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Jordan Higher left (σ,τ)- Centralizer on prime Γ-Rings

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Abstract: *Let M be a 2-torsion free prime Γ-ring. Then we prove that every Jordanhigher left (σ,τ)-centralizer on M is higher left (σ,τ)- centralizer on M. We also prove that with certain conditions every Jordan higher left (σ,τ)- centralizer on M is a Jordan triple higherleft(σ,τ)- centralizer of M.*
Keywords*: prime Γ–ring, higher left (σ,τ)- centralizer, Jordan higher left (σ,τ)- centralizer.*

I. Introduction

The definition of a Γ- ring was introduced by Nobusawa [5] and generalized by Barnes [6] as the following:

Let M and Γ be two additive abelian groups, then M is called Γ- ring if there exist a mapping M×Γ×M→M (the image of (a,α,b) being denoted by $a\alpha b$)
(where$a,b \in M$ and $\alpha \in \Gamma$) which satisfies the following conditions for all$a,b,c \in M$ and $\alpha,\beta \in \Gamma$

i) $(a + b)\alpha c= a\alpha c + a\alpha c$,$a(\alpha+\beta)b = a\alpha b + a\beta b$, $a\alpha(b + c) = a\alpha b + a\alpha c$
ii)$(a\alpha b)\beta c = a\alpha(b\beta c)$

Throughout this paper M is denote to a Γ- ring with center $Z(M)$ which equal to the set of all elements $a \in M$ such that $a\alpha b = b\alpha a$ for all $b \in M$ where $\alpha \in \Gamma$.
Now for any $a,b \in M$ and $\alpha \in\Gamma$, the symbol $[a,b]_\alpha$ will denoted to $a\alpha b - b\alpha a$ which is called the commutator [2]. M is said to be commutative Γ- ring if $[a,b]_\alpha = 0$ for all $a,b \in M$ and $\alpha \in\Gamma$[3]
A Γ- ringM is called prime if $a\Gamma M\Gamma b = \{0\}$ implies that $a = 0$ or $b = 0$ and it is called semi-prime if $a\Gamma M\Gamma a = \{0\}$ implies that $a = 0$. and a Γ- ring M is called 2-torsion free if $2a = 0$ implies that $a = 0$ for all $a \in M$ [4].

Throughout this paper we consider the Γ- ring M satisfy the following condition $a\alpha b\beta c = a\beta b\alpha c$ for all $a,b,c \in M$ and $\alpha,\beta \in \Gamma$ which will represented by $(*)$

In 2011 M.F. Hoque and A.C. Paul [3] also B. Zalar [1] defined a centralizer on Γ- ring as follows an additive mapping $T: M \to M$ is left (right) centralizer if $T(a\alpha b) = T(a)\alpha b$ $\big(T(a\alpha b) = a\alpha T(b)\big)$ holds for all $a,b \in M$ and $\alpha \in\Gamma$. In [3] , defined a Jordan centralizer on Γ- ring M as follows an additive mapping $T: M \to M$ is left (right) centralizer if $T(a\alpha a) = T(a)\alpha a$ $\big(T(a\alpha a) = a\alpha T(a)\big)$ holds for all $a \in M$ and $\alpha \in\Gamma$.
In this paper we introduce a new definition of higher left(σ,τ)-centralizer, Jordan higher left(σ,τ)-centralizer and jordan triple higher left(σ,τ)-centralizer on Γ- ring.

II. Higher (σ,τ)-Centralizer

In this section we will introduce the definitions of higher (σ,t) – centralizer ,Jordan higher (σ,t) – centralizer and Jordan triple higher (σ,t) – centralizer on M and other concepts which will be used in our work.

