IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 4 Ver. V (Jul - Aug. 2015), PP 68-74
www.iosrjournals.org

Support Strong Domination IN Fuzzy GRAPH

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Abstract: *Let $G = (\sigma, \mu)$ be a fuzzy graph. Let u be an element of V. Let $N(u) = \{v \in V : \mu(uv) = \sigma(u) \wedge \sigma(v)\}$. The fuzzy support of u is defined as the sum of the neighborhood degrees of the elements in N(u). In this research work we introduce the concept of support strong domination in fuzzy graphs. The fuzzy support of a vertex is defined and domination based on the fuzzy support is considered. Several results involving this new fuzzy domination parameter are established. We also obtain the fuzzy support strong domination number $\gamma_{f(supp)}$ for several classes of fuzzy graphs.*

I. Introduction and Definitions

Fuzzy concept is introduced in Graph theory. To work on domination in Fuzzy graphs, it is necessary to have a sound knowledge of fuzzy sets, Graph Theory and Domination Theory. Formally, a fuzzy graph $G = (V, \sigma, \mu)$ is a non-empty set V together with a pair of functions $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ such that $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$, for all $x, y \in V$. σ is called the fuzzy vertex set of G and μ is called the fuzzy edge set of G.

Definition 1.1. *A fuzzy graph $G = (\sigma, \mu)$ is a set with two functions $\sigma : V \to [0, 1]$ and $\mu : E \to [0, 1]$ such that $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$, for all $x, y \in V$.*

Definition 1.2. *Let $G = (\sigma, \mu)$ be a fuzzy graph on V and $V_1 \subseteq V$. Define σ_1 on V_1 by $\sigma_1(x) = \sigma(x)$ for all $x \in V_1$ and μ_1 on the collection E_1 of two element subsets of* ^{0}2000 Mathematics Subject Classification:

V_1 by $\mu_1(xy) = \mu(xy)$ for all $x, y \in V_1$. Then (σ_1, μ_1) is called the fuzzy subgraph of G induced by V_1 and is denoted by $\langle V_1 \rangle$.

Definition 1.3. *The order p and size q of a fuzzy graph $G = (\sigma, \mu)$ are defined to be $p = \sum_{x \in V} \sigma(x)$ and $q = \sum_{xy \in E} \mu(xy)$.*

Definition 1.4. *The effective degree of a vertex u is defined to be the sum of the weights of the effective edges incident at u and is denoted by dE(u).*

Definition 1.5. *Let $G = (V, E)$ be a graph. A subset S of V is called a dominating set in G if every vertex in $V \setminus S$ is adjacent to some vertex in S.*

Remark 1.6. *The domination number of G is the minimum cardinality taken over all dominating sets in G and is denoted by $\gamma(G)$ or γ.*

Definition 1.7. *Let x be an element of V. Let $N(x) = \{y \in V : \mu(xy) = \sigma(x) \wedge \sigma(y)\}$.*

The fuzzy support of u is defined as the sum of the neighbourhood degrees of the elements in N(u). That is $fuzzy\ supp(v) = \sum_{u \in N(v)} \sigma(u)$.

Definition 1.8. *Let $u, v \in V(G)$. u is said to fuzzy support strong dominate v if $\mu(uv) = \sigma(u) \wedge \sigma(v)$ and $fuzzy\ supp(u) \geq fuzzy\ supp(v)$.*

A subset D of V(G) is called a fuzzy support dominating set if for every $v \in V - D$ there exists $u \in D$ such that u fuzzy support strong dominates v.

Observation 1.9.
1. *If v is an isolated vertex, then fuzzy supp(v)=0.*
2. *If v is pendent vertex, then fuzzy supp(v)= deg(u), where u is the fuzzy supportof v.*
3. *A graph G is said to be fuzzy support regular if fuzzy supp(v) is constant, forall $v \in V$.*

Example: $K_{n;n}, K_n, C_n$.

