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Ruban's Cosmological Modelwith Bulk Stress In General Theory of Relativity

[1]V.G.Mete, [2]V.D.Elkar, [3]A.S.Nimkar
[1]*Department of Mathematics, R.D.I.K. & K.D. College, Badnera-Amravati,India.*
[2]*Department of Mathematics,J.D.Patil Sangludkar Mahavidyalaya,Daryapur,Dist.Amravati*
[3]*Department of Mathematics,Shri Dr.R.G.Rathod Arts and ScienceCollege,*
Murtizapur, Dist.Akola

***Abstract**: This paper deals with the study of Ruban's cosmological model in presence of bulk stress source in the framework of General theory of relativity. Exact solution of Einstein's field equations are obtained by using a supplementary condition between metric potentials and curvature parameter k. The viscosity coefficient of bulk viscous fluid is assumed to be a simple power function of mass density whereas the coefficient of shear viscosity is consider as proportional to the scale of expansion in the model. The time dependent Λ-term is found to be positive and is a decreasing function of time. Also some physical and geometrical properties of the model are discussed.*

***Keywords**: Ruban's space time, viscous fluid models: variables Λ-term.*

I. Introduction

To study the evolution of universe many researchers have constructed cosmological models containing viscous fluids. The effect of viscosity plays a very important role in the early evolution of the universe. There are many circumstances during the evolution of the universe in which bulk viscosity could arise. The presence of viscosity in the fluid introduces many interesting features in the dynamics of homogeneous cosmological model. The roles played by the viscosity and the consequent dissipative mechanism in cosmology have been discussed by several authors. The heat represented by the large entropy per baryon the microwave background provides a useful clue to the early universe and a possible explanation for this huge entropy per baryon is that it was generated by physical dissipative processes acting at the beginning to the evolution of the universe. These dissipative process may indeed be responsible for the smoothing out of initial anisotropies [1] Misner [2] suggested that the neutrino viscosity acting in the early era might have considerably reduced the present anisotropy of the black body radiation during the process of evolution. Many researchers [3-10] have shown interest to study bulk viscous cosmological model in general relativity. Weinberger [11], Heller and Klimek [12], Misner [13] have studied the effect of viscosity on the evolution of cosmological models. Collins and Stewart [14] have studied the effect of viscosity on the formation of galaxies.

In Einstein's theory of gravity newtonian gravitational constant G and cosmological constant Λ are considered as fundamental constants. The gravitational constant G plays the role of coupling constant between geometry of space and matter in Einstein's field equations. Recent cosmological observations shows that an accelerating universe with variables $G \& \Lambda$, generalized Einstein's theory of gravitation have been proposed by Lau[15]. The possibly of variables $G \& \Lambda$ in Einstein's theory has also been studied by Dersarkissian [16].In an evolving universe, it appears natural to look at this constant as a function time. Dirac [17] and Dicke [18] have suggested a time-varying gravitational constant. The Large Number Hypothesis (LNH) proposed by Dirac [19] leads to a cosmology where G varies with cosmic time. There have been many extensions of Einstein's theory of gravitation, with time dependent G, in order to achieve a possible unification of gravitation and elementary particle physics. The cosmological model with variable G and Λ have been recently studied by several authors. Some of the recent discussions on the cosmological constant and on cosmology with a time varying cosmological constant by Ratra and Peebles[20], Sahni and Starobinsky [21], Peebles [22], J.P.Singh et al. [23-24], M.K.Verma et al. [25] and Pradhan et al. [26].

Recently M.K.Verma and Shri Ram [27-28] studied spatially homogeneous bulk viscous fluid models with time dependent gravitational constant and cosmological term. Anirudh Pradhan et al. [29] have constructed accelerating Bianchi type-I universe with time varying G and Λ-term in general relativity. Also Lima and Nobre [30-32] studied the spatially inhomogeneous solutions of the Einstein's-Maxwell equations in the frame work of Ruban's metric. Recently Singh et al.[33] have investigated anisotropic Bianchi type II viscous fluid model with time dependent G and Λ. Bulk viscous anisotropic cosmological models with dynamical cosmological parameters G and Λ have been presented by Kotambkar et al.[34] . Harpreet et.al.[35] have investigated bulk viscous Bianchi type- I cosmological models with time dependent Λ- term in self creation theory of gravitation. Very recently Mete et.al. [36-39] have studied various cosmological models in presence of bulk viscous fluid.

