(i) a ≤ a, for all aϵP ie ≤ is reflexive,
(ii) a ≤ b and b ≤ a implies a = b ie ≤ is anti-symmetric, and
(iii) a ≤ b and b ≤ c implies a ≤ c ie ≤ is transitive.

2.5 Partially Ordered Set (POSET) : If ≤ is a partial ordering in P, then the ordered pair (P, ≤) is called a Partially Ordered Set or simply a POSET.

2.6 Lattice : A lattice is a partially ordered set (L,≤) in which every pair of elements a,b ϵ L has a greatest lower bound and a least upper bound.
The greatest lower bound of a and b is denoted by aΛb and the least upper bound of a and b is denoted by aV b

2.7 Lattice as an Algebraic System : A lattice is an algebraic system (L,Λ,V) with two binary operations Λ and V on L which are both commutative, associative and satisfy the absorption laws namely aΛ(aVb) = a and aV(aΛb) = a, for all a,bϵL
The operations Λ and V are called cap and cup respectively, or sometimes meet and join respectively.

2.8 Properties : We have the following properties in a lattice (L , Λ,V)
1.a Λ a = a 1'.a V a = a (Idempotent Law)

2.a Λ b = b Λ a 2'.a V b = b V a (Commutative Law)
3.(a Λ b)Λ c = a Λ(b Λ c) 3'.(a V b) V c = a V(b V c) (Associative Law)
4.a Λ (a V b) = a 4'.a V (a Λ b) = a, for all a,b,c∈ L (Absorption Law)

2.9 Complete Lattice : A lattice is called a complete lattice if each of its nonempty subsets has a least upper bound and a greatest lower bound.
Every finite lattice is a complete lattice and every complete lattice must have a least element and a greatest element.
The least and the greatest elements, if they exist, are called the bounds or units of the lattice and are denoted by 0 and 1 respectively.

2.10 Bounded Lattice : A lattice which has both elements 0 and 1 is called a bounded lattice. A bounded lattice is denoted by (L, Λ,V,0,1)
The bounds 0 and 1 of a lattice (L, Λ,V) satisfy the following identities.
For any a∈L, a V 0 = a a Λ 1 = a a V 1 = 1 a Λ 0 = 0

2.10.1 Example : For any set S, the lattice (P(S), ⊆) is a bounded lattice.Here for each A,B ∈ P(S), the least upper bound of A and B is A∪B and the greatest lower bound of A and B is A∩B. The bounds in this lattice are φ, the empty set and S, the universal set.

2.11 Complemented Lattice : Let (L, Λ,V,0,1) be a bounded lattice. An element a'∈L is called a complement of an element a∈L if a Λ a' = 0, a V a' = 1. A bounded lattice (L, Λ,V,0,1) is said to be a complemented lattice if every element of L has at least one complement. A complemented lattice is denoted by (L, Λ,V, ' ,0,1).

2.11.1 Example : For any set S, the lattice (P(S), ⊆) is a Complemented lattice.
For each A,B ∈ P(S), the least upper bound of A and B is A∪B and the greatest lower bound of A and B is A∩B.
The bounds in this lattice are φ, the empty set and S, the universal set.
Here for any A ∈ P(S), the complement of A in P(S) is S-A

2.12 Doubly Closed Space: A non-empty set D together with two binary operations denoted by + and . is called a Doubly Closed Space if (i) a.(b+c) = a.b + a.c and (ii) (a+b).c = a.c + b.c, for all a,b,c ∈ D
A Doubly closed space is denoted by (D, + , .)

Note 1: The axioms (i) a.(b+c) = a.b + a.c and (ii) (a+b).c = a.c + b.c ,for all a,b,c ∈ D are called the distributive properties of the Doubly Closed Space.

Note 2: The operations + and **.** need not be the usual addition and usual multiplication respectively.

2.12.1 Example : Let N be the set of all natural numbers.
Then (N, + , .), where + is the usual addition and . is the usual multiplication, is a Doubly closed space.
Similarly (Z, + , .), (Q, + , .), (R, + , .) and (C, + , .) are all Doubly closed spaces.

2.12.2 Example : (Z , + , –), where + is the usual addition and – is the usual subtraction , is not a Doubly closed space.
Even though + and – are binary operations in Z, (Z , + , –) is not a Doubly closed space because of the distributive properties of the Doubly Closed Space.
Take a = 15, b = 7, c = 4
Then a – (b + c) = 15 – (7 + 4)
= 15 – 11
= 4
But (a – b) + (a – c) = (15 – 7) + (15 – 4)
= 8 + 11
= 19
Therefore, a – (b + c) ≠ (a – b) + (a – c)
Therefore, (Z , + , –) is not a Doubly closed space.