4. *Every k-regular graph is a K^2-fuzzy support regular, but not the converse. A fuzzy support regular graph need to be regular.*

Example: $K_{1;n}, K_{m;n}$

II. Φ- Fuzzy Support Strong Domination

Let $\Phi : V \to R$ be a function which associates values to the vertices according to the influence enjoyed by the vertex. For any $v \in V$, the open Φ-Fuzzy Support Φ- fuzzy supp(v) is defined as Φ- fuzzy supp $(v) = \sum_{u \in N(v)} \Phi(u)$ and the closed Φ-Fuzzy Support Φ- fuzzy supp[v] is defined as Φ- fuzzy supp $[v] = \sum_{u \in N(v)} \Phi(u)$. u is said to open (closed) Φ-fuzzy support strong dominate v if $uv \in \mu(uv) = \sigma(u) \wedge \sigma(v)$ and Φ- fuzzy supp(u) ≥ Φ- fuzzy supp(v) (Φ-fuzzy supp[u]≥Φ- fuzzy supp[v]).

A subset D of V is said to open (closed)Φ- fuzzy support strong dominate v if for every $u \in V - D$, *there exists* $v \in D$ such that v is open (closed) Φ- fuzzy support strong dominates u.

The minimum cardinality of a Φ- fuzzy support strong open (closed) dom-inating set of G is called a Φ- fuzzy support strong open (closed) domination number of G and is denoted by $\gamma_{f(\Phi - supp)}[G]$.

Definition 2.1. *Let $G = (\sigma, \mu)$ be a fuzzy graph. Let $v \in V(G)$. Let $\Phi : V \to R$ be a map , the Φ-K-fuzzy support of v denoted by $\Phi_{f(supp)}(v)$ is defined as $\sum_{u \in N_k(v)} \Phi(u)$, where $N_k(v) = \{x \in V(G) : d(x, v) \leq k\}$. Let $u, v \in V(G)$. The vertex v Φ-k-fuzzy support strong dominates the vertex u if $uv \in \mu(G) = \{\mu(uv) = \sigma(u) \wedge \sigma(v)\}$ and $\Phi_{f(supp)}(v) \geq \Phi_{f(supp)}(u)$.*

A subset D of V is said to be a Φ-k-fuzzy support strong dominating set of G if for every $u \in V - D$, there exists a vertex $v \in D$ such that v Φ-k-fuzzy support strong dominates u.

The minimum cardinality of Φ-k-fuzzy support strong dominating set of Gis called a Φ-k-fuzzy support strong domination number of G and is denoted by $\gamma_{f(\Phi - supp)}(G)$.

Definition 2.2. *A fuzzy support strong (weak) dominating set of G is minimal if noproper subset of D is a fuzzy support strong (weak) dominating sets of G.*

Definition 2.3. *The minimum cardinality of a minimal fuzzy support strong(weak) dominating set of G is called the upper fuzzy support strong(weak) domination number of G and is denoted by $\gamma_{f(supp)}(G)(\gamma_{f(supp)}(G))$.*

Definition 2.4. *The minimum cardinality of a minimal fuzzy support (weak) domi- nating set of G is called the fuzzy support strong(weak) domination number of G and is denoted by $\Gamma_{f(supp)}(G)(\Gamma_{f(supp)}(G))$.*

Remark 2.5. *In a fuzzy support regular graph G,* $\Gamma_{f(supp)}(G) = \gamma(G)$.

Definition 2.6. *The (open) fuzzy support strong neighbourhood of a vertex v, denoted by* $N_{f(supp)}(v)$ *is defined as* $N_{f(supp)}(v) = \{u \in N(v) : fuzzy\ supp(u) \geq fuzzy\ supp(v)\}$

Definition 2.7. *The (open) fuzzy support weak neighbourhood of a vertex v, denoted by* $N_{f(supp}(v)$ *is defined as* $N_{f(supp}(v) = \{u \in N(v) : fuzzy\ supp(u) \leq fuzzy\ supp(v)\}$. *The (closed) fuzzy support weak neighbourhood of a vertex v, denoted by* $N_{f(supp)}[v]$ *is defined as* $N_{f(supp)}(v) \cup \{v\}$.

Fuzzy support strength and minimum (maximum) fuzzy support degrees of a graph are defined in the following:

$$fuzzy\ support\ strength\ of\ V = deg^{s}_{f(supp)}(v) = |N^{w}_{f(supp)}(v)|$$

$$\delta^{s}_{f(supp)}(G) = \min_{v \in V(G)} \{deg^{s}_{f(supp)}(v)\}$$

$$\Delta^{s}_{f(supp)}(G) = \max_{v \in V(G)} \{deg^{s}_{f(supp)}(v)\}$$

Definition 2.8. *A vertex u* $\in V(G)$ *is said to be a fuzzy support strong isolate of G if* $fuzzy\ supp(u) > fuzzy\ supp(v)$, *for every* $u \in N(v)$.