In this paper we discussed Ruban's cosmological model in presence of bulk stress source with time dependent Λ-term in the framework of general theory of relativity. This paper is organized as follows: In section-2 we have derive the field equations, section-3 deals with the solution of field equations in presence of viscous fluid. Some particular and special cases are discussed in Section-4 and Section-5.The last section contains concluding remark.

II. The Metric And Field Equations

Let us consider Ruban's line element[20]

$$ds^2 = dt^2 - Q^2(x,t)dx^2 - R^2(t)(dy^2 + h^2 dz^2) \quad , \tag{1}$$

where

$$h(y) = \frac{\sin \sqrt{k}\, y}{\sqrt{k}} = \begin{cases} \sin y & if \quad k = 1 \\ y & if \quad k = 0 \\ \sinh y & if \quad k = -1 \end{cases} \tag{2}$$

And k is the curvature parameter of the homogeneous 2-spaces t and x constants.
The functions Q and R are free and will be determined by the Einstein field equations (EFE) with cosmological constant.

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi T_{ij} + \Lambda g_{ij} \quad , \tag{3}$$

where R_i^j is the Ricci tensor; $R = g^{ij} R_{ij}$ is the Ricci scalar; and T_i^j is the energy momentum tensor of viscous fluid given by

$$T_i^j = (\rho + \overline{p}) v_i v^j - \overline{p} g_i^j + \eta g^{i\beta} (v_{i;\beta} + v_{\beta;i} - v_i v^\alpha v_{\beta;\alpha} - v_\beta v^\alpha v_{i;\alpha}) \ , \tag{4}$$

where

$$\overline{p} = p - (\xi - \frac{2}{3}\eta)\theta .$$

Here ρ, p, η and ξ are the energy density, pressure, coefficient of shear and bulk viscosities respectively.The semicolon (;) indicates covariant differentiation. The shear and bulk viscosity η and ξ are positive and may be either constant or function of time or energy such as

$$\eta = |a| \rho^\alpha , \quad \xi = |b| \rho^\beta ,$$

where a and b are constant. v_i is the flow vector satisfying the relations

$$g_{ij} v^i v^j = 1 \tag{5}$$

we choose the co ordinates to be commoving, so that

$$v^1 = 0 = v^2 = v^3, v^4 = 1 \tag{6}$$

The field equations (3) for the metric (1) with matter distribution (4) yield

$$\left(\frac{\dot{R}}{R}\right)^2 + 2\frac{\ddot{R}}{R} + \frac{k}{R^2} + \Lambda = 8\pi \left[-\overline{p} + 2\eta \frac{\dot{Q}}{Q} \right] \tag{7}$$

$$\frac{\dot{R}\dot{Q}}{RQ} + \frac{\ddot{R}}{R} + \frac{\ddot{Q}}{Q} + \Lambda = 8\pi \left[-\overline{p} + 2\eta \frac{\dot{R}}{R} \right] \tag{8}$$

$$2\frac{\dot{R}\dot{Q}}{RQ}+\left(\frac{\dot{R}}{R}\right)^2+\frac{k}{R^2}+\Lambda=8\pi\rho\ , \tag{9}$$

where the over head dot(.) at the symbol *Q* and *R* means time derivative.
The spatial volume for the model (1) is given by

$$V=hQR^2 \tag{10}$$

The scalar expansion θ, and shear scalar σ are define by

$$\theta=v^i_{;i}=\left(\frac{\dot{Q}}{Q}+2\frac{\dot{R}}{R}\right) \tag{11}$$

$$\sigma^2=\frac{1}{2}\sigma_{IJ}\sigma^{IJ} \tag{12}$$

III. Solution Of Field Equations

The field equations (7)-(9) are system of three equations with seven unknowns parameters R, Q, ρ, p, η, ξ and Λ. For complete determinacy of the system, extra conditions are needed. First we assume a relation in metric potential as

$$Q=(xR)^n \tag{13}$$

and secondly we assume that the coefficient of shear viscosity is proportional to the scale of expansion, i.e.