2.13 Bi-semi-group : A Doubly closed space (S , + , .) is called a Bi-semi-group if + and . are associative in D.

2.13.1 Example : (N, + , .), (Z, + , .), (Q, + , .), (R, + , .), and (C, + , .), where + is the usual addition and **.** is the usual multiplication, are all Bi-semi-groups.

2.14 Bi-monoid : A Bi-semi-group (M , + , .) is called a Bi-monoid if there exist elements denoted by 0 and 1 in S such that a+0=a=0+a, for all a∈M and a.1=a=1.a, for all a∈M .
The element 0 is called the identity element of M with respect to the binary operation + and the element 1 is called the identity element of M with respect to the binary operation**.**.

2.14.1 Example : Let W = {0,1,2,3,…}.Then (W, + , .), where + is the usual addition and . is the usual multiplication, is a Bi-monoid.

2.14.2 Example : Let Q'= Q^+ ∪{0}, where Q^+ is the set of all positive rational numbers. Then (Q', + , .) is a bi-monoid.

2.14.3 Example : R'= $R^+ \cup \{0\}$, where R^+ is the set of all positive real numbers. Then (R', + , .) is a bi-monoid.
2.14.4 Example : (Z, + , .), (Q, + , .), (R, + , .), and (C, + , .), where + is the usual addition and **.** is the usual multiplication, are all Bi-monoids.

III. Artex Spaces Over A Bi-Monoids

3.1 Artex Space Over a Bi-monoid : Let (M , + , .) be a bi-monoid with the identity elements 0 and 1 with respect to + and **.** respectively. A non-empty set A together with two binary operations ^ and v is said to be an Artex Space Over the Bi-monoid (M , + , .) if
1.(A, Λ , V) is a lattice and
2.for each m∊M , m≠0, and a∊A, there exists an element ma ∊ A satisfying the following conditions :
(i) m(a Λ b) = ma Λ mb
(ii) m(a V b) = ma V mb
(iii) ma Λ na ≤ (m +n)a and ma V na ≤ (m + n)a
(iv) (mn)a = m(na), for all m,n∊M, m≠0, n≠0, and a,b∊A
(v) 1.a = a, for all a ∊ A.
Here, ≤ is the partial order relation corresponding to the lattice (A, Λ , V) .The multiplication ma is called a **bi-monoid multiplication with an artex element** or simply bi-monoid multiplication in A.
3.2 Examples
3.2.1 Example : Let W = {0,1,2,3,…}.
Then (W , + , .) is a bi-monoid , where + and **.** are the usual addition and multiplication respectively.
Let Z be the set of all integers
Then (Z, ≤) is a lattice in which Λ and V are defined by a Λ b = minimum of {a,b} and a V b = maximum of {a,b}, for all a,b∊Z.
Clearly for each m∊W,m≠0, and for each a∊Z, ma∊Z.
Also
(i) m(a Λ b) = ma Λ mb
(ii) m(a V b) = ma V mb
(iii) ma Λ na ≤ (m +n)a and ma V na ≤ (m + n)a
(iv) (mn)a = m(na)
(v) 1.a = a , for all m,n∊W, m ≠ 0, n ≠ 0 and a,b∊Z
Therefore, Z is an Artex Space Over the Bi-monoid (W , + , .)
3.2.2 Example : As defined in Example 3.2.1, Q, the set of all rational numbers is an Artex space over W
3.2.3 Example : As defined in Example 3.2.1, R, the set of all real numbers is an Artex space over W.
3.2.4 Example : Let Q'= $Q^+ \cup \{0\}$, where Q^+ is the set of all positive rational numbers.
Then (Q', + , .) is a bi-monoid. Now as defined in Example 3.2.1, Q, the set of all rational numbers is an Artex space over Q'
3.2.5 Example : R'= $R^+ \cup \{0\}$, where R^+ is the set of all positive real numbers. Then (R', + , .) is a bi-monoid.
As defined in Example 3.2.1, R, the set of all real numbers is an Artex space over R'
3.3 Properties
Properties 3.3.1 : We have the following properties in a lattice (L , Λ,V)

1.a Λ a = a	1'.a V a = a
2.a Λ b = b Λ a	2'.a V b = b V a
3.(a Λ b)Λ c = a Λ(b Λ c)	3'.(a V b) V c = a V(b V c)
4.a Λ (a V b) = a	4'.a V (a Λ b) = a, for all a,b,c ∊ L

Therefore, we have the following properties in an Artex Space A over a bi-monoid M.