Definition (2.1): Let T= $(t_i)_{i\in N}$ be a family of additive mapping of a Γ−ring M into itself and σ,t are endomorphisms of M, then:-

i. T is called higher left (σ,t) – centralizer on M if
$T(a\alpha b) = \sum_{i=1}^{n} t_i\left(\sigma^i(a)\right)\alpha\tau^i(b)$
for all $a,b \in M$; $\alpha \in \Gamma$ and $n \in N$

ii. T is called Jordan higher left (σ,τ)- centralizer on M if
$T(a\alpha a) = \sum_{i=1}^{n} t_i\left(\sigma^i(a)\right)\alpha\tau^i(a)$
for all $a \in R$; $\alpha \in \Gamma$ and $n \in N$

iii. T is called Jordan triple higher left (σ,τ)- centralizer on M if
$T(a\alpha b\beta a) = \sum_{i=1}^{n} t_i\left(\sigma^i(a)\right)\alpha\tau^i(b)\beta\tau^i(a)$

for all $a, b \in M; \alpha, \beta \in \Gamma$ and $n \in N$

Lemma (2.2): Let $T=(t_i)_{i\in N}$ be a Jordan higher left (σ,τ)- centralizer on M then

i) $t_n(a\alpha b + b\alpha a) = \sum_{i=1}^{n} t_i\left(\sigma^i(a)\right)\alpha\tau^i(b) + t_i\left(\sigma^i(b)\right)\alpha\tau^i(a)$

ii) $t_n(a\alpha b\alpha c + c\alpha b\alpha a) = \sum_{i=1}^{n} t_i\left(\sigma^i(a)\right)\alpha\tau^i(b)\alpha\tau^i(c) + \sum_{i=1}^{n} t_i\left(\sigma^i(c)\alpha\tau^i(b)\alpha\tau^i(a)\right)$

iii) $t_n(a\alpha b\beta c + c\alpha b\beta a) = \sum_{i=1}^{n} t_i\left(\sigma^i(a)\right)\alpha\tau^i(b)_\beta\tau^i(c) + \sum_{i=1}^{n} t_i\left(\sigma^i(c)\alpha\tau^i(b)_\beta\tau^i(a)\right)$

Proof:-

i) $t_n((a+b)\alpha(a+b)) = \sum_{i=1}^{n} t_i\left(\sigma^i(a+b)\right)\alpha\tau^i(a+b)$

$= \sum_{i=1}^{n} t_i\left(\sigma^i(a)+\sigma^i(a)\right)\alpha(\tau^i(a)+\tau^i(b))$

$= \sum_{i=1}^{n} t_i\left(\sigma^i(a)\alpha\tau^i(a)\right) + \sum_{i=1}^{n} \tau_i\left(\sigma^i(a)\right)\alpha\,\tau^i(b) + \sum_{i=1}^{n} t_i\left(\sigma^i(b)\alpha\tau^i(a)\right) + \sum_{i=1}^{n} \tau_i\left(\sigma^i(b)\right)\alpha\,\tau^i(b)$ ... (1)

On the other hand

$t_n((a+b)\alpha(a+b)) = t_n(a\alpha a + a\alpha b + b\alpha a + b\alpha b)$

$= \sum_{i=1}^{n} t_i\left(\sigma^i(a)\alpha\tau^i(a)\right) + t_n(a\alpha b + b\alpha a) + \sum_{i=1}^{n} \tau_i\left(\sigma^i(b)\right)\alpha\,\tau^i(b)$ ... (2)

Comparing (1) and (2) we have

$$t_n(a\alpha b + b\alpha a) = \sum_{i=1}^{n} t_i\left(\sigma^i(a)\alpha\tau^i(b)\right) + \sum_{i=1}^{n} t_i\left(\sigma^i(b)\alpha\tau^i(a)\right)$$

ii) In Definition 2.1 (iii) replace a+c for a we get

$$t_n\big((a+c)\alpha b\alpha(a+c)\big) = \sum_{i=1}^{n} t_i\left(\sigma^i(a+c)\alpha\tau^i(b)\alpha\,\tau^i(a+c)\right)$$

$= \sum_{i=1}^{n} t_i\left(\sigma^i(a)+\sigma^i(c)\right)\ \alpha\tau^i(b)\alpha(\tau^i(a) + \tau^i(c))$