$$\eta\propto\theta\ , \tag{14}$$

where n is constant.
equation (7) and (8) leads to

$$\frac{\ddot{R}}{R}+\left(\frac{\dot{R}}{R}\right)^2+\frac{k}{R^2}-\frac{\dot{R}\dot{Q}}{RQ}-\frac{\ddot{Q}}{Q}=16\pi\eta\left(\frac{\dot{Q}}{Q}-\frac{\dot{R}}{R}\right) \tag{15}$$

Condition (14) leads to

$$\eta=l\left(\frac{\dot{Q}}{Q}+2\frac{\dot{R}}{R}\right)\ , \tag{16}$$

where l is proportionality constant.
equation (15) together with (13) and (16) yield

$$\ddot{R}+a\frac{\dot{R}^2}{R}=\frac{k}{(n-1)R} \tag{17}$$

where $a=(1+n)+16\pi l(n+2)$ (18)

we assume that

$$\dot{R}=f(R) \tag{19}$$

$$\ddot{R}=ff'\ , \tag{20}$$

when $f' = \frac{df}{dR}$

Equating (17), (19) and (20) we have

$$\frac{d}{dR}(f^2) + \frac{2a}{R}(f^2) = \frac{2k}{(n-1)R} \qquad (21)$$

From equation (21) we obtain

$$\left(\frac{dR}{dt}\right)^2 = \frac{k}{(n-1)a} + \frac{\beta}{R^{2a}} \quad , \qquad (22)$$

where β is the constant of integration.

After a suitable transformation of co ordinates, the metric (1) reduces to the form

$$ds^2 = \left[\frac{k}{(n-1)a} + \frac{\beta}{T^{2a}}\right]^{-1} dT^2 - x^{2n}T^{2n}dx^2 - T^2(dy^2 - h^2dz^2) \ , \qquad (23)$$

where $R = T$

The pressure and density for the model (23) are given by

$$8\pi p = \left\{K_1\left[\frac{k}{(n-1)a} + \frac{\beta}{T^{2a}}\right] - k\right\}\frac{1}{T^2} + \frac{2a\beta}{T^{2(a+1)}} + 8\pi\xi(n+2)\sqrt{\frac{k}{a(n-1)T^2} + \frac{\beta}{T^{2(a+1)}}} - \Lambda \qquad (24)$$

and

$$8\pi\rho = K_2\frac{k}{T^2} + \frac{(2n+1)\beta}{T^{2(a+1)}} + \Lambda \ , \qquad (25)$$

where

$$K_1 = \frac{\left[6n(n+2)l - 16\pi l(n+2)^2\right]}{3} - 1 \quad , K_2 = \frac{(2n+1)}{(n-1)a} + 1$$

For the specification of ξ , we assume that the fluid obeys an equation of state of the form

$$p = \gamma\rho \ , \qquad (26)$$

where $\gamma(0 \le \gamma \le 1)$ is constant.

Thus, given $\xi(t)$ we can solve for the cosmological parameters. In most of the investigations involving bulk viscosity is assumed to be a simple power function of the energy density (Pavon, [40]; Maartens,[41]; Zimdahl, [42])

$$\xi(t) = \xi_0\rho^m , \qquad (27)$$

where ξ_0 and m are constant. If $m = 1$ equation (27) may correspond to a radiative fluid

(Weinberg, [43]). However , more realistic models (Santos, [44]) are based on m lying in the regime $0 \le m \le \frac{1}{2}$.

Using (27) in (24), we obtain

$$8\pi p = \left\{K_1\left[\frac{k}{(n-1)a} + \frac{\beta}{T^{2a}}\right] - k\right\}\frac{1}{T^2} + \frac{2a\beta}{T^{2(a+1)}} + 8\pi\xi_0\rho^m(n+2)\sqrt{\frac{k}{a(n-1)T^2} + \frac{\beta}{T^{2(a+1)}}} - \Lambda \quad (28)$$

3.1.Model I: When $\xi(t)$ is constant

When $m=0$, equation (27) reduces to $\xi(t)=\xi_0=$constant. Hence in this case equation (28) with the use of (25) and (26), leads to

$$8\pi\rho(1+\gamma)=\left\{K_1\left[\frac{k}{(n-1)a}+\frac{\beta}{T^{2a}}\right]+\frac{(2n+1)k}{(n-1)a}\right\}\frac{1}{T^2}$$
$$+\frac{\beta(2n+2a+1)}{T^{2(a+1)}}+8\pi\xi_0(n+2)\sqrt{\frac{k}{a(n-1)T^2}+\frac{\beta}{T^{2(a+1)}}} \tag{29}$$