(i) m(a Λ a) = ma	(i)'.m(a V a) = ma
(ii) (m(a Λ b) =m(b Λ a)	(ii)'.m(a V b) = m(b V a)
(iii) m((a Λ b)Λ c)=m(a Λ(b Λ c))	(iii)'.m((a V b) V c) = m(a V(b V c))
(iv) m(a Λ (a V b)) = ma	(iv)'.m(a V (a Λ b)) = ma,

for all m∊M, m ≠ 0 and a,b,c ∊ A
3.4 SubArtex Space : Let (A, Λ ,V) be an Artex space over a bi-monoid. (M , + , .). Let S be a nonempty subset of A. Then S is said to be a SubArtex Space of A if (S, Λ ,V) itself is an Artex Space over M.
3.4.1 Example : As defined in Example 3.2.1, Z is an Artex Space over W = {0,1,2,3,…..} and W is a subset of Z. Also W itself is an Artex space over W under the operations defined in Z. Therefore, W is a SubArtex space of Z.

3.5 Complete Artex Space over a bi-monoid : An Artex space A over a bi- monoid M is said to be a Complete Artex Space over M if as a lattice, A is a complete lattice, that is each nonempty subset of A has a least upper bound and a greatest lower bound.

3.5.1 Remark : Every Complete Artex space must have a least element and a greatest element.

The least and the greatest elements, if they exist, are called the bounds or units of the Artex space and are denoted by 0 and 1 respectively.

3.6 Lower Bounded Artex Space over a bi-monoid : An Artex space A over a bi-monoid M is said to be a Lower Bounded Artex Space over M if as a lattice, A has the least element 0.

3.6.1 Example : Let A be the set of all constant sequences (x_n) in $[0,\infty)$

Let W = {0,1,2,3,…}.

Define $\leq$ ', an order relation, on A by for (x_n), (y_n) in A, $(x_n) \leq ' (y_n)$ means $x_n \leq y_n$, for each n

where $\leq$ is the usual relation " less than or equal to "

Therefore, A is an Artex space over W.

The sequence (0_n), where 0_n is 0 for all n, is a constant sequence belonging to A

Also $(0_n) \leq ' (x_n)$, for all the sequences (x_n) belonging to in A

Therefore, (0_n) is the least element of A.

That is, the sequence 0,0,0,…… is the least element of A

Hence A is a Lower Bounded Artex space over W.

3.7 Upper Bounded Artex Space over a bi-monoid : An Artex space A over a bi-monoid M is said to be an Upper Bounded Artex Space over M if as a lattice, A has the greatest element 1.

3.7.1 Example : Let A be the set of all constant sequences (x_n) in $(-\infty,0]$ and let W = {0,1,2,3,…}.

Define $\leq$' ,an order relation, on A by for (x_n), (y_n) in A, $(x_n) \leq' (y_n)$ means $x_n \leq y_n$, for n = 1,2,3,…, where $\leq$ is the usual relation " less than or equal to "

A is an Artex space over W.

Now, the sequence (1_n), where 1_n is 0, for all n, is a constant sequence belonging to A

Also $(x_n) \leq ' (1_n)$, for all the sequences (x_n) in A

Therefore, (1_n) is the greatest element of A.

That is, the sequence 0,0,0,….. is the greatest element of A

Hence A is an Upper Bounded Artex Space over W.

3.8 Bounded Artex Space over a bi-monoid : An Artex space A over a bi-monoid M is said to be a Bounded Artex Space over M if A is both a Lower bounded Artex Space over M and an Upper bounded Artex Space over M.

3.9 Completely Bounded Artex Space over a bi-monoid: A Bounded Artex Space A over a bi-monoid M is said to be a Completely Bounded Artex Space over M if (i) 0.a = 0, for all a ϵ A (ii) m.0 = 0, for all m ϵ M.

3.9.1 Note : While the least and the greatest elements of the Complemented Artex Space is denoted by 0 and 1, the identity elements of the bi-monoid (M , + , .) with respect to addition and multiplication are, if no confusion arises, also denoted by 0 and 1 respectively.

IV. The Sum Span Of A Sub Set Of An Artex Space Over A Bi-Monoid

4.1 Sum Combination : Let (A, Λ ,V) be a Completely Bounded Artex Space over a bi-monoid (M , + , .). Let $a_1, a_2, a_3, \ldots\ldots. a_n \in A$. Then any element of the form $m_1a_1Vm_2a_2Vm_3a_3V \ldots\ldots. V\, m_na_n$, where $m_i \in M$, is called a Sum Combination or Join Combination of $a_1, a_2, a_3, \ldots\ldots. a_n$ over the Artex Space A.

