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Effect of Viscous dissipation on Heat Transfer of Magneto-Williamson Nanofluid

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Abstract: *This study investigates the effect of the nano particle effect on magnetohydrodynamic boundary layer flow over a stretching surface with the effect of viscous dissipation. The governing partial differential equations are transformed to a system of ordinary differential equations and solved numerically using fifth order Runge-Kutta method integration scheme and Matlab bvp4c solver. The effects of the Non-Newtonian Williamson parameter, Prandtl number, Lewis number, the diffusivity ratio parameter, heat capacities ratio parameter, Eckert number, Schmidt number on the fluid properties as well as on the skin friction and Nusselt number coefficients are determined and shown graphically.*

Keywords: *MHD, nanoparticle, viscous dissipation, Williamson fluid model, heat transfer.*

I. Introduction

Nanoparticles have one dimension that measures 100 nanometers or less. The properties of a lot of conventional materials change when formed from nanoparticles. This is typically because nanoparticles contain a greater surface area per weight than larger particles which causes them to be more reactive to some other molecules. Choi [1] investigated the theoretical learn of the thermal conductivity of nanofluids with Copper nanophase meterials and he estimated the potential profit of the fluids and also he shown that one of the benefits of nanofluids will be dramatic reductions in heat exchanger pumping power. The characteristic feature of nanofluids is thermal conductivity enhancement, a phenomenon observed by Masuda et al. [2]. This phenomenon suggests the opportunity of using nanofluids in advanced nuclear systems [3]. A comprehensive survey of convective transport in nanofluids was made by Buongiorno [4], who says that a satisfactory explanation for the abnormal increase of the thermal conductivity and viscosity is yet to be found. He focused on added heat transfer enhancement observed in convective situations. Kuznetsov and Nield [5] have examined the influence of nanoparticles on natural convection boundary-layer flow past a vertical plate using a model in which Brownian motion and thermophoresis are accounted for. The authors have assumed the simplest possible boundary conditions, namely those in which both the temperature and the nanoparticle fraction are constant along the wall. Furthermore, Nield and Kuznetsov [6, 7] have studied the Cheng and Minkowycz [8] problem of natural convection past a vertical plate in a porous medium saturated by a nanofluid and used for the nanofluid incorporates the effects of Brownian motion and thermophoresis for the porous medium.

The problem of viscous flow and heat transfer over a stretching sheet has important industrial applications, for example, in metallurgical processes, such as drawing of continuous filaments through quiescent fluids, annealing and tinning of copper wires, glass blowing, manufacturing of plastic and rubber sheets, crystal growing, and continuous cooling and fiber spinning, in addition to wide-ranging applications in many engineering processes, such as polymer extrusion, wire drawing, continuous casting, manufacturing of foods and paper, glass fiber production, stretching of plastic films, and many others. During the manufacture of these sheets, the melt issues from a slit and is subsequently stretched to achieve the desired thickness. The final product with the desired characteristics strictly depends upon the stretching rate, the rate of cooling in the process, and the process of stretching. In view of these applications, Sakiadis [9, 10] investigated the boundary layer on a moving continuous flat surface and a moving continuous cylindrical surface, for both laminar and turbulent flow in the boundary layer. Syahira Mansur and Anuar Ishak [11] found that the local Nusselt number and the local Sherwood number as well as the temperature and concentration profiles for some values of the convective parameter, stretching/shrinking parameter, Brownian motion parameter, and thermophoresis parameter. Nadeem and Hussain [12] analyze the nano particle effect on boundary layer flow of Williamson fluid over a stretching surface.

Dissipation is the process of converting mechanical energy of downward-flowing water into thermal and acoustical energy. Viscous dissipation is of interest for many applications. Significant temperature rises are observed in polymer processing flows such as injection modelling or extrusion at high rates. Aerodynamic heating in the thin boundary layer around high speed aircraft raises the temperature of the skin. In a completely different application, the dissipation function is used to characterize the viscosity of dilute suspensions Einstein [13]. Viscous dissipation for a fluid with suspended particles is equated to the viscous dissipation in a pure Newtonian fluid, both being in the same flow (same macroscopic velocity gradient). Vajravelu and Hadjinicolaou [14] studied the heat transfer characteristics over a stretching surface with viscous dissipation in the presence of internal heat generation or absorption. Recently, Yohannes et al [15] analyzes the thermal boundary layer thickness increases with increasing the values of Eckert number. More recently, Zaimi et al. [16] presents a similarity solution of the boundary layer flow and heat transfer over a nonlinearly stretching/shrinking sheet immersed in a nanofluid with suction effect.

However, the interactions of magnetohydrodynamic boundary layer flow on heat transfer of Williamson nano fluid flow with viscous dissipation. The governing boundary layer equations have been transformed to a two-point boundary value problem in similarity variables and the resultant problem is solved numerically using the fourth order Runge-Kutta method along with shooting technique. The effects of various governing parameters on the fluid velocity, temperature, nanoparticle volume friction, reduced Nusselt number and nanoparticle volume friction gradient are shown in figures and analyzed in detail.

II. Mathematical Formulation

Let us consider the two-dimensional steady flow of an incompressible, viscous dissipative nano Williamson fluid over a stretching surface. The plate is stretched along x-axis with a velocity *Bx*, where *B>0* is stretching parameter. The fluid velocity, temperature and nanoparticle concentration near surface are assumed to be *Uw*, *Tw* and *Cw*, respectively.

For Williamson fluid model is defined in (Dapra [17]) as

$$\tau = \left[\mu_0 + \frac{(\mu_0 - \mu_\infty)}{1 - \Gamma \dot{\gamma}} \right] \Lambda_1 \qquad (2.1)$$

where τ is extra stress tensor, μ_0 is limiting viscosity at zero shear rate and μ_∞ is limiting viscosity at infinite shear rate, $\Gamma > 0$ is a time constant, Λ_1 is the first Rivlin–Erickson tensor and $\dot{\gamma}$ is defined as follows:

$$\dot{\gamma} = \sqrt{\frac{1}{2}\pi} \qquad (2.2)$$

$$\pi = trace\left(\Lambda_1^2\right) \qquad (2.3)$$

Here we considered the case for which $\mu_\infty = 0$ and $\Gamma \dot{\gamma} < 1$. Thus Eq. (2.1) can be written as

$$\tau = \left[\frac{\mu_0}{1 - \Gamma \dot{\gamma}} \right] \Lambda_1 \qquad (2.4)$$

or by using binomial expansion we get

$$\tau = \mu_0 \left[1 + \Gamma \dot{\gamma} \right] \Lambda_1 \qquad (2.5)$$

The above model reduces to Newtonian for $\Gamma = 0$

The equations governing the flow are

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2.6)$$

Momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \sqrt{2}\nu\Gamma \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} \qquad (2.7)$$

Energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\rho_p c_p}{\rho c}\left[D_B \frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2 \right] + \frac{\mu}{\rho c}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (2.8)$$

Volumetric species equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{2.9}$$

The boundary conditions are

$$u=U_w, v=v_w, T=T_w, C=C_w \quad \text{at} \quad y=0$$
$$u\to 0, T\to T_\infty, C\to C_\infty \quad \text{as} \quad y\to\infty \tag{2.10}$$

Since the surface is stretched with velocity *Bx*, thus $U_w = Bx$ and u and *v* are horizontal and vertical components of velocity, ν is kinematic viscosity, v_w is the suction or injuction velocity with $v_w<0$ for suction and $v_w>0$ for injection. α is the nanofluid thermal diffusivity. ρ is nanofluid density, ρc and $\rho_p c_p$ are heat capacities of nanofluid and nanoparticles, respectively, *T* is temperature, *k* is nanofluid thermal conductivity, D_B is Brownian diffusion coefficient, *C* is nanoparticle volumetric fraction, D_T is thermophoretic diffusion coefficient and T_∞ is the ambient fluid temperature.

In order to transform the equations (2.6) to (2.10) into a set of ordinary differential equations, the following similarity transformations and dimensionless variables are introduced.

$$u=Bxf'(\eta), v=-\sqrt{B\nu}\,f(\eta), \eta=y\sqrt{\frac{B}{\nu}}$$
$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}, \mathrm{Pr}=\frac{\nu}{\alpha}$$
$$Le=\frac{\alpha}{D_B}, \lambda=\Gamma x\sqrt{\frac{2B^3}{\nu}}, Sc=\frac{\nu}{D_B}, Ec=\frac{U_w}{c(T_w-T_\infty)} \tag{2.11}$$
$$Nc=\frac{\rho_p c_p(C_w-C_\infty)}{\rho c}, Nt=\frac{D_B T_\infty(C_w-C_\infty)}{D_T(T_w-T_\infty)}$$

where $f(\eta)$ is the dimensionless stream function, θ - the dimensionless temperature, ϕ - the dimensionless nanoparticle volume fraction, η - the similarity variable, λ - the Non-Newtonian williamson parameter, Le - the Lewis number, Nc - the heat capacities ratio, Nt - the diffusivity ratio, *Ec*-the Eckert number, *Pr* - the Prandtl number, Sc - the Schmidt number.

In view of the equation (2.11), the equations (2.7) to (2.10) transform into

$$f'''+ff''-f'^2+\lambda f''f'''=0 \tag{2.12}$$
$$\theta''+\mathrm{Pr}\,f\theta'+\frac{Nc}{Le}\theta'\phi'+\frac{Nc}{LeNt}\theta'^2+Ecf''^2=0 \tag{2.13}$$
$$\phi''+Scf\phi'+\frac{1}{Nt}\theta''=0 \tag{2.15}$$

The transformed boundary conditions can be written as

$$f=S, f'=1, \theta=1, \phi=1 \quad \text{at} \quad \eta=0$$
$$f'\to 0, \theta\to 0, \phi\to 0 \quad \text{as} \quad \eta\to\infty \tag{2.16}$$

Where the constant parameter $S=-v_w/\sqrt{B\nu}$ corresponds to the suction (S>0) and injection (S<0) or the withdrawal of the fluid, respectively.

If we put $\lambda=0$, our problem reduces to the one for Newtonian nano and for $D_B = D_T = 0$ in Eq. (2.8) our heat equation reduce to the classical boundary layer heat equation in the absence of viscous dissipation. Physical quantities of interest are Local skin friction coefficient C_f, Local Nusselt number Nu and Local Sherwood number Sh.

$$C_f = \frac{\tau_w}{\rho U_w^2}, Nu = \frac{-x}{(T_w - T_\infty)}\left(\frac{\partial T}{\partial y}\right)_{y=0}, Sh = \frac{-x}{(C_w - C_\infty)}\left(\frac{\partial C}{\partial y}\right)_{y=0} \quad (2.17)$$

or by introducing the transformations (15), we have

$$\sqrt{\mathrm{Re}}C_f = \left[\left(f'' + \frac{\lambda}{2} f''^2\right)\right]_{\eta=0}, \frac{Nu}{\sqrt{\mathrm{Re}}} = -\theta'(0), \frac{Sh}{\sqrt{\mathrm{Re}}} = -\phi'(0) \quad (2.18)$$

Where $\mathrm{Re} = \frac{Bx^2}{\nu}$ is the local Reynolds number.

III. Solution Of The Problem

The set of non-linear coupled differential Eqs. (2.12)-(2.15) subject to the boundary conditions Eq. (2.16) constitute a two-point boundary value problem. In order to solve these equations numerically we follow most efficient numerical shooting technique with fifth-order Runge-Kutta-integration scheme. In this method it is most important to choose the appropriate finite values of $\eta \to \infty$. To select η_∞ we begin with some initial guess value and solve the problem with some particular set of parameters to obtain f'', θ' and ϕ'. The solution process is repeated with another large value of η_∞ until two successive values of f'', θ' and ϕ' differ only after desired digit signifying the limit of the boundary along η. The last value of η_∞ is chosen as appropriate value of the limit $\eta \to \infty$ for that particular set of parameters. The four ordinary differential Eqs. (2.12)-(2.15) were first formulated as a set of seven first-order simultaneous equations of seven unknowns following the method of superposition [18]. Thus, we set

$$y_1 = f, y_2 = f', y_3 = f'', y_4 = \theta, y_5 = \theta', y_6 = \phi, y_7 = \phi'$$

$$y_1' = y_2, y_2' = y_3$$

$$y_1(0) = S, y_2(0) = 1$$

$$y_3' = \frac{1}{1 + \lambda y_3}\left(y_2^2 - y_1 y_3\right)$$

$$y_3(0) = \delta_1$$

$$y_4' = y_5, y_4(0) = 1$$

$$y_5' = -\left[\Pr y_1 y_5 + Ec y_3^2 + \frac{Nc}{Le} y_5 y_7 + \frac{Nc}{LeNt} y_5^2\right]$$

$$y_5(0) = \delta_2$$

$$y_6' = y_7, y_6(0) = 1$$

$$y_7' = -\left[Sc y_1 y_6 + \frac{1}{Nt} y_5'\right]$$

$$y_7(0) = \delta_3$$

Eqs. (2.12)-(2.15) then reduced into a system of ordinary differential equations, i.e., where δ_1, δ_2 and δ_3 are determined such that it satisfies $y_2(\infty) \to 0, y_4(\infty) \to 0$ and $y_6(\infty) \to 0$. The shooting method is used to guess δ_1, δ_2 and δ_3 until the boundary conditions $y_2(\infty) \to 0, y_4(\infty) \to 0$ and $y_6(\infty) \to 0$ are satisfied. Then the resulting differential equations can be integrated by fourth-order Runge-Kutta scheme. The above procedure is repeated until we get the results up to the desired degree of accuracy, 10^{-6}

IV. Results And Discussion

In order to get a clear insight of the physical problem, the velocity, temperature and nanoparticle volume friction have been discussed by assigning numerical values to the governing parameters encountered in the problem. Numerical computations are shown from figs.1-13.