$= \sum_{i=1}^{n} t_i\left(\sigma^i(a)\alpha\tau^i(b)\alpha\,\tau^i(a)\right) +$

i=1ntiσiaατi(b)α τic+i=1nτiσicατi(b)α τia+i=1nτiσicατi(b)α τic …(1)

on the other hand

$$t_n\big((a+c)\alpha b\beta(a+c)\big) = t_n\big((a\alpha b\alpha a) + (a\alpha b\alpha c + c\alpha b\alpha a) + c\alpha b\alpha c\big)$$

$$\sum_{i=1}^{n} t_i\left(\sigma^i(a)\right)\alpha\tau^i(b)\alpha\,\tau^i(a) + t_n(a\alpha b\alpha c + c\alpha b\alpha a) + \sum_{i=1}^{n} t_i\left(\sigma^i(c)\alpha\,\tau^i(b)\alpha\tau^i(c)\right) \quad ...(2)$$

Comparing (1) and (2) we have

$$t_n(a\alpha b\alpha c + c\alpha b\alpha a) = \sum_{i=1}^{n} t_i\left(\sigma^i(a)\right)\alpha\tau^i(b)\alpha\,\tau^i(c) + \sum_{i=1}^{n} t_i\left(\sigma^i(c)\right)\alpha\,\tau^i(b)\alpha\tau^i(a)$$

iv) In Definition 2.1 (iii) replace a+c for a we get

$$t_n\big((a+c)\alpha b\beta(a+c)\big) = \sum_{i=1}^{n} t_i\left(\sigma^i(a+c)\alpha\tau^i(b)\beta\,\tau^i(a+c)\right)$$

$= \sum_{i=1}^{n} t_i\left(\sigma^i(a)+\sigma^i(c)\right)\ \alpha\tau^i(b)\beta(\tau^i(a) + \tau^i(c))$

$= \sum_{i=1}^{n} t_i\left(\sigma^i(a)\alpha\tau^i(b)\beta\,\tau^i(a)\right) +$

i=1ntiσiaατi(b)β τic+i=1nτiσicατi(b)β τia+i=1nτiσicατi(b)β τic …(1)

On the other hand

$$t_n\big((a+c)\alpha b\beta(a+c)\big) = t_n\big((a\alpha b\beta a) + (a\alpha b\beta c + c\alpha b\beta a) + c\alpha b\beta c\big)$$

$$\sum_{i=1}^{n} t_i \left(\sigma^i(a)\right) \alpha\tau^i(b)\beta\, \tau^i(a) + t_n(a\alpha b\beta c + c\alpha b\beta a) + \sum_{i=1}^{n} t_i \left(\sigma^i(c)\alpha\, \tau^i(b)\beta\tau^i(c)\right) \dots (2)$$

Comparing (1) and (2) we have

$$t_n(a\alpha b\beta c + c\alpha b\beta a) = \sum_{i=1}^{n} t_i \left(\sigma^i(a)\right) \alpha\tau^i(b)\beta\, \tau^i(c) + \sum_{i=1}^{n} t_i \left(\sigma^i(c)\right) \alpha\, \tau^i(b)\beta\tau^i(a) \blacksquare$$

Definition (2.3).:- Let T=$(t_i)_{i\in N}$ be a Jordan higher left (σ ,τ) - centralizer of a $\Gamma - ring\ M$ then for all $a, b \in M$; $\alpha \in \Gamma$and $n \in N$

$$\delta_n(a, b)_\alpha = t_n(a\alpha b) - \sum_{i=1}^{n} t_i \left(\sigma^i(a)\right) \alpha\tau^i(b)$$