4.2 The Sum Span of a subset of a Completely Bounded Artex Space over a Bi-monoid : Let (A, Λ ,V) be a Completely Bounded Artex Space over a bi-monoid (M , + , .) and W be a nonempty finite subset of A. Then the Sum Span of W or Join Span of W denoted by S[W] is defined to be the set of all sum combinations of elements of W. That is, if W = { $a_1, a_2, a_3, \ldots\ldots. a_n$ }, then S[W] = {$m_1a_1Vm_2a_2Vm_3a_3V \ldots\ldots Vm_na_n$ / $m_i \in$ M}.

4.3 Propositions

Proposition 4.3.1: Let (A, Λ ,V) be a Completely Bounded Artex Space over a bi-monoid (M , + , .) and W be a nonempty finite subset of A. Then W ⊆ S [W]

Proof : Let (A, Λ ,V) be a Completely Bounded Artex Space over a bi-monoid (M , + , .)

Let W = { $a_1, a_2, a_3, \ldots\ldots. a_n$ }be a finite subset of A.

S[W] = {$m_1a_1Vm_2a_2Vm_3a_3V \ldots\ldots Vm_na_n$ / $m_i \in$ M}.

Since (A, Λ ,V) is a Completely Bounded Artex Space over a bi-monoid (M , + , .), the least element of A and the greatest element of A will exist in A.

Let 0 and 1 be the least and the greatest elements of A respectively.

For any aϵA, a V 0 = a a Λ 1 = a a V 1 = 1 a Λ 0 = 0.

Without any confusion the identity elements with respect to + and . in M will also be denoted by 0 and 1 respectively .
Let $a_i \in W$.
Then $a_i = 0.a_1 V 0.a_2 V \ldots 0.a_3 V 1.a_i V 0.a_{i+1} V \ldots V 0.a_n \in S[W]$
Therefore, $W \subseteq S[W]$.

Proposition 4.3.2 : Let (A, Λ ,V) be a Completely Bounded Artex Space over a bi-monoid (M , + , .). Let W and V be any two nonempty finite subsets of A. Then W ⊆ V implies S[W] ⊆ S[V] .

Proof : Let (A, Λ ,V) be a Completely Bounded Artex Space over a bi-monoid (M , + , .)
Let W and V be any two nonempty finite subsets of A such that $W = \{ a_1, a_2, a_3, \ldots a_n \}$ and $V = \{ a_1, a_2, a_3, \ldots a_n, b_1, b_2, b_3, \ldots b_k \}$
Then W ⊆ V
Let x ϵ S[W]
Then $x = m_1a_1 V m_2a_2 V m_3a_3 V \ldots V m_na_n$, where $m_i \in M$
We have W ⊆ V and $a_1, a_2, a_3, \ldots a_n \in V$.
Therefore, $x = m_1a_1 V m_2a_2 V m_3a_3 V \ldots V m_na_n V 0.b_1 V 0.b_2 V \ldots V 0.b_k \in S[V]$.
(Since for any aϵA, a V 0 = a)
Therefore, S[W] ⊆ S[V]
Hence, if W ⊆ V, then S[W] ⊆ S[V] .

Proposition 4.3.3 : Let (A, Λ ,V) be a Completely Bounded Artex Space over a bi-monoid (M , + , .). Let W and V be any two nonempty finite subsets of A. Then S[W∪V] = S[W] V S[V] .

Proof : Let (A, Λ ,V) be a Completely Bounded Artex Space over a bi-monoid (M , + , .)
Let W and V be any two nonempty finite subsets of A such that $W = \{ a_1, a_2, a_3, \ldots a_n \}$ and $V = \{ b_1, b_2, b_3, \ldots b_k \}$
Let x ϵ S[W∪V]
Then $x = m_1a_1 V m_2a_2 V m_3a_3 V \ldots V m_na_n V m_{n+1}b_1 V m_{n+2}b_2 V \ldots V m_{n+k}b_k$
Let x = w Vv , where $w = m_1a_1 V m_2a_2 V m_3a_3 V \ldots V m_na_n$ and $v = m_{n+1}b_1 V m_{n+2}b_2 V \ldots V m_{n+k}b_k$
Clearly $w = m_1a_1 V m_2a_2 V m_3a_3 V \ldots V m_na_n \in S[W]$ and
$v = m_{n+1}b_1 V m_{n+2}b_2 V \ldots V m_{n+k}b_k \in S[V]$
Therefore, x = w Vv ϵ S[W] V S[V]
Therefore, S[W∪V] ⊆ S[W] V S[V] ------------------ (i)
Conversely, let x ϵ S[W] V S[V]
Then x = w Vv, where wϵ S[W] and v ϵ S[V]
Then $w = m_1a_1 V m_2a_2 V m_3a_3 V \ldots V m_na_n \in S[W]$ and
$v = m_{n+1}b_1 V m_{n+2}b_2 V \ldots V m_{n+k}b_k \in S[V]$, where $m_i \in M$.
Now x = w Vv
$x = m_1a_1 V m_2a_2 V m_3a_3 V \ldots V m_na_n V m_{n+1}b_1 V m_{n+2}b_2 V \ldots V m_{n+k}b_k \in S[W \cup V]$
S[W] V S[V] ⊆ S[W∪V] ---------------------- (ii)
From (i) and (ii) we have S[W∪V] = S[W] V S[V].