Eliminating $\rho(t)$ between (25) and (29), we obtain

$$(1+\gamma)\Lambda=\left\{K_1\left[\frac{k}{(n-1)a}+\frac{\beta}{T^{2a}}\right]-\left[1+\gamma-\frac{(2n+1)\gamma}{(n-1)a}\right]k\right\}\frac{1}{T^2}+\frac{\beta[2a-(2n+1)\gamma]}{T^{2(a+1)}}$$
$$+8\pi\xi_0(n+2)\sqrt{\frac{k}{a(n-1)T^2}+\frac{\beta}{T^{2(a+1)}}} \tag{30}$$

3.2. Model II : When $\xi(t)\propto\rho$

When $m=1$, equation (27) reduces to $\xi(t)=\xi_0\rho$, hence in this case equation (28) with the use of (25) and (26), leads to

$$8\pi\rho=\frac{1}{\left[(1+\gamma)-\xi_0(n+2)\sqrt{\frac{k}{a(n-1)T^2}+\frac{\beta}{T^{2(a+1)}}}\right]}$$
$$\times\left[\left\{K_1\left[\frac{k}{(n-1)a}+\frac{\beta}{T^{2a}}\right]+\frac{(2n+1)k}{(n-1)a}\right\}\frac{1}{T^2}+\frac{\beta(2n+2a+1)}{T^{2(a+1)}}\right] \tag{31}$$

Eliminating $\rho(t)$ between (25) and (31),we obtain

$$\Lambda=\frac{1}{\left[(1+\gamma)-\xi_0(n+2)\sqrt{\frac{k}{a(n-1)T^2}+\frac{\beta}{T^{2(a+1)}}}\right]}$$
$$\times\left[\left\{K_1\left[\frac{k}{(n-1)a}+\frac{\beta}{T^{2a}}\right]+\frac{(2n+1)k}{(n-1)a}\right\}\frac{1}{T^2}+\frac{\beta(2n+2a+1)}{T^{2(a+1)}}\right]-\left[K_2\frac{k}{T^2}+\frac{(2n+1)\beta}{T^{2(a+1)}}\right] \tag{32}$$

From equations (30) and (32), we observe that when $a>0$ the time dependent Λ-term is a decreasing function of time and approaches a small value in the present epoch.

Some physical aspects of the models.

With regards to the kinematical properties of the velocity vector v^i in metric (23) the scalar expansion (θ) and shear scalar (σ) of the fluid are given by

$$\theta=(n+2)\sqrt{\frac{k}{a(n-1)T^2}+\frac{\beta}{T^{(a+1)}}} \tag{33}$$

$$\sigma=\frac{(n-1)}{\sqrt{3}}\left(\frac{k}{a(n-1)T^2}+\frac{\beta}{T^{(a+1)}}\right) \tag{34}$$

IV. Particular Models

If we set $n = 2$ then the geometry of space time (23) reduces to the form

$$ds^2 = \left[\frac{k}{3+64\pi l} + \frac{\beta}{T^{2(3+64\pi l)}}\right]^{-1} dT^2 - x^4 T^4 dx^2 - T^2(dy^2 - h^2 dz^2) \tag{35}$$

where β is an integrating constant.

The pressure and density of the model (35) are given by

$$8\pi p = \left\{\left[16l\left(1-\frac{16\pi}{3}\right)-1\right]\times\left[\frac{k}{3+64\pi l} + \frac{\beta}{T^{2(3+64\pi l)}}\right] - k\right\}\frac{1}{T^2} + \frac{2\beta(3+64\pi l)}{T^{2(4+64\pi l)}} + 32\pi\xi\sqrt{\frac{k}{(3+64\pi l)T^2} + \frac{\beta}{T^{2(4+64\pi l)}}} - \Lambda \tag{36}$$

$$8\pi\rho = \left(\frac{5}{3+64\pi l}+1\right)\frac{k}{T^2} + \frac{5\beta}{T^{2(4+64\pi l)}} + \Lambda \tag{37}$$