Now we present the properties of $\delta_n(a, b)_\alpha$

Lemma (2.4):- Let T=$(t_i)_{i\in N}$be a Jordan higher left (σ ,τ) - centralizer of a $\Gamma - ring$ M then for all a,b ∈M , $\alpha \in \Gamma$and n ∈ N

i- $\delta_n(a, b)_\alpha = -\delta_n(b, a)$

ii- $\delta_n(a + b, c)_\alpha = \delta_n(a, b)_\alpha + \delta_n(b, c)_\alpha$

iii- $\delta_n(a, b + c)_\alpha = \delta_n(a, b)_\alpha + \delta_n(a, c)_\alpha$

Proof:

i- Since

$$t_n(a\alpha b + b\alpha a) = \sum_{i=1}^{n} t_i \left(\sigma^i(a)\right) \alpha\tau^i(b) + \sum_{i=1}^{n} t_i \left(\sigma^i(b)\right) \alpha\tau^i(a)$$

then

$$t_n(a\alpha b) - \sum_{i=1}^{n} t_i \left(\sigma^i(a)\right) \alpha\tau^i(b) = -t_n(b\alpha a) + \sum_{i=1}^{n} t_i \left(\sigma^i(b)\right) \alpha\tau^i(a)$$

So that

$$\delta_n(a, b)_\alpha = -\delta_n(b\, , a)$$

ii- $\delta_n(a + b, c)_\alpha = t_n((a + b)\alpha c) - \sum_{i=1}^{n} t_i \left(\sigma^i(a + b)\right) \alpha\tau^i(c)$

$$= t_n(a\alpha c) + t_n(b\alpha c) - \sum_{i=1}^{n} t_i \left(\sigma^i(a) + \sigma^i(b)\right) \alpha\tau^i(c)$$

$$= t_n(a\alpha c) + t_n(b\alpha c) - \sum_{i=1}^{n} t_i \left(\sigma^i(a)\right) \alpha\tau^i(c) - \sum_{i=1}^{n} t_i \left(\sigma^i(b)\right) \alpha\tau^i(c)$$

$$= t_n(a\alpha c) - \sum_{i=1}^{n} t_i \left(\sigma^i(a)\right) \alpha\tau^i(c) + t_n(b\alpha c) - \sum_{i=1}^{n} t_i \left(\sigma^i(b)\right) \alpha\tau^i(c)$$

$$= \delta_n(a, c)_\alpha + \delta_n(b, c)_\alpha$$

iii- $\delta_n(a, b + c)_\alpha = t_n(a\alpha(b + c)) - \sum_{i=1}^{n} t_i \left(\sigma^i(a)\right) \alpha T^i(b + c)$

$$= t_n(a\alpha b) + t_n(a\alpha c) - \sum_{i=1}^{n} t_i \left(\sigma^i(a)\right) \alpha(T^i(b) + T^i(c))$$

=$t_n(a\alpha b) - \sum_{i=1}^{n} t_i \left(\sigma^i(a)\right) \alpha\tau^i\big(b) + t_n(a\alpha c)\big) - \sum_{i=1}^{n} t_i \left(\sigma^i(a)\right) \alpha\tau^i(c)$

=$\delta_n(a, b)_\alpha$+ $\delta_n(a, c)_\alpha$

Remark (2.5):- Note that T=$(t_i)_{i\in N}$ is higher left (σ ,τ) - centralizer of a $\Gamma - ring$M if and only if $\delta_n(a, c)_\alpha = 0$

Lemma (2.6): Let T=$(t_i)_{i\in N}$ be a Jordan higher left (σ ,τ) – centralizers of a 2-torsion free prime Γ-ring M then for all a,b, m $\in M$,$\alpha,\beta\in\Gamma$and $n\in N$

$$\delta_n(a,b)_\alpha\beta\,\tau^n(m)\beta[\tau^n(a),\tau^n(b)]_\alpha=0$$

Proof:- We prove by induction on $n\in N$

Let $\omega=a\alpha b\beta m\beta b\alpha a+b\alpha a\beta a\alpha b$

$t(\omega)=t(a\alpha(\beta m\beta b)\alpha a+b\alpha(a\beta m\beta a)\alpha b)$

$=t(\sigma(a)\alpha\tau(b\beta m\beta b))\alpha\tau(a)+t(\sigma(b))\alpha\tau(a\beta m\beta a)\alpha\tau(b)$