4.4 Examples

4.4.1 Example : Let $R' = R^+ \cup \{0\}$, where R^+ is the set of all positive real numbers and let W = {0,1,2,3,…..}
(R’, ≤) is a lattice in which Λ and V are defined by a Λ b = mini {a,b} and a V b = maxi {a,b} , for all a,b ϵ R’.
Here ma is the usual multiplication of a by m.
Clearly for each m ϵ W,m≠0, and for each a ϵ R’, ma ϵ R’.
Also,
(i) m(a Λ b) = ma Λ mb
(ii) m(a V b) = ma V mb
(iii) ma Λ na ≤ (m +n)a and ma V na ≤ (m + n)a
(iv) (mn)a = m(na) , for all m,n ϵ W, m≠0, n≠0, and a,b ϵ R’
(v) 1.a = a , for all a ϵ R’
Therefore, R’ is an Artex Space Over the bi-monoid (W , + , .)
Generally, if Λ_1, Λ_2, and Λ_3 are the cap operations of A , B and C respectively and if V_1, V_2, and V_3 are the cup operations of A , B and C respectively, then the cap of A×B×C denoted by Λ and the cup of A×B×C denoted by V are defined
$x \Lambda y = (a_1,b_1,c_1) \Lambda (a_2,b_2, c_2) = (a_1 \Lambda_1 a_2 \Lambda_1 a_3 , b_1 \Lambda_2 b_2 \Lambda_2 b_3, c_1 \Lambda_3 c_2 \Lambda_3 c_3)$ and
$x V y = (a_1,b_1,c_1) V (a_2,b_2, c_2) = (a_1 V_1 a_2 V_1 a_3 , b_1 V_2 b_2 V_2 b_3, c_1 V_3 c_2 V_3 c_3)$
Here, Λ_1, Λ_2, and Λ_3 denote the same meaning minimum of two elements in R’ and V_1, V_2, and V_3 denote the same meaning maximum of two elements in R’

Therefore, $R'^3 = R' \times R' \times R'$ is an Artex Space over W, where cap and cup operations are denoted by Λ and V respectively.

Let H = { (1,0,0) } and let T = { (0,1,0) }

Now S[H] = { (m,0,0) / m∈R' } and S[T] = { (0,n,0) / n∈R' }

S[H] V S[T] = { (m,0,0) / m∈R' } V { (0,n,0) / n∈R' }

= {(m $V_1$0 ,0V_2n, 0$V_3$0)}

={(m,n,0)} (since m$V_1$0 = max.{m,0} = m, 0V_2n=max.{0,n}=n and 0$V_3$0=max.{0,0}=0)

S[H] V S[T] = {(m,n,0) / m,n∈R'} ----------- (i)

Now H ∪ T = { (1,0,0), (0,1,0) }

Let m,n ∈ M, m≠0,n≠0

Then m(1,0,0) V n(0,1,0) = (m,0,0) V (0,n,0)

= (m $V_1$0 ,0 V_2 n ,V_3 0)

=(m,n,0) (since m$V_1$0 = max.{m,0} = m, 0V_2n=max.{0,n}=n and 0$V_3$0=max.{0,0}=0)

Therefore, S[H∪T] = {(m,n,0) / m,n∈R'} ----------- (ii)

From equations (i) and (ii) we have S[H∪T] = S[H] V S[T]

4.4.2 Example : Let H = { (1,0,0) } and let T = { (1,0,0),(0,1,0) }

Clearly H ⊆ T

Now S[H] = { (a,0,0) / a ∈ R' }

and S[T] = { (a,0,0), (0,b,0) / a, b ∈ R' }

Therefore, S[H] ⊆ S[T].

V. Conclusion

Sum Combination of elements of an Artex Space over a bi-monoid , Sum Span of a finite subset of a completely bounded artex space over a bi-monoid will create a dimension in the theory of Artex spaces over bi-monoids. Interested researcher can do wonders if they work very hard in this field..

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