4.1.Model- I : When $\xi(t)$ is constant

When $m = 0$, equation (27) reduces to $\xi(t) = \xi_0 =$ constant. Hence in this case equation (36) with the use of (37) and (26), leads to

$$8\pi(1+\gamma)\rho = \left\{\left[16l\left(1-\frac{16\pi}{3}\right)-1\right]\times\left[\frac{k}{3+64\pi l} + \frac{\beta}{T^{2(3+64\pi l)}}\right] + \frac{5k}{3+64\pi l}\right\}\frac{1}{T^2} + \frac{\beta(11+128\pi l)}{T^{2(4+64\pi l)}} + 32\pi\xi_0\sqrt{\frac{k}{(3+64\pi l)T^2} + \frac{\beta}{T^{2(4+64\pi l)}}} \tag{38}$$

Eliminating $\rho(t)$ between (37) and (38), we obtain

$$(1+\gamma)\Lambda = \left\{\left[16l\left(1-\frac{16\pi}{3}\right)-1\right]\times\left[\frac{k}{3+64\pi l} + \frac{\beta}{T^{2(3+64\pi l)}}\right] - \frac{5\gamma k}{3+64\pi l} - (1+\gamma)k\right\}\frac{1}{T^2} + \frac{\beta(6+128\pi l - 5\gamma)}{T^{2(4+64\pi l)}} + \frac{32\pi\xi_0}{T}\sqrt{\frac{k}{(3+64\pi l)} + \frac{\beta}{T^{6(1+16\pi l)}}} \tag{39}$$

4.2. Model- II: When $\xi(t) \propto \rho$

When $m = 1$, equation (27) reduces to $\xi(t) = \xi_0\rho$, hence in this case equation (36) with the use of (37) and (26), leads to

$$8\pi\rho = \frac{1}{\left[(1+\gamma) - \frac{4\pi\xi_0}{T}\sqrt{\frac{k}{3+64\pi l} + \frac{\beta}{T^{6(1+16\pi l)}}}\right]} \times\left\{\left[16l\left(1-\frac{16\pi}{3}\right)-1\right]\times\left[\frac{k}{3+64\pi l} + \frac{\beta}{T^{2(3+64\pi l)}}\right] + \frac{5k}{3+64\pi l}\right\}\frac{1}{T^2} + \frac{\beta(11+128\pi l)}{T^{2(4+64\pi l)}} \tag{40}$$

Eliminating $\rho(t)$ between (37) and (40),we obtain

$$\Lambda = \frac{1}{\left[(1+\gamma) - \frac{4\pi\xi_0}{T}\sqrt{\frac{k}{3+64\pi l} + \frac{\beta}{T^{6(1+16\pi l)}}}\right]}$$
$$\times\left\{\left[16l\left(1-\frac{16\pi}{3}\right)-1\right]\times\left[\frac{k}{3+64\pi l}+\frac{\beta}{T^{2(3+64\pi l)}}\right]+\frac{5k}{3+64\pi l}\right\}\frac{1}{T^2}+\frac{\beta(11+128\pi l)}{T^{2(4+64\pi l)}}$$
$$-\left(\frac{5}{3+64\pi l}+1\right)\frac{k}{T^2}-\frac{5\beta}{T^{8(1+16\pi l)}} \tag{41}$$

From equations (39) and (41), we observe that the time dependent Λ-term is a decreasing function of time and approaches a small value in the present epoch.

Some physical aspects of the models.

With regards to the kinematical properties of the velocity vector v^i in metric (23) the scalar expansion (θ) and shear scalar (σ) of the fluid are given by

$$\theta = 4\sqrt{\frac{k}{(3+64\pi l)T^2} + \frac{\beta}{T^{8(1+64\pi l)}}} \tag{42}$$

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{k}{(3+64\pi l)T^2} + \frac{\beta}{T^{8(1+64\pi l)}}\right) \tag{43}$$

V. Special Models

If we set $n = 2$ and $l = -\frac{1}{32\pi}$, equation (22) leads to

$$\frac{RdR}{\sqrt{kR^2+\beta}} = dt \tag{44}$$

which on integration gives

$$R^2 = kt^2 + b' \quad , \tag{45}$$

where $b' = \frac{1}{k}(b_1^2 - \beta)$ and b_1 is an integrating constant, hence we obtain