$=t(\sigma(a)\alpha\tau(b)\beta\tau(m)\beta\tau(b)\alpha\tau(a))+t(\sigma(b))\alpha\tau(a)+$
$t(\sigma(b)\alpha\tau(a)\beta\tau(m)\beta\tau(a)\alpha\tau(b))$ ... (1)

On the other hand

$t(\omega)=t((a\alpha b)\beta m\beta(b\alpha a)+(b\alpha a)\beta m\beta(a\alpha b))$

$=t(\sigma(a\alpha b))\beta\tau(m)\beta\tau(b\alpha a)+t(\sigma(b\alpha a))\beta\tau(m)\beta\tau(a\alpha b)$

$=t(\sigma(a)\alpha\sigma(b))\beta\tau(m)\beta\tau(b\alpha a)+t(\sigma(b)\alpha\sigma(a))\beta\tau(m)\beta\tau(a\alpha b)$

$=t(\sigma(\sigma(a))\alpha\tau(\sigma(b))\beta\tau(m)\beta\tau(b\alpha a)+t(\sigma(\sigma(b)))\alpha\tau(\sigma(a))\beta\tau(m)\tau(a\alpha b)$

$=t(\sigma(a)\alpha\tau(b)\beta\tau(m)\beta\tau(b\alpha a)+t(\sigma(b\tau(m))\alpha\tau(a)\beta\tau(m)\beta\tau(a\alpha b))$

$=t(a\alpha b)\beta\tau(m)\beta\tau(b\alpha a)+t(b\alpha a)\beta\tau(m)\beta\tau(a\alpha b)$

$=t(a\alpha b)_\beta\tau(m)_\beta\tau(b)\alpha\tau(a)+t(b\alpha a)_\beta\tau(m)_\beta\tau(a)\alpha\tau(b)$ ... (2)

Comparing (1)*and* (2) *we have*

$0=(t(a\alpha b)-t(\sigma(a)\alpha\tau(b))\beta\tau(m)\beta\tau(b)\alpha\tau(a)+(t(b\alpha a)-t(\sigma(b)\alpha\tau(a))\beta\tau(m)\beta\tau(a)\alpha\tau(b)$

$=\delta(a,b)_\alpha\beta\tau(m)\beta\tau(b)\alpha\tau(a)+\delta(b,a)_\alpha\beta\tau(m)\beta\tau(a)\alpha\tau(b)$

$=\delta(a,b)_\alpha\beta\tau(m)\beta\tau(b)\alpha\tau(a)-\delta(a,b)_\alpha\beta\tau(m)\beta\tau(a)\alpha\tau(b)$

$=\delta(a,b)_\alpha\beta\tau(m)\beta[\tau(a),\tau(b)]_\alpha$

Then we can assume that

$$\delta_s(a,b)_\alpha\beta\tau^s(m)\beta[\tau^s(a),\tau^s(b)]_\alpha=0$$

for all $a,b,m\in M,\alpha,\beta\in\Gamma$ *and* $s,n\in N,s<n$

Now

$$t_n(w)=t_n(a\alpha(b\beta m\beta b)\alpha a+b\alpha(a\beta m\beta a)\alpha b)$$
$$=\sum_{i=1}^{n}t_i\left(\sigma^i(a)\right)\alpha\tau^i(b\beta m\beta b)\alpha\tau^i(a)+\sum_{i=1}^{n}t_i\left(\sigma^i(b)\right)\alpha\tau^i(a\beta m\beta a)\alpha\tau^i(b)$$

$$=\sum_{i=1}^{n}t_i\left(\sigma^i(a)\right)\alpha t^i(b)\beta t^i(m)\beta t^i(b)\alpha t^i(a)+\sum_{i=1}^{n}t_i\left(\sigma^i(b)\right)\alpha t^i(a)\beta t^i(m)\beta t^i(a)\alpha t^i(b)$$

$$=(\sum_{i=1}^{n}t_i\left(\sigma^i(a)\right)\alpha t^i(b))\beta\tau^n(m)\beta\tau^n(b)\alpha\tau(a)$$
$$+(\sum_{i=1}^{n}t_i\left(\sigma^i(b)\right)\alpha\tau^i(a))\beta\tau^n(m)\beta\tau^n(a)\alpha\tau^n(b)$$

