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Semi-symmetry type Sasakian manifolds

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Abstract: *Recently the present author introduced the notion of generalized quasi-conformal curvature tensor which bridges Conformal curvature tensor, Concircular curvature tensor, Projective curvature tensor and Conharmonic curvature tensor. This article attempts to charectrize Sasakian manifolds with* $\omega(X,Y)\cdot W=0$. *Based on this curvature conditions, we obtained and tabled the expression for the Ricci tensor for the respective semi-symmetry type Sasakian manifolds.*

I. Introduction

In tune with Yano and Sawaki [9], recently the present authors [6] have defined and studied *generalized quasi-conformal curvature tensor* W, in the context of $N(k,\mu)$-manifold. The beauty of *generalized quasi-conformal curvature tensor* lies in the fact that it has the flavour of Riemann curvature tensor R, conformal curvature tensor C [10], conharmonic curvature tensor $\hat{C}$ [15], concircular curvature tensor $E($ [8], p.84), projective curvature tensor P ([8], p.84) and m-projective curvature tensor H [5] as special cases.

An Sasakian manifold is said to be semi-symmetry type (respectively Ricci semi-symmetry type) if the *generalized quasi-conformal curvature tensor* W (respectively Ricci tensor S) admits the condition

$$\omega(X,Y)\cdot W=0 \quad \left(\text{respectively } W(X,Y)\cdot S=0\right), \text{ for any } X,Y \text{ on } M,$$

(1)

where the dot means that $\omega(X,Y)$ acts on W (respectively on S) as derivation. Here ω and W stand for *generalized quasi-conformal curvature tensor* with the associated scalar triples $(\bar{a},\bar{b},\bar{c})$ and (a,b,c) respectively. In particular, manifold satisfying the condition $R(X,Y)\cdot R=0$ (obtained from 1 by setting $\bar{a}=\bar{b}=\bar{c}=0=a=b=c$) is said to be semi-symmetric in the sense of Cartan ([4], P- 265 and named by N. S. Sinjukov [12]). A full classification of such space is given by Z. I. Szabö ([16]). This type of the manifolds have been studied by several authors such as Papantoniou [1], Perrone [3] and the references therein.

Our paper is structured as follows. Section 2 is a very brief account of Sasakian manifolds. Definition and some basic results of the *generalized quasi-conformal curvature tensor* are discussed in section 3. In section 4, we investigate Sasakian manifolds with $\omega(X,Y)\cdot W=0$. Based on this curvature condition, we obtained and tabled the expressions for the Ricci tensor.

II. Sasakian manifolds

Let $M^{2n+1}(\phi,\xi,\eta,g)$ be a Sasakian manifold with the structure (ϕ,ξ,η,g).
Then the following relation hold[2]:

$$\eta(X)=g(X,\xi), \quad \phi^2=-I+\eta\circ\xi, \qquad (2)$$

$$\eta(\xi)=1, \quad \eta\cdot\phi=0, \qquad (3)$$

$$\nabla_X\xi=-\phi X, \qquad (4)$$

$$R(\xi,X)Y=(\nabla_X\phi)(Y)=g(X,Y)\xi-\eta(Y)X, \qquad (5)$$

$$g(\phi X,\phi Y)=g(X,Y)-\eta(X)\eta(Y), \qquad (6)$$

$$(\nabla_X \eta)(Y) = -g(\phi X, Y), \qquad (7)$$

$$R(X,Y)\xi = [\eta(Y)X - \eta(X)Y], \qquad (8)$$

$$\eta(R(X,Y)Z) = [g(Y,Z)\eta(X) - g(X,Z)\eta(Y)], \qquad (9)$$

$$S(\phi X, \phi Y) = S(X,Y) - 2n\eta(X)\eta(Y), \quad (10)$$

$$S(X,\xi) = 2n\eta(X), \qquad (11)$$

for any vector fields X, Y on M, where ∇ denotes the operator of covariant differentiation with respect to g, ϕ is a skew--symmetric tensor field of type $(1,1)$, S is the Ricci tensor of type $(0,2)$ and R is the Riemannian curvature tensor of the manifold.