Figs. 1(a)-(c) shows the effect of the Non-Newtonian Williamson parameter on the velocity, temperature and mass volume fraction profiles. It is observed that the velocity of the fluid decreases with an increase the Non-Newtonian Williamson parameter and temperature of the fluid as well as mass volume friction of the fluid increases the Non-Newtonian Williamson parameter. Figs. 2(a)-(c) shows the effect of the suction parameter on the velocity, temperature and mass volume fraction profiles. It is observed that the velocity of the fluid decreases with an increase the suction parameter and temperature of the fluid as well as mass volume friction of the fluid increases the suction parameter. The effect of viscous dissipation on temperature and mass volume friction is shown in figs. 3(a)&(b). Increases in Ec the temperature of the fluid is decreases as well as opposite results were found in mass volume friction.

The effect of Nc on temperature and mass volume friction is shown in figs. 4(a)&(b). An increase in Nc the temperature of the fluid is increases as well as opposite results were found in mass volume friction. From fig. 5(a) & 5(b) show that the effect of Lewis number (Le) on temperature and mass volume friction. Since Lewis number is the ratio of nanoparticle thermal diffusivity to Brownian diffusivity. It is observe that the temperature of the fluid decreases where as mass volume friction increases with an increase in the Lewis number. The effect of the Prandtl number (Pr) on temperature is shown in fig.6. Since the Prandtl number is the ratio of momentum diffusivity to the nanofluid thermal diffusivity. It is noticed that temperature of the fluid increases with an increases the Prandtl number. The variation of mass volume friction verses diffusivity ratio parameter (Nt) is plotted in fig. 7. Since the diffusivity ratio is the ratio of Brownian diffusivity to the thermophoretic diffusivity. It is seen that as Nt increases the mass volume friction of the fluid decreases. From fig.8 show that the variation of the Schmidt number (Sc) to the nanoparticle volume friction. Since Schmidt number is the ratio of the momentum diffusivity to Brownian diffusivity. It is seen that nanoparticle volume friction decreases with increases the Schmidt number.

Fig.9 shows the effects of S and λ on skin friction. From fig.9 it is seen that the skin friction increases with an increase S or λ. The variations of Ec and λ on reduced Nusselt number is shown in fig.10. It is observed that the reduced Nusselt number increases with an increase the parameter Ec and decrease with an increasing the parameter λ. The effect of Ec and λ on Sherwood number is shown in fig.11. it is found that the Sherwood number reduces with an increase in the parameters Ec or λ. Fig.12 shows the effects of Nt and Nc on local Nusselt number. From fig.12 it is seen that the local Nusselt number increases with an increase Nt whereas decreases with the influence of Nc. The variations of Nt and Nc on reduced Sherwood number is shown in fig.13. It is observed that the reduced Sherwood number increases with an increase the parameter Nt or Nc. Table 1 is shows to compare our results for the viscous case in the absence of nanoparticles and viscous dissipation. These results are found to be in good agreement.

V. Conclusions

In this paper numerically investigated the nanoparticle effect on magnetohydrodynamic boundary layer flow of Williamson fluid over a stretching surface in the presence of viscous dissipation. The important findings of the paper are:

- The velocity of the fluid decreases with an increase of the Non-Newtonian Williamson parameter.
- The fluid temperature and mass volume friction increases with the influence of Non-Newtonian Williamson parameter.
- The nanoparticle volume friction enhances the viscous dissipation.
- The influence of heat capacities ratio parameter or viscous dissipation reduces the heat transfer coefficient.
- nanoparticle volume friction gradient enhances the diffusivity ratio parameter and heat capacities ratio parameter.