$=t_n(a\alpha b)\beta\tau^n(m)\beta\tau^n(b)\alpha\tau^n(a)+t_n(b\alpha a)\beta\tau^n(m)\beta\tau^n(a)\alpha\tau^n(b)$ ... (1)

Now

$t_n(w)=t_n((a\alpha b)\beta m\beta(b\alpha a)+(b\alpha a)\beta m\beta(a\alpha b))$

$=t_n((a\alpha b)\beta\tau^n(m)\beta\tau^n(\mathrm{b})\alpha\tau^n(a)+t_n(b\alpha a)\beta\tau^n(m)\beta\tau^n(a)\alpha\tau^n(b)$ ... (2)

Compare (1) and (2) we have

$0=(t_n(a\alpha b)-\sum_{i=1}^{n}t_i(\sigma^i(a))\alpha\tau^i(b)\beta\tau^n(m)\beta\tau^n(b)\alpha\tau^n(a))$

$$+(t_n(b\alpha a)-\sum_{i=1}^{n}t_i(\sigma^i(a))\alpha\tau^i(a)\beta\tau^n(m)\beta\tau^n(a)\alpha\tau^n(b)$$

$=\delta_n(a,b)_\alpha\beta t^n(m)\beta[t^n(a),t^n(b)]_\alpha$

III. The Main Result

In this section, we introduce our main results, we have prove that every Jordan higher left (σ,τ)- centralizer of a 2-torsion free prime Γ- ring M is higher left (σ,τ)- centralizer of M. and we prove that under certain conditions a Jordan higher left (σ,τ)- centralizer of a prime $\Gamma-ring$ M is Jordan triple higher left (σ,τ)- centralizer of M.

Theorem (3.1):- Let T=$(t_i)_{i\in N}$ be a Jordan higher left (σ,τ)- centralizer of a prime $\Gamma-ring$ M then for all $a,b,c,m \in M;\alpha,\beta \in \Gamma$ and $n \in N$

$$\delta_n(a+c,b)_\alpha\beta\tau^n(m)\beta[\tau^n(c),\tau^n(d)]_\alpha = 0$$

Proof: - In Lemma (2.6) replace a+c for a

$\delta_n(a+c,b)_\alpha\beta\tau^n(m)\beta[\tau^n(a+c),\tau^n(b)]_\alpha = 0$

$0 = \delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(a+c),\tau^n(b)]_\alpha + \delta_n(c,b)_\alpha\beta\tau^n(m)\beta[\tau^n(a+c),\tau^n(b)]_\alpha$

$0 = \delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(a),\tau^n(b)]_\alpha + \delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(c),\tau^n(b)]_\alpha$
$+\delta_n(c,b)_\alpha\beta\tau^n(m)\beta[\tau^n(a),\tau^n(b)]_\alpha + \delta_n(c,b)_\alpha\beta\tau^n(m)\beta[\tau^n(c),\tau^n(b)]_\alpha$

By Lemma 3 we get

$$\delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(c),\tau^n(b)]_\alpha + \delta_n(c,d)_\alpha\beta\tau^n(m)\beta[\tau^n(a),\tau^n(b)]_\alpha = 0$$
$$\delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(c),\tau^n(b)]_\alpha = -\delta_n(c,d)_\alpha\beta\tau^n(m)\beta[\tau^n(a),\tau^n(b)]_\alpha$$

Since M is $\Gamma-ring$

$0 = \delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(c),\tau^n(b)]_\alpha\beta\tau^n(m)\beta\delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(c),\tau^n(b)]_\alpha =$

$-\delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(c),\tau^n(b)]_\alpha\beta\tau^n(m)\beta\delta_n(c,b)\beta[\tau^n(c),\tau^n(b)]_\alpha = 0$