III. The generalized quasi-conformal curvature tensor

The *generalized quasi-conformal curvature tensor* is defined as

$$\begin{aligned} W(X,Y)Z &= \frac{2n-1}{2n+1}[(1+2na-b)-\{1+2n(a+b)\}c]C(X,Y)Z \\ &+[1-b+2na]E(X,Y)Z + 2n(b-a)P(X,Y)Z \\ &+\frac{2n-1}{2n+1}(c-1)\{1+2n(a+b)\}\hat{C}(X,Y)Z \qquad (12) \end{aligned}$$

for all X, Y & $Z \in \chi(M)$, the set of all vector field of the manifold M, where a, b & c are real constants. And C, E, P and $\hat{C}$ stand for Conformal, Concircular, Projective and Conharmonic curvature tensor respectively. These curvature tensor are defined as follows

$$\begin{aligned} C(X,Y) &= R(X,Y) - \frac{1}{2n-1}[(X\wedge_g QY)+(QX\wedge_g Y)] \\ &+\frac{r}{2n(2n-1)}[X\wedge_g Y], \qquad (13) \end{aligned}$$

$$E(X,Y) = R(X,Y) - \frac{r}{2n(2n+1)}[X\wedge_g Y], \qquad (14)$$

$$P(X,Y) = R(X,Y) - \frac{1}{2n}[X\wedge_g QY], \quad (15)$$

$$\hat{C}(X,Y) = R(X,Y) - \frac{1}{2n-1}[(X\wedge_g QY)+(QX\wedge_g Y)] \qquad (16)$$

for all X, Y & $Z \in \chi(M)$, where R, S, Q & r being Christoffel Riemannian curvature tensor, Ricci tensor, Ricci operator and scalar curvature respectively.

In particular, the *generalized quasi-conformal curvature tensor* W induced to be

(1) Riemann curvature tensor R, if $a = b = c = 0$,

(2) conformal curvature tensor C, if $a = b = -\frac{1}{2n-1}$, $c = 1$,

(3) conharmonic curvature tensor $\hat{C}$, if $a = b = -\frac{1}{2n-1}$, $c = 0$,

(4) concircular curvature tensor E, if $a = b = 0$ and $c = 1$,

(5) projective curvature tensor P, if $a = -\frac{1}{2n}$, $b = 0$, $c = 0$ and

(6) m-projective curvature tensor H, if $a = b = -\frac{1}{4n}$, $c = 0$.

The m-projective curvature tensor is introduced by G. P. Pokhariyal & R. S. Mishra [5]. Which is defined as follows

$$H(X,Y)Z = R(X,Y) - \frac{1}{4n}[S(Y,Z)X - S(X,Z)Y$$
$$+ g(Y,Z)QX - g(X,Z)QY]. \quad (17)$$

Using (13), (14), (15) and (16) in (12), the *generalized quasi-conformal curvature tensor* W becomes

$$W(X,Y)Z = R(X,Y)Z + a[S(Y,Z)X - S(X,Z)Y]$$
$$+ b[g(Y,Z)QX - g(X,Z)QY]$$
$$-\frac{cr}{2n+1}\left(\frac{1}{2n} + a + b\right)[g(Y,Z)X - g(X,Z)Y]. \quad (18)$$

IV. Sasakian manifolds with semi-symmetry type curvature condition

Definition 4.1 *A* $(2n+1)$ *-dimensional* $(n > 1)$ *Sasakian manifold is said to be semi-symmetry type if the condition* $\omega(X,Y)\cdot W = 0$ *holds, for any vector fields* X, Y *on the manifold and* $\omega(X,Y)$ *acts on* W *as derivation, where* ω *and* W *stand for generalized quasi-conformal curvature tensor with the associated scalar triples* $(\bar{a},\bar{b},\bar{c})$ *and* (a,b,c) *respectively.*

Now, let us consider a $(2n+1)$-dimensional Sasakian manifold M, satisfying the condition

$$(\omega(X,Y)\cdot W)(Z,U)V = 0. \quad (19)$$
$$\text{i.e.}\ \omega(X,Y)W(Z,U)V = W(\omega(X,Y)Z,U)V + W(Z,\omega(X,Y)U)V$$
$$+ W(Z,U)\omega(X,Y)V \quad (20)$$

which is equivalent to

$$g(\omega(\xi,X)W(Y,Z)U,\xi) - g(W(\omega(\xi,X)Y,Z)U,\xi)$$
$$- g(W(Y,\omega(\xi,X)Z)U,\xi) - g(W(Y,Z)\omega(\xi,X)U,\xi) = 0. \quad (21)$$