If G has a fuzzy support strong isolate vertex, then $\delta^{w}_{f(supp)}(G) = 0$.

Definition 2.9. *A vertex u* $\in V(G)$ *is said to be anti fuzzy support strong isolate (or, fuzzy support weak isolate) of G if* $fuzzy\ supp(u) < fuzzy\ supp(v)$, *for every* $u \in N(v)$.

If G has an anti fuzzy support strong isolate vertex, then $\delta^{s}_{f(supp)}(G) = 0$.

Definition 2.10. *A subset D of V is called a total fuzzy support strong dominating set of G if* $\bigcup_{v \in D} N_{f(supp)}(v) = V$.

Remark 2.11. *Since any super set of a fuzzy support strong dominating set is also a fuzzy support strong dominating set, the class of all fuzzy support strong dominating sets of G has super hereditary property. Hence a fuzzy support strong dominating set D is minimal if and only if it is 1-minimal. That is, for every u* $\in D$, *D-{u} is not a fuzzy support strong dominating set.*

Definition 2.12. *Let* $S \subseteq V$. *Let* $v \in S$. *The fuzzy support strong private neighbour of a vertex v with respect to a set S, denoted by fuzy suppn[v,S] is defined as,*

$$\begin{aligned} fuzzy\ suppn[v,S] &= \{x \in S : x \in N^{w}_{f(supp)}[v]\ and\ x \notin N^{w}_{f(supp)}(u), \forall u \in S - \{v\}\} \\ &= N^{w}_{f(supp)}[v] - N^{w}_{f(supp)}[S - \{v\}] \end{aligned}$$

III. Main Results

Observation 3.1. *Let* $G = (\sigma, \mu)$ *be a fuzzy graph. Let* $u \in V(G)$. *If* $\langle N[u] \rangle$ *is complete, then for every* $v \in N(u)$, $fuzzy\ supp(v) \geq fuzzy\ supp(u)$.

$$\mathit{fuzzy\ supp}(v) = \sum_{x \epsilon N[u]; x=v} \sigma(x) + \sum_{x \epsilon N(v)-N[v]} \sigma(x)$$

$$= |N(u)| + \sum_{x \epsilon N[u]; x=v} \sigma(x) + \sum_{x \epsilon N(v)-N[u]} \sigma(x)$$

$$\text{Fuzzysupp(u)} = \sum_{x \epsilon N(u)} \sigma(x)$$

$$= \sum_{x \epsilon N(u); x=u} \sigma(x) + \sigma(v)$$

$$= \sum_{x \epsilon N(u); x=u} \sigma(x) + |N(u)| + |N(v) - N[u]|$$

$$\leq \sum_{x \epsilon N(u); x=v} \sigma(x) + |N(u)| + \sum_{x \epsilon N(v)-N[u]} \sigma(x)$$

$$= \mathit{fuzzy\ supp}(v)$$

Observation 3.2. *Let $G = (\sigma, \mu)$ be a fuzzy graph. Let u be a full degree vertex of G. Then fuzzy supp(u) $\geq$ fuzzy supp(v), for all $v \epsilon V(G)$.*

Proof. : Let $v \epsilon N(u)$. Let v be not adjacent to vertices $w_1, w_2, ..., w_k (k \geq 0)$.

$$\text{fuzzy supp(u)} = \sum_{x \epsilon N(v)} \sigma(x)$$

$$= \sum_{x \epsilon \{w_1; w_2; ::::; w_k; u; v\}} \sigma(x) + \sigma(u)$$

$$= \sum_{x \epsilon \{w_1; w_2; ::::; w_k; u; v\}} \sigma(x) + n - 1$$

$$\mathit{fuzzy\ supp}(u) = \sum_{x \epsilon V - \{u\}} \sigma(x)$$

$$= \sum_{x \epsilon \{w_1; w_2; ::::; w_k; u; v\}} \sigma(x) + \sigma(w_1) + \sigma(w_2) + ... + \sigma(w_k) + \sigma(v)$$

$$\geq \sum_{x \epsilon \{w_1; w_2; ::::; w_k; u; v\}} \sigma(x) + \sigma(v) + k$$

$$= \sum_{x \epsilon \{w_1; w_2; ::::; w_k; u; v\}} \sigma(x) + n - k - 1 + k$$

$$= \sum_{x \epsilon \{w_1; w_2; ::::; w_k; u; v\}} \sigma(x) + n - 1$$

$$= \mathit{fuzzy\ supp}(v).$$

Corollary 3.3. *If G has a full degree vertex, then $\gamma_{f(supp)}(G) = 1$.* ^{s}