$$Q = x^2R^2 = x^2(kt^2+b') \ . \tag{46}$$

Using the transformations

$$T = kt^2 + b', x = X, y = Y$$

the metric (1) takes the form

$$ds^2 = \frac{1}{4k(T-b')}dT^2 - X^2T^2dX^2 - T(dY^2 + h^2dZ^2) \tag{47}$$

The pressure and density for the model (47) are given by

$$8\pi p = \beta\left(\frac{11}{3} - \frac{1}{2\pi}\right)\frac{1}{T^4} + k\left(\frac{5}{3} - \frac{1}{2\pi} - k\right)\frac{1}{T^2} + \frac{32\pi\xi}{T}\left(\sqrt{k + \frac{\beta}{T^2}}\right) - \Lambda \tag{48}$$

and

$$8\pi\rho = \left(6k + \frac{5\beta}{T^2}\right)\frac{1}{T^2} + \Lambda \tag{49}$$

5.1.Model -I : When $\xi(t)$ is constant

When $m = 0$, equation (27) reduces to $\xi(t) = \xi_0 =$ constant. Hence in this case equation (48) with the use of (26) and (49), leads to

$$8\pi(1+\gamma)\rho = \beta\left(\frac{26}{3} - \frac{1}{2\pi}\right)\frac{1}{T^4} + k\left(\frac{23}{3} - \frac{1}{2\pi} - k\right)\frac{1}{T^2} + \frac{32\pi\xi_0}{T}\left(\sqrt{k + \frac{\beta}{T^2}}\right) \tag{50}$$

Eliminating $\rho(t)$ between (49) and (50), we obtain

$$(1+\gamma)\Lambda = \beta\left(\frac{11}{3} - \frac{1}{2\pi} - 5\gamma\right)\frac{1}{T^4} + k\left(\frac{5}{3} - \frac{1}{2\pi} - k - 6\gamma\right)\frac{1}{T^2} + \frac{32\pi\xi_0}{T}\left(\sqrt{k + \frac{\beta}{T^2}}\right) \tag{51}$$

5.2. Model- II: When $\xi(t) \propto \rho$

When $m = 1$, equation (27) reduces to $\xi(t) = \xi_0\rho$, hence in this case equation (48) with the use of (26) and (49), leads to

$$8\pi\rho = \frac{1}{\left[(1+\gamma) - \frac{4\xi_0}{T}\sqrt{k + \frac{\beta}{T^2}}\right]} \times \left[\left(\frac{26}{3} - \frac{1}{2\pi}\right)\frac{\beta}{T^4} + \left(\frac{23}{3} - \frac{1}{2\pi} - k\right)\frac{k}{T^2}\right] \tag{52}$$

Eliminating $\rho(t)$ between (51) and (52), we obtain

$$\Lambda = \frac{1}{\left[(1+\gamma) - \frac{4\xi_0}{T}\sqrt{k + \frac{\beta}{T^2}}\right]} \times \left[\left(\frac{26}{3} - \frac{1}{2\pi}\right)\frac{\beta}{T^4} + \left(\frac{23}{3} - \frac{1}{2\pi} - k\right)\frac{k}{T^2}\right] - \left(6k + \frac{5\beta}{T^2}\right)\frac{1}{T^2} \tag{53}$$

From equations (51) and (53), we observe that the time dependent Λ-term is a decreasing function of time and approaches a small value in the present epoch.

Some physical aspects of the models.

With regards to the kinematical properties of the velocity vector v^i in metric (23) the scalar expansion (θ) and shear scalar (σ) of the fluid are given by

$$\theta = 4\sqrt{\frac{k}{T^2} + \frac{\beta}{T^4}} \tag{54}$$

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{k}{T^2} + \frac{\beta}{T^4}\right) \tag{55}$$

VI. Conclusion

In this paper, we have obtained Ruban's cosmological model in the presence of bulk stress source with time dependent Λ-term. In all these models we observe that they do not approach isotropy for large value of time T.For simplicity fluid obey an equation of state of the form $p = \gamma\rho$ and bulk viscosity is assume to be a simple power function of energy density given by $\xi(t) = \xi\rho^m$.

The time dependent Λ-term in all models are decreasing function of time and they all approaches small positive value as time in progress(i.e. present epoch).It is observe that θ is decreasing function of T and approaches to zero as $T \to \infty$ also in all models energy density and pressure tends to zero as $T \to \infty$ and

$\frac{\sigma}{\theta} \neq 0$ .The model is not isotropy for large value of T .The model is expanding, shearing and non rotating and has no initial singularity for $k = -1,\ 0, +1$.

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