Putting $X = Y = e_i$ in (21) where $\{e_1, e_2, e_3, ..., e_{2n}, e_{2n+1} = \xi\}$ is an orthonormal basis of the tangent space at each point of the manifold M and taking the summation over i, $1 \le i \le 2n+1$, we get

$$\sum_{i=1}^{2n+1}[g(\omega(\xi,e_i)W(e_i,Z)U,\xi) - g(W(\omega(\xi,e_i)e_i,Z)U,\xi)$$
$$- g(W(e_i,\omega(\xi,e_i)Z)U,\xi) - g(W(e_i,Z)\omega(\xi,e_i)U,\xi)] = 0. \quad (22)$$

From the equation (18), we can easily bring out the followings

$$\eta(W(\xi,U)Z)$$
$$= \left[\frac{cr}{2n+1}\left(\frac{1}{2n} + a + b\right) - 2n(a-b) - 1\right]\eta(Z)\eta(U)$$
$$+ \left[1 + 2nb - \frac{cr}{2n+1}\left(\frac{1}{2n} + a + b\right)\right]g(Z,U) + aS(Z,U). \quad (23)$$

$$\sum_{i=1}^{2n+1}\overline{W}(e_i,Z,U,e_i)$$
$$= (1 - b + 2na)S(Z,U) + \left\{br - \frac{2ncr}{2n+1}\left(\frac{1}{2n} + a + b\right)\right\}g(Z,U). \quad (24)$$

$$\sum_{i=1}^{2n+1}\eta(W(e_i,Z)e_i)$$
$$=\left[-2n(1-a+2nb)-\left\{ar-\frac{2ncr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right\}\right]\eta(Z). \qquad (25)$$

$$\sum_{i=1}^{2n+1}S(W(e_i,Z)U,e_i)$$
$$=\left\{ar+\frac{cr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right\}S(Z,U)-(a+b-1)S^2(Z,U)$$
$$+\left\{b\|Q\|^2-\frac{cr^2}{2n+1}\left(\frac{1}{2n}+a+b\right)\right\}g(Z,U). \qquad (26)$$

$$\sum_{i=1}^{2n+1}\eta(e_i)\eta(W(Qe_i,Z)U)$$
$$=2n\left[1+2nb-\frac{cr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right]g(Z,U)+2naS(Z,U)$$
$$-2n\left[1+2n(a+b)-\frac{cr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right]\eta(Z)\eta(U). \qquad (27)$$

$$\sum_{i=1}^{2n+1}S(e_i,Z)\eta(W(e_i,\xi)U)$$
$$=2n\left[1+2n(a+b)-\frac{cr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right]\eta(Z)\eta(U)$$
$$-\left\{1+2nb-\frac{cr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right\}S(Z,U)-aS^2(Z,U). \qquad (28)$$

Now, $\sum_{i=1}^{2n+1}g(\omega(\xi,e_i)W(e_i,Z)U,\xi)$
$$=\left[1+2n\bar{b}-\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n}+\bar{a}+\bar{b}\right)\right]\overline{W}(e_i,Z,U,e_i)+\bar{a}S(W(e_i,Z)U,e_i)$$
$$+\left[\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n}+\bar{a}+\bar{b}\right)-2n\bar{b}-1-2n\bar{a}\right]\eta(W(\xi,U)Z). \qquad (29)$$

In view of (24) & (26), (29) becomes

$$g(\omega(\xi,e_i)W(e_i,Z)U,\xi)$$
$$=\left[\left\{1+2n\bar{b}-\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n}+\bar{a}+\bar{b}\right)\right\}(1+2na-b)\right.$$
$$\left.+\bar{a}\left\{ar+\frac{cr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right\}\right]S(Z,U)+\bar{a}(1-a-b)S^2(Z,U)$$

$$+\left\{\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n}+\bar{a}+\bar{b}\right)-2n(\bar{a}+\bar{b})-1\right\}\eta(W(\xi,U)Z)$$
$$+\left[\left\{1+2n\bar{b}-\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n}+\bar{a}+\bar{b}\right)\right\}\left\{br-\frac{2ncr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right\}\right.$$
$$\left.+\bar{a}\left\{b\|Q\|^2-\frac{cr^2}{2n+1}\left(\frac{1}{2n}+a+b\right)\right\}\right]g(Z,U). \quad (30)$$

