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An Optimization of Fuzzy Integrated Production Model for A Deteriorating Inventory Item

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Abstract: *We develop an integrated production model for a deteriorating item in a two-echelon supply chain. The supplier's production batch size is restricted to an integer multiple of the discrete delivery, lot quantity to the buyer. Exact cost functions for the supplier is developed. It leads to the determination of individual optimal policies.*

Keywords : *Inventory, Graded mean Integration Representation, Lagrangean method.*

I. Introduction

In real life, it is not uncommon for inventory items, such as milk, fruit, blood pharmaceutical product, vegetables etc, to decay or deteriorate over time. It is important to point out that the total cost function of the integrated production inventory system contains two decision variables N and q. In this paper to derive the optimal lot size for the models considering in an integrated production model for a deteriorating inventory item. Chen (1985) function principle is proposed for arithmetic operation of fuzzy number and Lagrangean method is used for optimization .

1. The fuzzy Arithmetical operations under function principle.

Function principle is proposed to be as the fuzzy arithmetical operations by trapezoidal fuzzy numbers. We define some fuzzy arithmetical operations under function principle as follows.

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ & $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers. Then

i) The addition of $\tilde{A}$ *and* $\tilde{B}$ *is*

$$\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$$

where a_1, a_2, a_3, a_4, b_1, b_2, b_3 and b_4 are any real numbers.

ii) The multiplication of $\tilde{A}$ *and* $\tilde{B}$ *is*

$$\tilde{A} \otimes \tilde{B} = (c_1, c_2, c_3, c_4) \text{ where}$$

$$T = \{a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4\}$$

$$T_1 = \{a_2 b_2, a_2 b_3, a_3 b_2, a_3 b_3\}$$

C_1 = min T, C_2=min T_1, C_3=max T, C_4=max T_1

If a_1, a_2, a_3, a_4, a_4, b_1, b_2, b_3, and b_4 are all zero positive real numbers then

$$\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$$

iii) $-\tilde{B} = (-b_4, -b_3, -b_2, -b_1)$ then the subtraction of $\tilde{A}$ *and* $\tilde{B}$ *is*

$$\tilde{A} \ominus \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1) \text{ where}$$

a_1, a_2, a_3, a_4, a_4, b_1, b_2, b_3, and b_4 are any real numbers

iv) $\frac{1}{\tilde{B}} = \tilde{B}^{-1} = \left(\frac{1}{b_4}, \frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1}\right)$ where b_1, b_2, b_3, b_4 are all positive real numbers. If a_1, a_2, a_3, a_4, b_1, b_2, b_3 and b_4 are all non zero positive real numbers then the division $\tilde{A}$ *and* $\tilde{B}$ is

$$\tilde{A} \oslash \tilde{B} = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right)$$

v) Let $\alpha \in R$, then $i)\ \alpha \geq 0, \alpha \otimes \tilde{A} = (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4)$

$ii)\ \alpha \geq 0, \alpha \otimes \tilde{A} = (\alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1)$

1.1 Extension of the Lagrangean method.

Step (1) : Solve the unconstrained problem Min *y=f(x)*. If the resulting optimum satisfies all the constraints, stop because all constraints are redundant. Otherwise set K=1 and go to step 2.

Step (2) : Activate any K constraints [(ie) convert them into equality] and optimize *f(x)* subject to the K active constraints by the Lagrangean method. If the resulting solution is feasible with respect to the remaining constraints and repeat the step. If all sets of active constraints taken K at a time are considered without encountering a feasible solution, go to step 3.

Step (3): If K=m,.stop; no feasible solution exists. Otherwise set K=K+1 and go to step 2.

1.2 Methodology

Graded mean integration representation method.

Chen & Hsieh (1999) introduced Graded Mean Integration representations method based on the integral value of graded mean h-level of generalized fuzzy number for defuzzifying generalized fuzzy number. Throughout this paper, we only use popular trapezoidal fuzzy number as the type of all fuzzy parameters in our proposed fuzzy production inventory models. Let $\tilde{B}$ be a trapezoidal fuzzy number and be denoted as

$$\tilde{B} = (b_1, b_2, b_3, b_4) \qquad (1)$$

Then we can get the graded mean integration representation of $\tilde{B}$ by the formula is

$$P(\tilde{B}) = \frac{b_1 + 2b_2 + 2b_3 + b_4}{6} \qquad (2)$$

1.3. Notations:

The following notations are used throughout to develop an integrated inventory model.

D → Demand rate in units/time unit

N → The number of deliveries per production batch cycle.

C → Setup cost for a production batch ($/Setup)

q → delivery lotsize

d → the items deterioration rate

H_s → inventory holding cost in $/unit/time unit.

C_d → the cost of deterioration per unit ($)

P → Production rate (units/time unit)

Tc (q, N) → Suppliers cost function

II. An Integrated Production Model For A Deteriorating Inventory Item.

In this section we develop an integrated inventory optimization method using Lagrangean method. The supplier's lotsize (q) is optimized using a concept of Lagrangean method.