Hence by primness we get

$\delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(c),\tau^n(b)]_\alpha = 0$ ... (1)

Now replace b+d for b in Lemma (2.6), we get :-

$\delta_n(a,b+d)_\alpha\beta\tau^n(m)\beta[\tau^n(c),\tau^n(b+d)]_\alpha = 0$

$\delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(a),\tau^n(b+d)]_\alpha + \delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(a),\tau^n(b+d)]_\alpha = 0$

$\delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(a),\tau^n(b)]_\alpha + \delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(a),\tau^n(d)]_\alpha$
$+\delta_n(a,d)_\alpha\beta\tau^n(m)\beta[\tau^n(a),\tau^n(b)]_\alpha + \delta_n(a,d)_\alpha\beta\tau^n(m)\beta[\tau^n(a),\tau^n(d)]_\alpha = 0$

By Lemma (2.6) we get:-

$\delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(a),\tau^n(d)]_\alpha + \delta_n(a,d)_\alpha\beta\tau^n(m)\beta[\tau^n(a),\tau^n(d)]_\alpha = 0$

Hence

$\delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(a),\tau^n(d)]_\alpha = -\delta_n(a,d)_\alpha\beta\tau^n(m)\beta[\tau^n(a),\tau^n(d)]_\alpha$

Since M is $\Gamma-ring$ we can conclude

$\delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(a),\tau^n(d)]_\alpha\beta\tau^n(m)\beta\delta_n(a,b)_\alpha\beta\tau^n(m)[\tau^n(a),\tau^n(d)]_\alpha = 0$

So

$-\delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(a),\tau^n(d)]_\alpha\beta\tau^n(m)\beta\delta_n(a,b)_\alpha\beta\tau^n(m)[\tau^n(a),\tau^n(d)]_\alpha = 0$

Since M is prime we get

$\delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(a),\tau^n(d)]_\alpha = 0$ ... (2)

Thus

$$\delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(a+c),\tau^n(b+d)]_\alpha = 0$$

$\delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(a),\tau^n(b)]_\alpha + \delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(a),\tau^n(d)]_\alpha$
$+\delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(c),\tau^n(b)]_\alpha + \delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(c),\tau^n(d)]_\alpha = 0$

from Lemma (2.6) and by 1 and 2 we get the result

$$\delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(c),\tau^n(d)]_\alpha = 0$$

Theorem (3.2): Every Jordan higher left (σ,τ)- centralizer of a 2-torsion free prime $\Gamma-ring$ M is higher left (σ,τ)- centralizer of M.

Proof:-Let $T=(t_i)_{i\in N}$ be a Jordan higher left (σ,τ)-centralizer of a prime $\Gamma-ring$ M.

from Theorem (3.1) we have

$$\delta_n(a,b)_\alpha\beta\tau^n(m)\beta[\tau^n(c),\tau^n(d)]_\alpha = 0$$

for all $a,b,c,d,m \in$ M , and $\alpha,\beta \in \Gamma$ and $n \in N$

since M is prime $\Gamma-ring$ we have

either $\delta_n(a,b)_\alpha = 0$ or $[\tau^n(c),\tau^n(d)]_\alpha = 0$

if $[\tau^n(c),\tau^n(d)]_\alpha \neq 0$ for all $c,d \in M,\alpha \in \Gamma\ and\ n \in$ N

then $\delta_n(a,b)_\alpha = 0$ and hence T is higher left (σ,τ)- centralizer of M.

if $[\tau^n(c),\tau^n(d)]_\alpha = 0$ for all $c,d \in M, n \in$N and $\alpha \in \Gamma$

then M is commutative $\Gamma-ring$ and by Lemma (2.2) (i)

we have

$$t_n(2a\alpha b) = 2\sum_{i=1}^{n} t_i\left(\sigma^i(a)\right)\alpha\tau^i(b)$$

and since M is a 2-torsion free we get the required result. ■

Theorem (3.3):- Let $T=(t_i)_{i\in N}$ be a Jordan higher left (σ,τ)-centralizer of a prime $\Gamma-ring$ M such that $\tau^i\sigma^i=\tau^i$ and $\sigma^{2i}=\sigma^i$ then T is Jordan triple higher left (σ,τ)- centralizer of M.