In consequence of of (23)-(28), we obtain the followings

$$\sum_{i=1}^{2n+1}g(W(\omega(\xi,e_i)e_i,Z)U,\xi)$$
$$=\left[2n\left\{1+2n\bar{b}-\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n}+\bar{a}+\bar{b}\right)\right\}+\bar{a}r+2n(\bar{b}-\bar{a})\right]\eta(W(\xi,U)Z)$$
$$-2n\bar{b}\left[1+2nb-\frac{cr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right]g(Z,U)-2na\bar{b}S(Z,U)$$
$$+2n\bar{b}\left[1+2n(a+b)-\frac{cr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right]\eta(Z)\eta(U). \quad (31)$$

$$g(W(e_i,\omega(\xi,e_i)Z)U,\xi)$$
$$=\left\{\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n}+\bar{a}+\bar{b}\right)-2n\bar{b}-1\right\}\eta(W(\xi,U)Z)$$
$$+2n\bar{a}\left[1+2n(a+b)-\frac{cr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right]\eta(Z)\eta(U)$$
$$-\bar{a}\left\{1+2nb-\frac{cr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right\}S(Z,U)-\bar{a}aS^2(Z,U). \quad (32)$$

$$\sum_{i=1}^{2n+1}g(W(e_i,Z)\omega(\xi,e_i)U,\xi)$$
$$=\left\{\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n}+\bar{a}+\bar{b}\right)-2n\bar{a}-1\right\}\eta(W(e_i,Z)e_i)\eta(U), \quad (33)$$

which is equivalemt to

$$\sum_{i=1}^{2n+1}g(W(e_i,Z)\omega(\xi,e_i)U,\xi)$$
$$=-\left\{\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n}+\bar{a}+\bar{b}\right)-2n\bar{a}-1\right\}\left[2n(1-a+2nb)\right.$$
$$\left.+\left\{ar-\frac{2ncr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right\}\right]\eta(U)\eta(Z). \quad (34)$$

By virtue of (30), (31), (32) & (34), we obtain from (22) that

$$\Bigg[\Bigg\{1+2n\bar{b}-\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n}+\bar{a}+\bar{b}\right)\Bigg\}(1-b)+\bar{a}(1+2nb)\Bigg]S(Z,U)$$
$$+\Bigg[\Bigg\{2n\,(1-a+2nb)+ar-\frac{2ncr}{2n+1}\left(\frac{1}{2n}+a+b\right)\Bigg\}\times$$
$$\Bigg\{\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n}+\bar{a}+\bar{b}\right)-2n\bar{a}-1\Bigg\}+\Bigg\{1+2n\,(a+b)-\frac{cr}{2n+1}\left(\frac{1}{2n}+a+b\right)\Bigg\}\times$$
$$\Bigg\{2n\{1+2n\bar{b}-\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n}+\bar{a}+\bar{b}\right\}+\bar{a}r-2n\bar{a}\Bigg\}\Bigg]\eta(U)\eta(Z)$$
$$+\Bigg[\Bigg\{1+2n\bar{b}-\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n}+\bar{a}+\bar{b}\right)\Bigg\}\{br-2n-4n^2b\}$$
$$+\bar{a}\{\;b\|Q\|^2-r-2nrb\}\;\Bigg]g(Z,U)+\bar{a}(1-b)S^2(Z,U)=0. \quad (35)$$

Theorem 4.2 *Let* $(M^{2n+1}, g),\ \ n>1$ *be an Sasakian manifold. Then for respective semi-symmetry type conditions, the Ricci tensor of the manifold* M *takes the respective forms as follows-*