2.1. Mathametical Model

The total cost of the supplier's cost function can be written as

$$TC(q,N) = \left(\frac{D}{Nq} + \frac{d}{2N}\right)C + (H_s + C_d d)\, q\left[\frac{D}{P} - \frac{1}{2} + \frac{N}{2} - \frac{DN}{2P}\right] \qquad (3)$$

The objective is to find the optimal lotsize (q). The necessary conditions for minimum is

$$\frac{\partial TC(q,N)}{\partial q} = 0$$

Therefore the optimal lotsize (q) is

$$q^{*}=\sqrt{\frac{2DC}{N\left(H_s+C_d d\right)\left(\frac{(2-N)D}{P}+N-1\right)}} \qquad (4)$$

Throughout this paper, we use the following variables in order to simplify the treatment of the fuzzy inventory models $\tilde{D}$, $\tilde{C}$, $\tilde{H}_s$, $\tilde{C}_d$, $\tilde{P}$, $\tilde{d}$ are fuzzy parameters. The fuzzy total cost of the integrated inventory production model for a deteriorating item is

$$\widetilde{TC}(q,N)=\Bigg\{\left(\frac{D_1}{Nq}+\frac{d_1}{2N}\right)C_1+\left(H_{S_1}+C_{d_1}d_1\right)q\left[\frac{D_1}{P_1}-\frac{1}{2}+\frac{N}{2}-\frac{D_1N}{2P_1}\right],$$

$$\left(\frac{D_2}{Nq}+\frac{d_2}{2N}\right)C_2+\left(H_{s_2}+C_{d_2}d_2\right)q\left(\frac{D_2}{P_2}-\frac{1}{2}+\frac{N}{2}-\frac{D_2N}{2P_2}\right),$$

$$\left(\frac{D_3}{Nq}+\frac{d_3}{2N}\right)C_3+\left(H_{S_3}+C_{d_3}d_3\right)q\left(\frac{D_3}{P_3}-\frac{1}{2}+\frac{N}{2}-\frac{D_3N}{2P_3}\right),$$

$$\left(\frac{D_4}{Nq}+\frac{d_4}{2N}\right)C_4+\left(H_{S_4}+C_{d_4}d_4\right)q\left(\frac{D_4}{P_4}-\frac{1}{2}-\frac{N}{2}-\frac{D_4N}{2P_4}\right)\Bigg\}$$

$\widetilde{TC}$ (q,N) = [((D ⊘ (N⊗q))+(d_1 ⊘ (2⊗N)))⊗C_1+((H_s⊕(C_d⊗d))⊗q⊗((D ⊘ P)⊕N ⊖ 1)/2 ⊖ (D⊗N)⊖(2⊗P))]

Where, ⊘, ⊗, ⊕, ⊖ the arithmetical operations under function principle

Suppose
$$\tilde{D}=\left(D_1,D_2,D_3,D_4\right),\ \tilde{N}=\left(N_1,N_2,N_3,N_4\right)$$
$$\tilde{H}_s=\left(H_{s_1},H_{s_2},H_{s_3},H_{s_4}\right),\ \tilde{d}=\left(d_1,d_2,d_3,d_4\right)$$
$$\tilde{P}=\left(P_1,P_2,P_3,P_4\right),\ \tilde{C}_d=\left(C_{d_1},C_{d_2},C_{d_3},C_{d_4}\right)$$

are non negative trapezoidal fuzzy numbers. Then we solve the optimal lotsize formula as the following steps. Second we defuzzify the fuzzy total cost of the integrated production inventory system for a deteriorating item by (3). Graded Mean Integration Representation of $\widetilde{TC}(q,N)$ *is*

$$P\left(\widetilde{TC}(q,N)\right)=\frac{1}{6}\Bigg[\left(\frac{D_1}{Nq}+\frac{d_1}{2N}\right)C_1+\left(H_{s_1}+C_{d_1}d_1\right)q\left(\frac{D_1}{P_1}-\frac{1}{2}+\frac{N}{2}-\frac{D_1N}{2P_1}\right)$$

$$+\frac{2D_2C_2}{Nq}+\frac{2d_2C_2}{2N}+2\left(H_{s_2}+C_{d_2}d_2\right)q\left(\frac{D_2}{P_2}-\frac{1}{2}+\frac{N}{2}-\frac{D_2N}{2P_2}\right)$$

$$+\frac{2D_3C_3}{Nq}+\frac{2d_3C_3}{2N}+2\left(H_{s_3}+C_{d_3}d_3\right)q\left(\frac{D_3}{P_3}-\frac{1}{2}+\frac{N}{2}-\frac{D_3N}{2P_3}\right)$$

$$+\frac{D_4C_4}{Nq}+\frac{d_4C_4}{2N}+\left(H_{s_4}+C_{d_4}d_4\right)q\left(\frac{D_4}{P_4}-\frac{1}{2}+\frac{N}{2}-\frac{D_4N}{2P_4}\right)\Bigg]$$