Proof:-replace b by $a\beta b+b\beta$a in Definition 2.1 then

Then

$$t_n(a\alpha(a\beta b+b\beta a)+(a\beta b+b\beta a)\alpha a$$

$$=\sum_{i=1}^{n} t_i\left(\sigma^i(a)\right)\alpha\tau^i(a\beta b+b\beta a)+t_i\left(\sigma^i(a\beta b+b\beta a)\right)\alpha\tau^i(a)$$

$$=\sum_{i=1}^{n} t_i\left(\sigma^i(a)\right)\alpha\tau^i(a)\beta\tau^i(b)+t_i\left(\sigma^i(a)\right)\alpha\tau^i(b)\beta\tau^i(a)$$

$$+\sum_{i=1}^{n} t_i\left(\sigma^i(a)\right)\beta\sigma^i(b)\alpha\tau^i(a)+t_i\left(\sigma^i(b)\beta\sigma^i(a)\right)\alpha\tau^i(a)$$

$$=\sum_{i=1}^{n} t_i\left(\sigma^i(a)\right)\alpha\tau^i(a)\beta\tau^i(b)+t_i\left(\sigma^i(a)\right)\alpha\tau^i(b)\beta\tau^i(a)$$

$$+\sum_{i=1}^{n} t_i(\sigma^i(\sigma^i(a)))\beta\tau^i(w^i(b))\alpha\tau^i(a)+t_i\big(\sigma^i(\sigma^i((b)))\beta\tau^i(\sigma^i(a))\big)\alpha\tau^i(a)$$

by hypothesis we have

$$t_n(a\alpha(a\beta b+b\beta a)+(a\beta b+b\beta a)\alpha a)$$

$$=\sum_{i=1}^{n} t_i\left(\sigma^i(a)\right)\alpha\tau^i(a)\beta\tau^i(b)+t_i\left(\sigma^i(a)\right)\alpha\tau^i(b)\beta\tau^i(a)\ +$$

$$+\sum_{i=1}^{n} t_i\left(\sigma^i(a)\right)\beta\tau^i(b)\alpha\tau^i(a)+t_i\left(\sigma^i(b)\right)\beta\tau^i(a)\alpha\tau^i(a) \quad ...(1)$$

on the other hand

$$t_n(a\alpha(a\beta b+b\beta a)+(a\beta b+b\beta a)\alpha a)$$

$$=t_n(a\alpha a\beta b+a\alpha b\beta a+a\beta b\alpha a+b\beta a\alpha a)$$

$$=\sum_{i=1}^{n} t_i\left(\sigma^i(a)\right)\alpha\tau^i(a)\beta\tau^i(b)+t_n(a\alpha b\beta a+a\beta b\alpha a)+\sum_{i=1}^{n} t_i\left(\sigma^i(b)\right)\beta\tau^i(a)\alpha\tau^i(a) \ ...(2)$$

Comparing (1) and (2) we get

$$t_n(a\alpha b\beta a+a\beta b\alpha a)$$

$$=\sum_{i=1}^{n} t_i\left(\sigma^i(a)\right)\alpha\tau^i(b)\beta\tau^i(a)+t_i\left(\sigma^i(a)\right)\beta\tau^i(b)\alpha\tau^i(a)$$

since M satisfying (*) we have

$$t_n(2a\alpha b\beta a)=2\sum_{i=1}^{n} t_i\left(\sigma^i(a)\right)\alpha\tau^i(b)\beta\tau^i(a)$$

since M is 2-torsion free $\Gamma-$ ring we have

$$t_n(a\alpha b\beta a)=\sum_{i=1}^{n} t_i\left(\sigma^i(a)\right)\alpha\tau^i(b)\beta\tau^i(a)$$

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