Curvature condition	Expression for Ricci tensor
$R(X,Y)\cdot R=0$	$S=2ng.$
$R(X,Y)\cdot C=0$	$S=\left(\frac{r}{2n}-1\right)g-\left\{\frac{r}{2n}-(2n+1)\right\}\eta\otimes\eta.$
$R(X,Y)\cdot \hat{C}=0$	$S=\left(\frac{r}{2n}-1\right)g-\left\{\frac{r}{2n}-(2n+1)\right\}\eta\otimes\eta.$
$R(X,Y)\cdot E=0$	$S=2ng.$
$R(X,Y)\cdot P=0$	$S=2ng-\left(\frac{r}{2n}-2n-1\right)\eta\otimes\eta.$
$R(X,Y)\cdot H=0$	$S=\left(\frac{r+4n^2}{4n+1}\right)g-\left\{\frac{r-2n\,(2n+1)}{4n+1}\right\}\eta\otimes\eta.$
$E(X,Y)\cdot R=0$	$S=2ng.$
$E(X,Y)\cdot C=0$	$S=\left(\frac{r}{2n}-1\right)g-\left\{\frac{r}{2n}-(2n+1)\right\}\eta\otimes\eta.$
$E(X,Y)\cdot \hat{C}=0$	$S=\left(\frac{r}{2n}-1\right)g-\left\{\frac{r}{2n}-(2n+1)\right\}\eta\otimes\eta.$
$E(X,Y)\cdot E=0$	$S=2ng.$
$E(X,Y)\cdot P=0$	$S=2ng-\left\{\frac{r}{2n}-2\right\}\eta\otimes\eta.$
$E(X,Y)\cdot H=0$	$S=\left(\frac{r+4n^2}{4n+1}\right)g-\left\{\frac{r-2n(2n+1)}{4n+1}\right\}\eta\otimes\eta.$
$\hat{C}(X,Y)\cdot R=0$	$\left\{\frac{r}{2n}-2\right\}S=-2ng+\{r-2n\}\eta\otimes\eta+S^2.$

$\hat{C}(X,Y)\cdot\hat{C}=0$	$\{r-(2n-1)\}S = \left[\left\{\frac{r}{2n}-2n\right\}\{(r-2n)+r-\lVert Q\rVert^2\}\right]g + \left[\left\{\frac{r}{2n}-1\right\}\{2n(2n+1)-r\}-(2n+1)\{r-2n\}\right]\eta\otimes\eta$.
$\hat{C}(X,Y)\cdot C=0$	$\{r-(2n-1)\}S = \left[\left\{\frac{r}{2n}-2n\right\}\{(r-2n)+r-\lVert Q\rVert^2\}\right]g + \left[\left\{\frac{r}{2n}-1\right\}\{2n(2n+1)-r\}+\left\{\frac{r}{2n}-(2n+1)\right\}\{r-2n\}\right]\eta\otimes\eta + 2nS^2$

$\hat{C}(X,Y)\cdot E=0$	$2S=-S^2+[r+2n]g+(2n-r)\left\{\frac{r}{2n(2n+1)}-1\right\}\eta\otimes\eta$.
$\hat{C}(X,Y)\cdot P=0$	$\left\{\frac{r}{2n}-2\right\}S=-2ng+S^2+\left\{\frac{r}{2n}-1\right\}\times\left\{(2n+1)-\frac{r}{2n}\right\}\eta\otimes\eta$.
$\hat{C}(X,Y)\cdot H=0$	$\frac{(4n+1)+2n}{4n}S=\left[\left(\frac{r}{4n}+n\right)+\frac{r}{2}+\frac{1}{4n}\lVert Q\rVert^2\right]g - \left(1+\frac{1}{4n}\right)S^2+\left(\frac{r}{4n}-\frac{2n+1}{2}\right)\eta\otimes\eta$.
$P(X,Y)\cdot R=0$	$\left(\frac{2n-1}{2n}\right)S=\left(-\frac{r}{2n}+2n\right)g+\frac{1}{2n}S^2$.
$P(X,Y)\cdot\hat{C}=0$	$\left(\frac{4n^2+1}{2n}\right)S=S^2+\left\{(r-2n)+\frac{1}{2n}(r-\lVert Q\rVert^2)\right\}g + (2n+1)\left\{-2n+\left(1-\frac{r}{2n}\right)\right\}\eta\otimes\eta$.
$P(X,Y)\cdot C=0$	$\left(\frac{4n^2+1}{2n}\right)S=S^2+\left\{(r-2n)+\frac{1}{2n}(r-\lVert Q\rVert^2)\right\}g + (2n+1)\left[\left(1-\frac{r}{2n}\right)-\{r+2n\}\right]\eta\otimes\eta$.
$P(X,Y)\cdot E=0$	$\left\{\frac{2n-1}{2n}\right\}S=\left(-\frac{r}{2n}+2n\right)g+\frac{1}{2n}S^2 + \frac{1}{2n}\left(2n-\frac{r}{2n+1}\right)\left\{\frac{r}{2n}-(2n+1)\right\}\eta\otimes\eta$.
$P(X,Y)\cdot P=0$	$\left\{\frac{2n-1}{2n}\right\}S=\frac{1}{2n}S^2+\left(-\frac{r}{2n}+2n\right)g$.
$P(X,Y)\cdot H=0$	$S=\frac{1}{2n}\left(1+\frac{1}{4n}\right)S^2+\left\{n-\frac{1}{8n^2}\lVert Q\rVert^2\right\}g$.