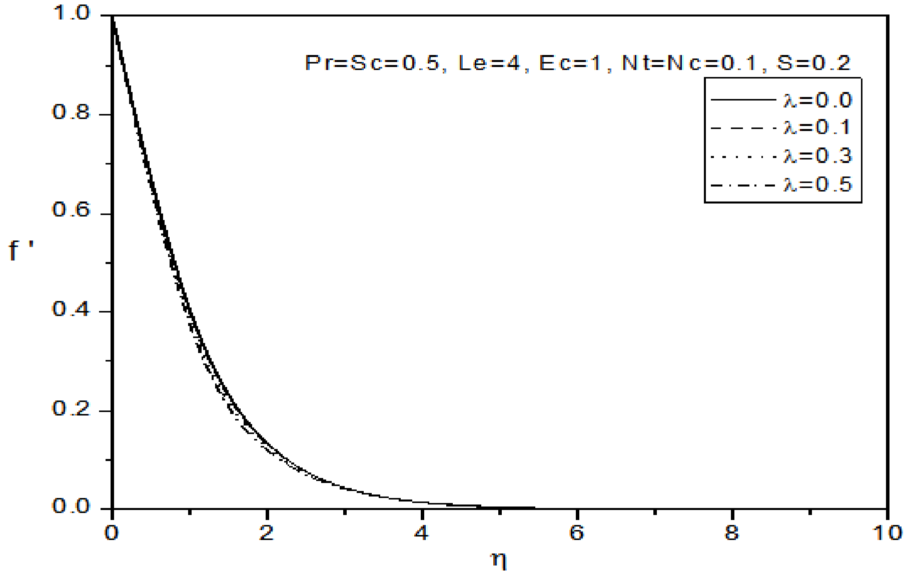

Fig.1(a) Velocity for different values of λ

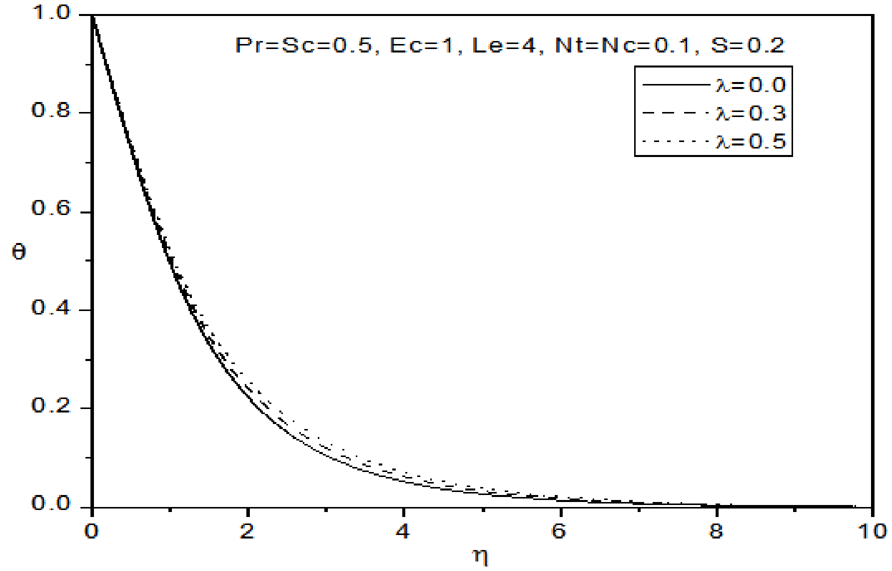

Fig.1(b) Temperature for different values of λ

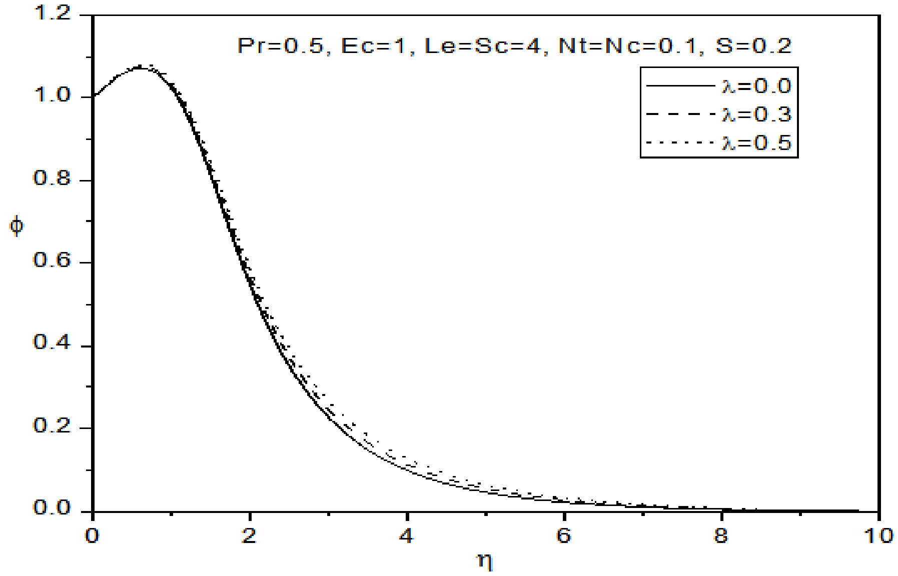

Fig.1(c) Nanoparticle volume fraction for different values of λ

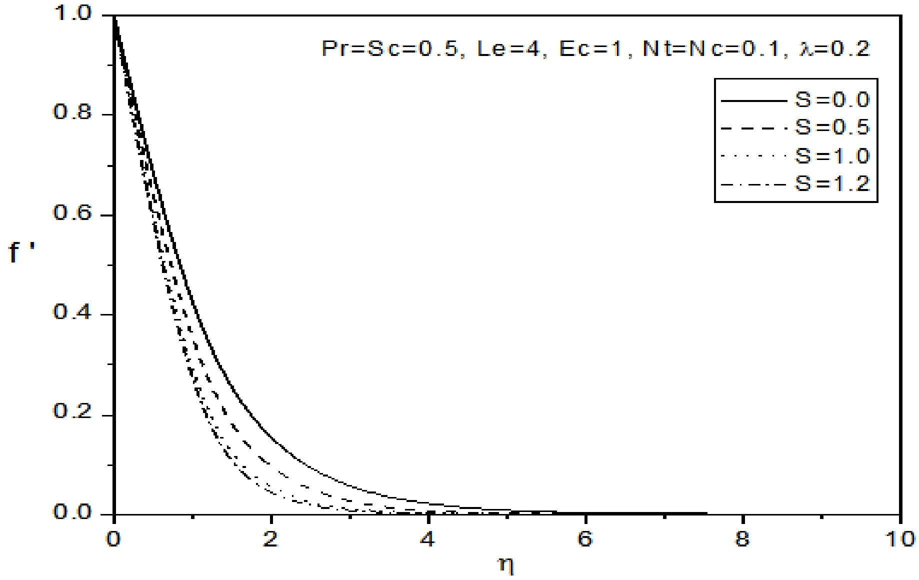

Fig.2 (a) Velocity for different values of *S*

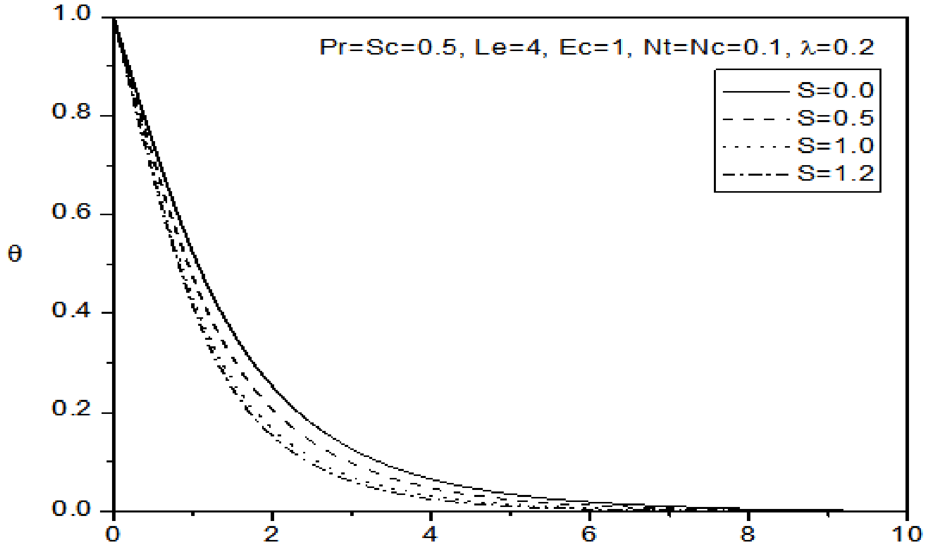

Fig.2(b) Temperature for different values of *S*