Definition 3.4. *A subset S of V is called a fuzzy support strong set of G if for everyu ϵ S, fuzzy supp(u) $\geq$ fuzzy supp(v), for all $v \epsilon N(u)$.*

Definition 3.5. *A subset S of V is called a fuzzy support weak set of G if for everyu ϵ S, fuzzy supp(u) $\leq$ fuzzy supp(v), for all $v \epsilon N(u)$.*

Definition 3.6. *A subset S of V is called a fuzzy support strong independent set ofG if S is independent and S is fuzzy support strong. The maximum cardinality of a fuzzy support strong independent of G is denoted by $\beta_{f(supp-s)}(G)$.*

Definition 3.7. *A subset S of V is called a fuzzy support weak independent set of G if S is independent and S is fuzzy support weak. The maximum cardinality of a fuzzy support weak independent of G is* *denoted by* $\beta_{f(supp-w)}(G)$.

Definition 3.6. *A subset S of V is called a fuzzy support strong independent set ofG if S is independent and S is fuzzy support strong. The maximum cardinality of a fuzzy support strong independent of G is denoted by* $\beta_{f(supp-s)}(G)$.

Definition 3.7. *A subset S of V is called a fuzzy support weak independent set of G if S is independent and S is fuzzy support weak. The maximum cardinality of a fuzzy support weak independent of G is denoted by* $\beta_{f(supp-w)}(G)$.

Definition 3.8. *A vertex v is fuzzy support strong (weak) if* $fuzzy\ supp(v) \geq fuzzy\ supp(u),\ \forall v \in N(u)$

$(fuzzy\ supp(v) \leq fuzzy\ supp(u),\ \forall v \in N(u))$.

Definition 3.9. *A vertex v is fuzzy support balanced, if it is neither fuzzy support strong nor fuzzy support weak.*

Definition 3.10. *A vertex v is fuzzy support regular, if fuzzy supp(v)= fuzzy supp(u),* $\forall v \in N(u)$.

Definition 3.11. *A subset S of V is called fuzzy support balanced (fuzzy support regular) if every vertex of S is fuzzy support balanced (fuzzy support regular).*

Definition 3.12. *The maximum Cardinalities of a fuzzy support strong, fuzzy support weak, fuzzy support balanced and fuzzy support regular set of G are respectively denoted by* $fuzzy\ S_{st}(G), fuzzy\ S_{wk}(G), fuzzy\ S_b(G), fuzzy\ S_r(G)$.

Definition 3.13. *Let* $S_{f(supp)}, W_{f(supp)}, B_{f(supp)}$ *and* $R_{f(supp)}$ *denote respectively, the maximum fuzzy support strong, fuzzy support weak, fuzzy support balanced and fuzzy support regular sets of G. Then*

$$V(G) = S_{f(supp)} \cup W_{f(supp)} \cup B_{f(supp)}$$
$$R_{f(supp)} = S_{f(supp)} \cap W_{f(supp)}$$

If G is fuzzy support regular, then

$$B_{f(supp)} = S_{f(supp)} = W_{f(supp)} = R_{f(supp)}$$
$$Let,\ C_{f(supp)} = S_{f(supp)} - R_{f(supp)}$$
$$and\ D_{f(supp)} = W_{f(supp)} - R_{f(supp)}$$
$$Then V = C_{f(supp)} \cup D_{f(supp)} \cup B_{f(supp)} \cup R_{f(supp)}$$

where $C_{f(supp)}, D_{f(supp)}, R_{f(supp)}$ *and* $B_{f(supp)}$ *are all disjoint.*

Observation 3.14. *Let* $G = (\sigma, \mu)$ *be a fuzzy graph with n vertices. Then* $n = fuzzyS_{st}(G) + fuzzyS_{wk}(G) + fuzzyS_b(G) - fuzzyS_r(G)$