$H(X,Y)\cdot R=0$	$\left\{\frac{2n-1}{4n}\right\}S=\left(-\frac{r}{4n}+n\right)g+\left(\frac{r}{4n}-\frac{1}{2}\right)\eta\otimes\eta+\frac{1}{4n}S^2$.
$H(X,Y)\cdot \hat{C}=0$	$\left(\frac{4n^2+1}{4n}\right)S=\frac{1}{2n}S^2+\left\{\frac{1}{2}(r-2n)+\frac{1}{4n}(r-\lVert Q\rVert^2)\right\}g$ $+\left[-\frac{r}{2}+(2n+1)\left\{\frac{2n+1}{2}-\frac{r}{4n}\right\}\right]\eta\otimes\eta$
$H(X,Y)\cdot E=0$	$\left\{\frac{2n-1}{4n}\right\}S=\left(-\frac{r}{4n}+n\right)g$ $+\left[\frac{(n+1)r}{2n(2n+1)}-\frac{r^2}{8n^2(2n+1)}+\frac{1}{2}\right]\eta\otimes\eta+\frac{1}{4n}S^2.$
$H(X,Y)\cdot P=0$	$\left\{\frac{2n-1}{4n}\right\}S=\left(-\frac{r}{4n}+n\right)g$ $+\left(-\frac{1}{2}\right)\left\{\frac{r}{2n}-(2n+1)\right\}\eta\otimes\eta+\frac{1}{4n}S^2$.
$H(X,Y)\cdot H=0$	$S=\left\{-\frac{r}{4n}-\frac{1}{8n^2}\lVert Q\rVert^2+(\frac{r}{4n}+n)\right\}g+\frac{1}{2n}\left(1+\frac{1}{4n}\right)S^2$ $+\frac{1}{2}\left\{2n+1-\frac{r}{2n}\right\}\eta\otimes\eta.$
$C(X,Y)\cdot R=0$	$\left\{\frac{r}{2n}-2\right\}S=-2ng+\{r-2n\}\eta\otimes\eta+S^2$.
$C(X,Y)\cdot \hat{C}=0$	$\{r-(2n-1)\}S=\left[\left\{\frac{r}{2n}-2n\right\}\{(r-2n)+r-\lVert Q\rVert^2\}\right]g$ $+\left[\left\{\frac{r}{2n}-1\right\}\{2n(2n+1)-r\}-(2n+1)\{r-2n\}\right]\eta\otimes\eta+2nS^2$.
$C(X,Y)\cdot P=0$	$\left\{\frac{r}{2n}-2\right\}S=+\left\{\frac{r}{2n}-1\right\}\left\{(2n+1)-\frac{r}{2n}\right\}\eta\otimes\eta-2ng+S^2$.
$C(X,Y)\cdot E=0$	$\left\{\frac{r}{2n}-2\right\}S=-2ng+\{r-2n\}\eta\otimes\eta+S^2$.
$C(X,Y)\cdot C=0$	$\{r-(2n-1)\}S=\left[\left\{\frac{r}{2n}-2n\right\}\{(r-2n)+r-\lVert Q\rVert^2\}\right]g$ $+\left[\left\{\frac{r}{2n}-1\right\}\{2n(2n+1)-r\}+\left\{\frac{r}{2n}-(2n+1)\right\}\{r-2n\}\right]\eta\otimes\eta$ $+2nS^2$

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