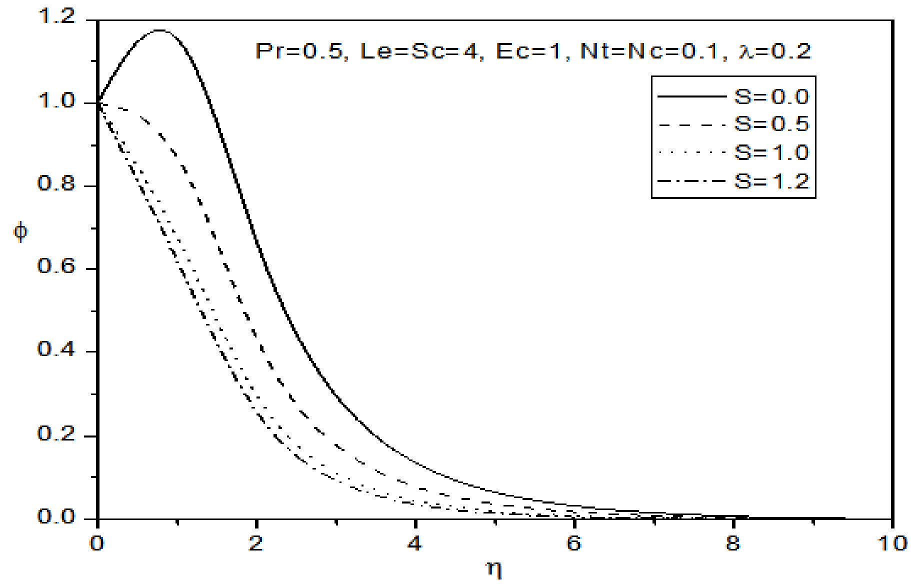

Fig.2(c) Nanoparticle volume fraction for different values of *S*

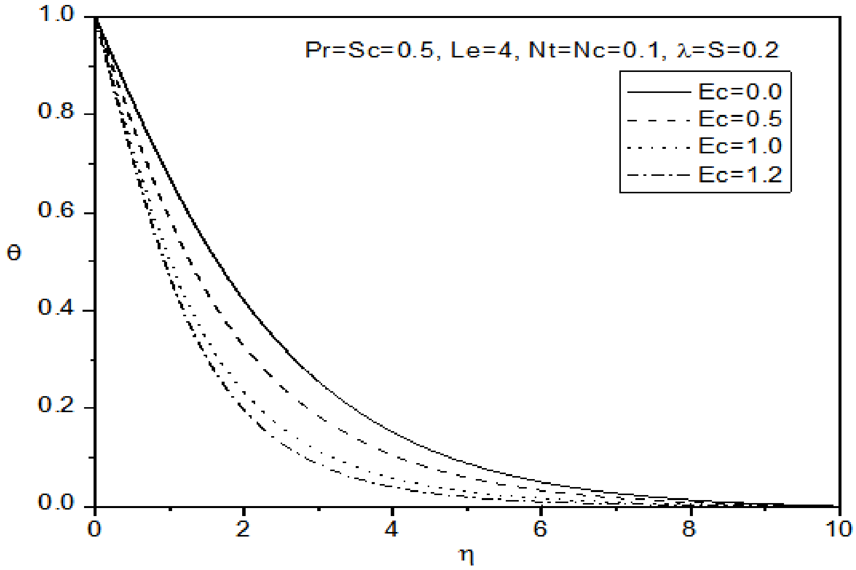

Fig.3(a) Temperature for different values of *Ec*

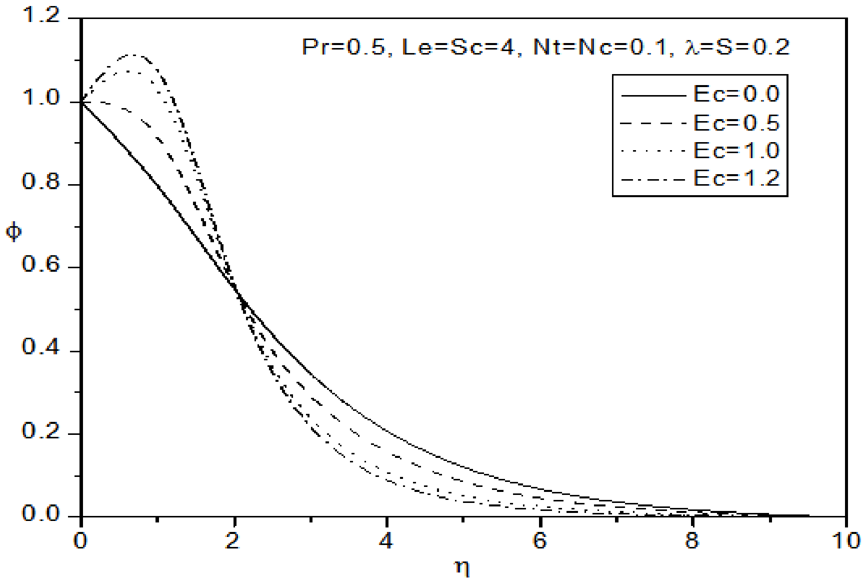

Fig.3(b) Nanoparticle volume fraction for different values of *Ec*

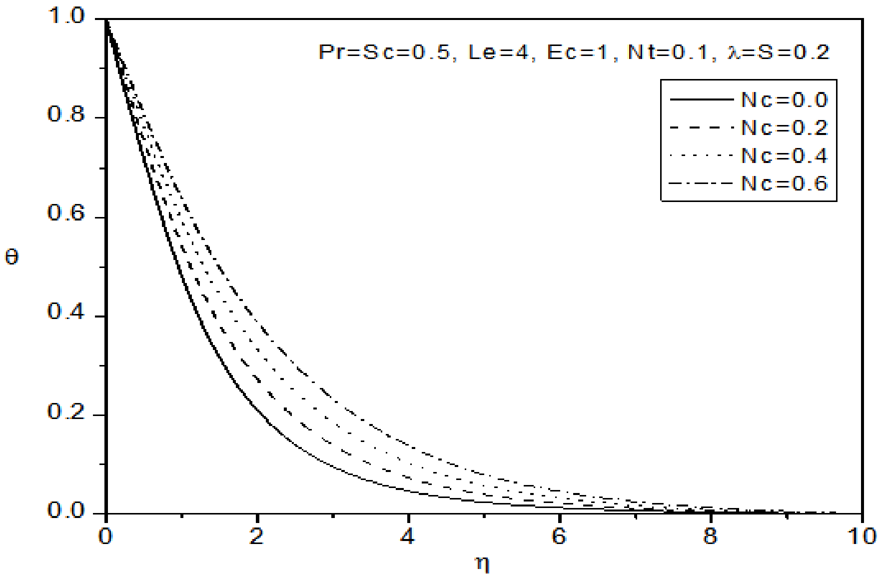

Fig.4(a) Temperature for different values of *Nc*

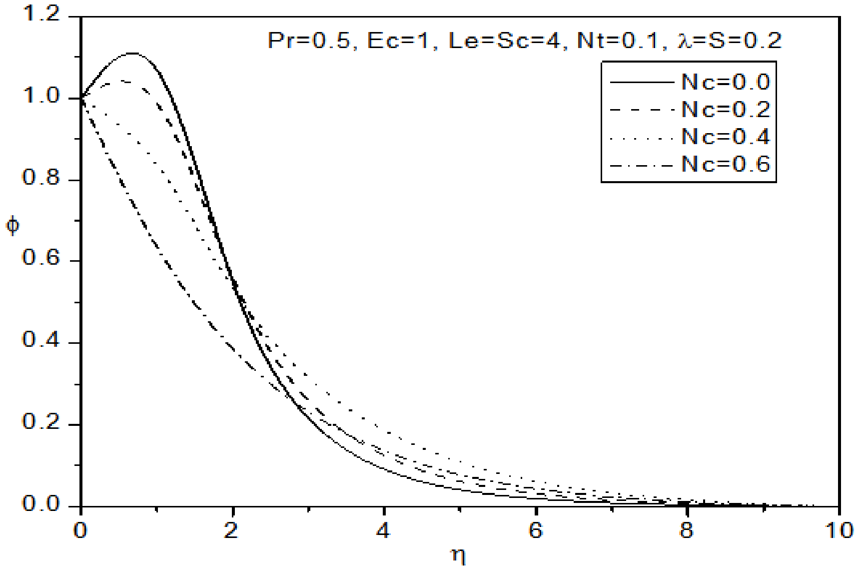

Fig.4(b) Nanoparticle volume fraction for different values of *Nc*