Proof. :

$$
\begin{aligned}
|B_{f(supp)}| &= |(S_{f(supp} \cup W_{f(supp)})^C| \\
fuzzyS_b(G) &= |V(G)| - |(S_{f(supp)} \cup W_{f(supp)})| \\
&= n - [|S_{f(supp)}| + |W_{f(supp)}| - |(S_{f(supp)} \cap W_{f(supp)})|] \\
&= n - [fuzzyS_{st}(G) + fuzzyS_{wk}(G) - R_{f(supp)}(G)] \\
&= n - [fuzzyS_{st}(G) + fuzzyS_{wk}(G) - fuzzyS_r(G)] \\
Therefore,\ n &= fuzzyS_{st}(G) + fuzzyS_{wk}(G) + fuzzyS_b(G) - fuzzyS_r(G)
\end{aligned}
$$

Definition 3.15. *A subset S of V(G) is a fuzzy support strong (weak) vertex cover if every edge e=xy in E(G) is incident with a vertex of S and fuzzy support of any vertex*

u of S is greater(less) than or equal to the fuzzy support of any v in $N(u) \cap (V - S)$.

The minimum cardinality of a fuzzy support strong (weak) vertex cover of fuzzy graph G is denoted by $\alpha_0^{f(s-supp)}(G)(\alpha_0^{f(w-supp)}(G))$.

Theorem 3.16. *For any subset S of V, S is a fuzzy support strong independent set if and only if V-S is a fuzzy support weak vertex cover.*

Proof. : Let S be a fuzzy support strong independent set.

To prove that, V-S is a fuzzy weak vertex cover.

Let E(G)=uv be an edge of G. Let without loss if generality, $fuzzy\ supp(u) \geq fuzzy\ supp(v)$.

Since S is an independent set, u and v both cannot belong to S. Therefore u or vϵ V-S. Let xϵ V-S. To show that, $fuzzy\, supp(x) \leq fuzzy\, supp(y)$, $\forall y\ \epsilon N(x) \cap S$.

Since $y\ \epsilon\ S$, $fuzzy\ supp(y) \geq fuzzy\ supp(x)$. Hence the claim. Conversely, Suppose V-S is a fuzzy support weak vertex over. Let $x, y\ \epsilon S$.

If x, y are adjacent, then V-S is not a fuzzy support weak vertex cover. Which is a contradiction. Therefore S is independent. Let $x\ \epsilon\ S$ and $xy\ \epsilon\ E(G)$. Then

$y\ \epsilon V - S$. Therefore $fuzzy\, supp(x) \geq fuzzy\ supp(y)$

Therefore S is a fuzzy support strong independent set.

Theorem 3.17. *For any fuzzy graph G,* $\frac{n}{1+\Delta^s_{f(supp)}(G)} \leq \gamma^s_{f(supp)}(G) \leq n - \Delta^s_{f(supp)}(G)$

Proof. : Let u be a vertex in V(G) with fuzzy support strength $\Delta^s_{f(supp)}(G)$.

Then $V - N^w_{f(supp)}(u)$ fuzzy support strong dominates G. Therefore Each vertex v can fuzzy support $\gamma_f(G) \leq |V - N^w_{f(supp)}(G)| = n - \Delta^s_{f(supp)}(G)$. dominate atmost itself and $\Delta^s_{f(supp)}(G)$ vertices. Therefore at least $\frac{n}{1+\Delta^s_{f(supp)}(G)}$ vertices are required to fuzzy support strong dominate G. Therefore $\gamma^s_{f(supp)}(G) \geq \frac{n}{1+\Delta^s_{f(supp)}(G)}$ Hence the theorem.

IV. Applications

There are many origins to the domination theory. The earliest ideas of dominating sets date back, to the origin of game of Chess in India. In this game, one studies of chess pieces which cover various opposing pieces or various squares of the board.

Besides this paper also contained application to Surveillance networks and game theory.

In society as well as in administration, the influence of the individual depends on the strength that he derives from his supporter. In times of made the individual has to depend more on his supporter, than on himself.

In the fuzzy graph model, as influence function may be defined on the vertex set which gives a measure of the influence of the vertices. The fuzzy support of a vertex then, will be given by sum of the influences of the neighbours of the vertex.

Domination using the fuzzy support strength may be defined by adjacency and superiority of the fuzzy support strength.

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