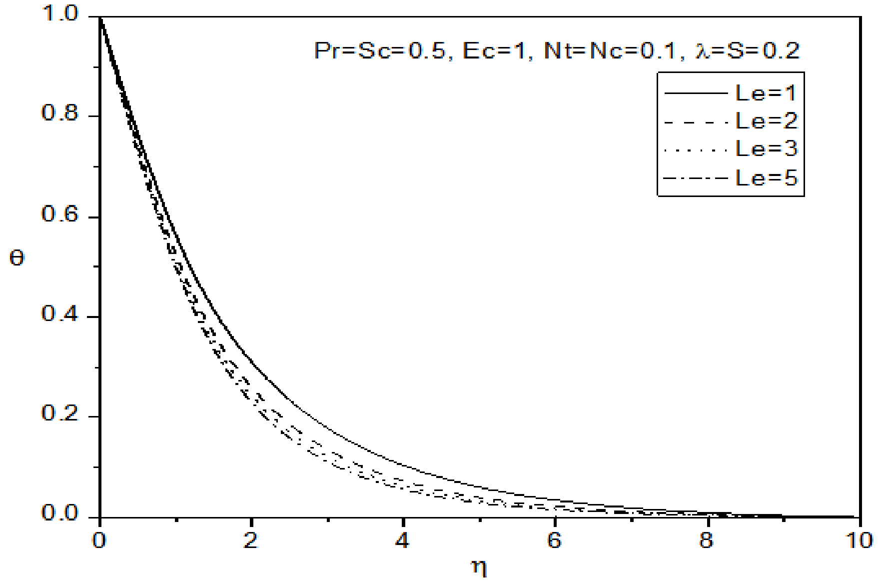

Fig.5(a) Temperature for different values of *Le*

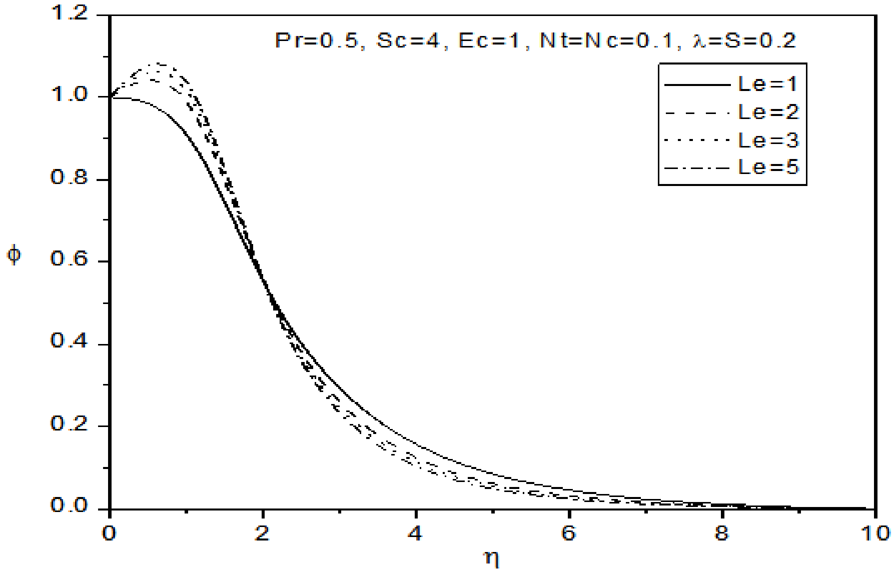

Fig.5(b) Nanoparticle volume fraction for different values of *Le*

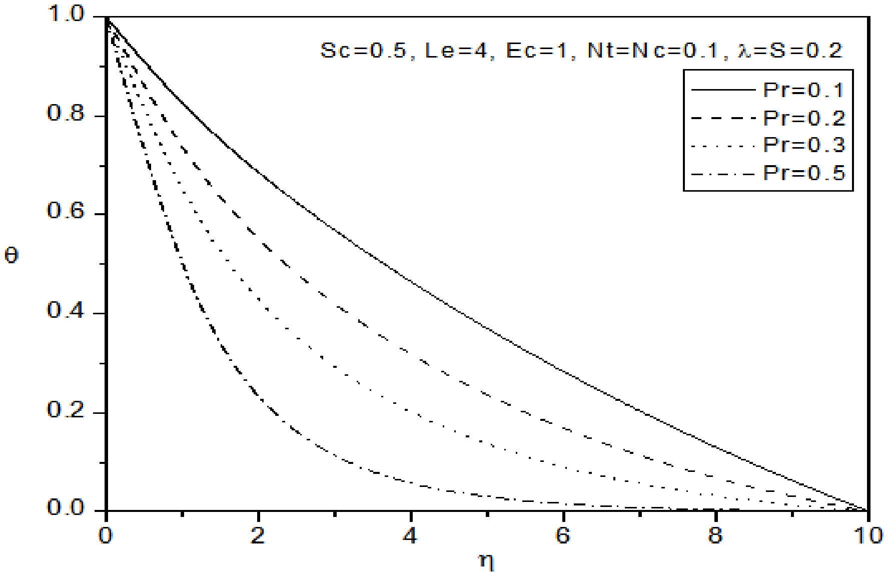

Fig.6 Temperature for different values of *Pr*

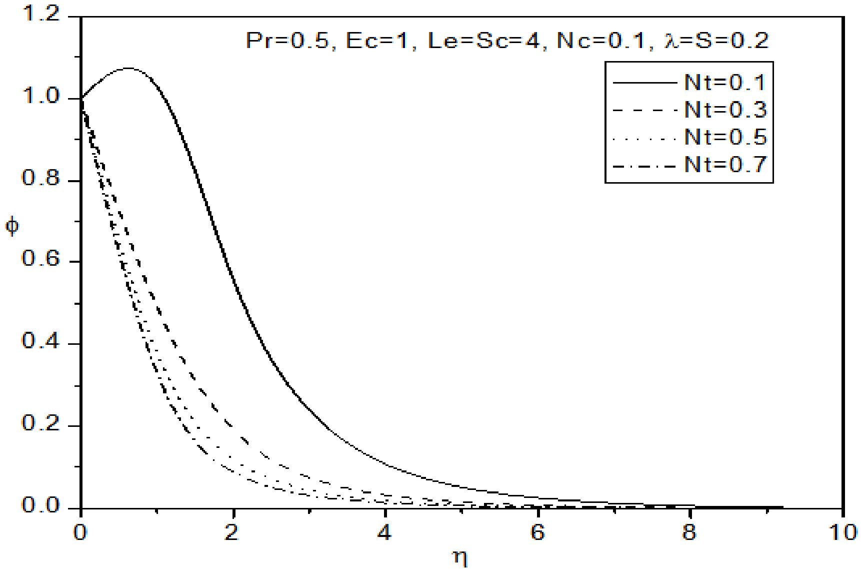

Fig.7 Nanoparticle volume fraction for different values of *Nt*

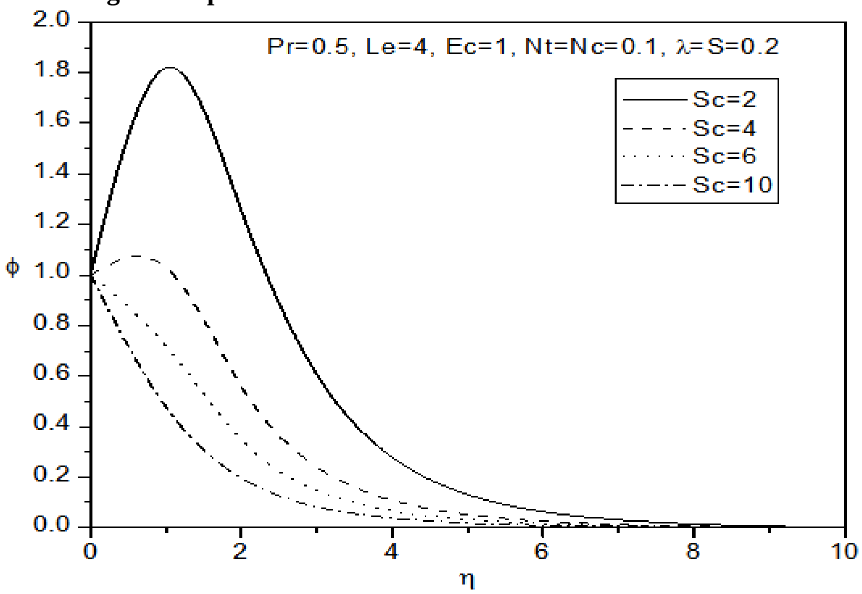

Fig.8 Nanoparticle volume fraction for different values of Sc

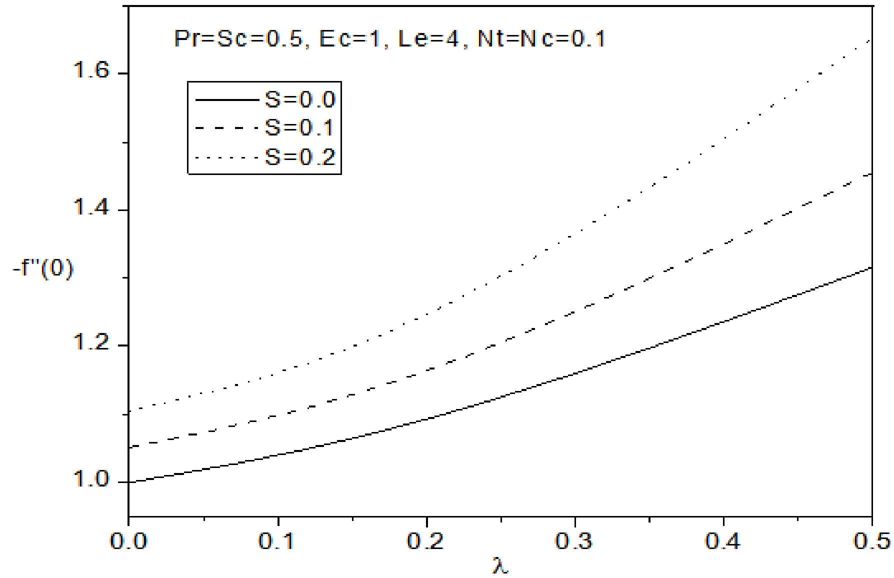

Fig.9 Effect of λ and S on the reduced skin friction

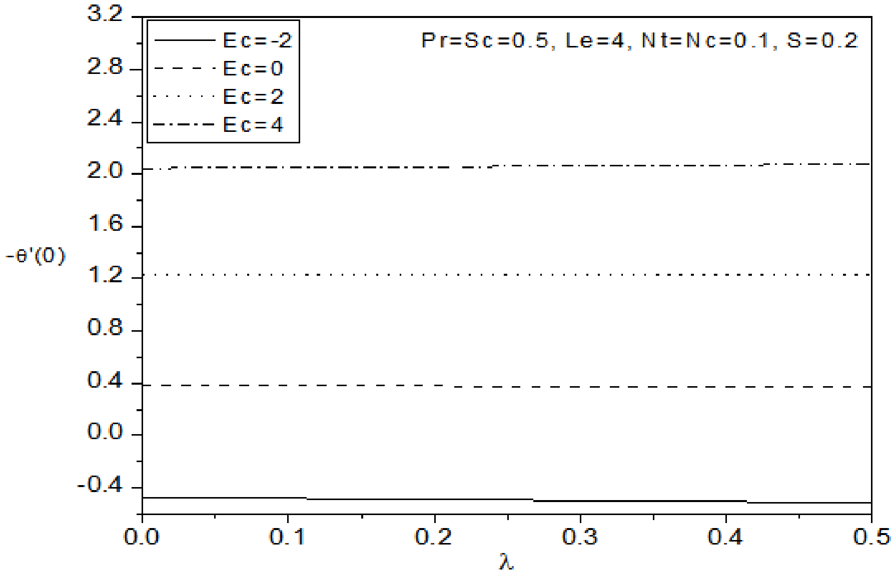

Fig.10 Effect of λ and Ec on the reduced Nusselt number

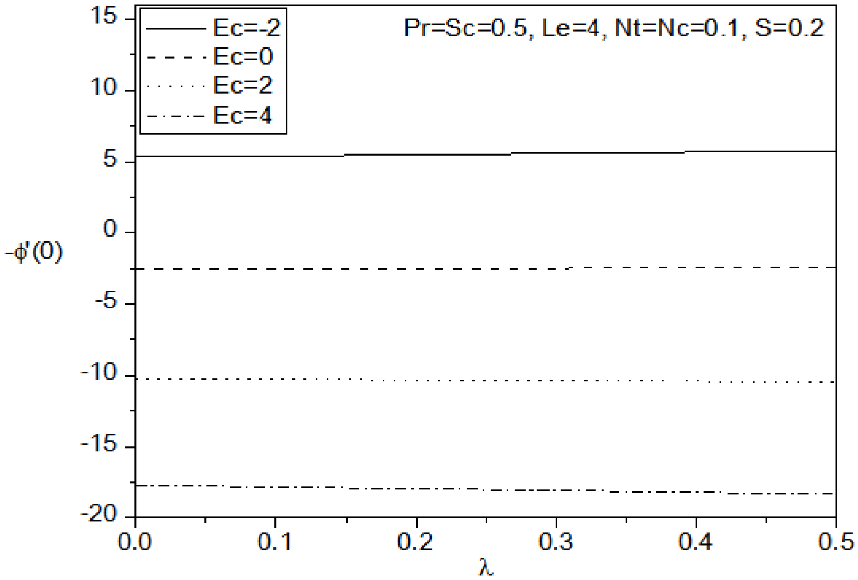

Fig.11 Effect of λ and Ec on the local Sherwood number

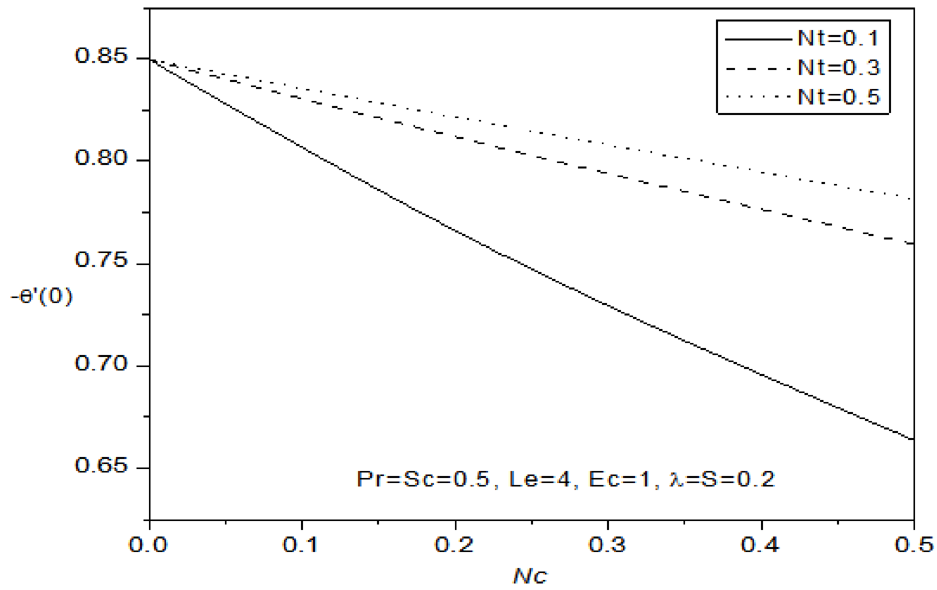

Fig.12 Effect of *Nt* and *Nc* on the reduced Nusselt number

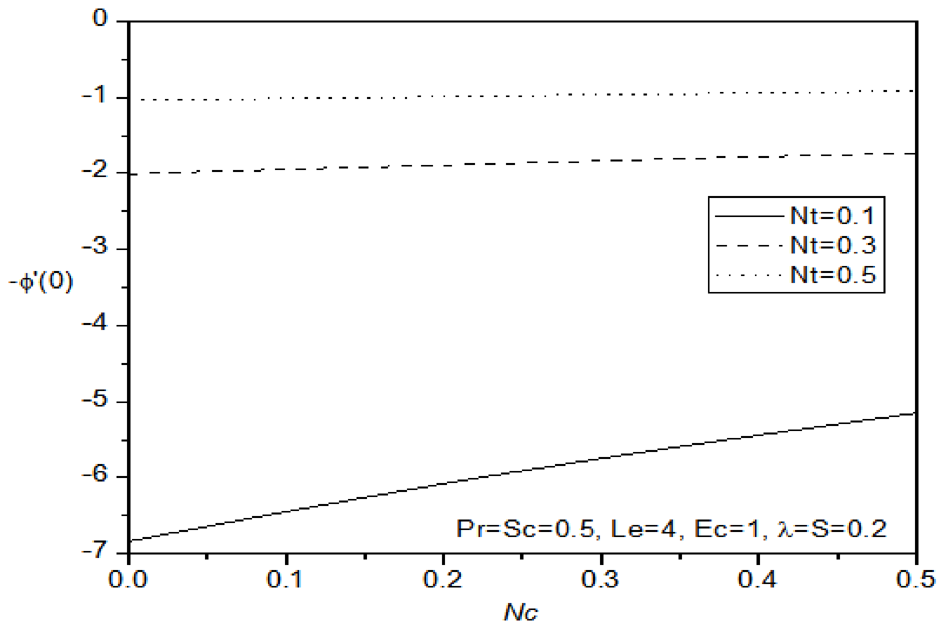

Fig.13 Effect of *Nt* and *Nc* on the local Sherwood number

Table 1 Comparison for viscous case $-\theta'(0)$ with Pr for λ =Nc=Nt=Le=Ec=Sc=0

Pr	$-\theta'(0)$					
	Present results		Nadeem and Hussain [12]	Khan and Pop [19]	Golra and Sidawi [20]	Wang [21]
	RKF5	bvp4c				
0.07	0.066	0.0723	0.066	0.066	0.066	0.066
0.2	0.169522	0.1695	0.169	0.169	0.169	0.169
0.7	0.453916	0.4539	0.454	0.454	0.454	0.454
2.0	0.911358	0.9114	0.911	0.911	0.911	0.911

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