We can get the optimal lotsize q* when $P(\tilde{T}C(q,N))$ is minimization. In order to find the minimization of $P(\tilde{T}c(q,N))$ the derivative of $P(\tilde{T}C(q,N))$ with q is $\frac{\partial q\tilde{T}C(q,N)}{\partial q}=0$ we find the optimal lotsize q

$$q^{*}=\sqrt{\frac{\frac{D_1C_1}{N}+\frac{2D_2C_2}{N}+\frac{2D_3C_3}{N}+\frac{D_4C_4}{N}}{\frac{1}{2}\left\{\left(H_{s_1}+C_{d_1}d_1\right)\left(\frac{(2-N)D_1}{P_1}-1+N\right)+2\left(H_{s_2}+C_{d_2}d_2\right)\left(\frac{(2-N)D_2}{P_2}-1+N\right)+2\left(H_{s_3}+C_{d_3}d_3\right)\left(\frac{(2-N)D_3}{P_3}-1+N\right)+\left(H_{s_4}+C_{d_d}d_4\right)\left(\frac{(2-N)D_4}{P_4}-1+N\right)\right\}}}$$

2.2 Fuzzy integrated production model for deteriorating inventory item.

In this section we introduce the fuzzy inventory integrated models by changing the crisp lotsize in section 1 into fuzzy lotsize. Suppose fuzzy lotsize $\tilde{q}$ be the trapezoidal fuzzy number $\tilde{q}=(q_1,q_2,q_3,q_4)$ with $o\le q_1\le q_2\le q_3\le q_4$. Then we get the fuzzy total inventory cost of the integrated production model for deteriorating inventory item.

$$P(\tilde{T}C(\tilde{q},N))=\left\{\begin{array}{l}\frac{D_1C_1}{Nq_4}+\frac{d_1e_1}{2N}+\left(H_{s_1}+C_{d_1}d_1\right)q_1\left[\frac{D_1}{P_4}-\frac{1}{2}+\frac{N}{2}-\frac{D_1N}{2P_4}\right],\\ \frac{D_2C_2}{Nq_3}+\frac{d_2e_2}{2N}+\left(H_{s_2}+C_{d_2}d_2\right)q_2\left[\frac{D_2}{P_3}-\frac{1}{2}+\frac{N}{2}-\frac{D_2N}{2P_3}\right],\\ \frac{D_3C_3}{Nq_2}+\frac{d_3e_3}{2N}+\left(H_{s_3}+C_{d_3}d_3\right)q_3\left[\frac{D_3}{P_2}-\frac{1}{2}+\frac{N}{2}-\frac{D_3N}{2P_2}\right],\\ \frac{D_4C_4}{Nq_1}+\frac{d_4e_4}{2N}+\left(H_{s_4}+C_{d_4}d_4\right)q_4\left[\frac{D_4}{P_1}-\frac{1}{2}+\frac{N}{2}-\frac{D_4N}{2P_1}\right]\end{array}\right\}$$

We can apply the Graded Mean integration representation if $P(\tilde{T}C(\tilde{q},N))$ by formula (2) with $o\le q_1\le q_2\le q_3\le q_4$. It will not change the meaning of the formula if we replace inequality conditions $o\le q_1\le q_2\le q_3\le q_4$ into the following inequality $q_2-q_1\ge 0, q_3-q_2\ge 0, q_4-q_3\ge 0, q_1\ge 0$. In the following steps, extension of the lagrangean method is used to find the solutions of q_1, q_2, q_3 and q_4 to minimize $P(\tilde{T}C(\tilde{q},N))$.

Step 1: Solve the unconstraint problem. To find the min $\left[P(\tilde{T}C(\tilde{q},N))\right]$. We have to find the derivative of min $\left[P(\tilde{T}C(\tilde{q},N))\right]$ with respect to q_1, q_2, q_3, q_4.

$$\frac{\partial P}{\partial q_1}=\frac{1}{6}\left\{\left(H_{S_1}+C_{d_1}d_1\right)\left[\frac{D_1}{P_4}\left(1-\frac{N}{2}\right)+\frac{1}{2}(N-1)\right]-\left(\frac{1}{q_1^2}\right)\left(\frac{D_4C_4}{N}\right)\right\}$$

$$\frac{\partial P}{\partial q_2}=\frac{1}{6}\left\{2\left(H_{S_2}+C_{d_2}d_2\right)\left[\frac{D_2}{P_3}\left(1-\frac{N}{2}\right)+\frac{1}{2}(N-1)\right]-\left(\frac{2}{q_2^2}\right)\left(\frac{D_3C_3}{N}\right)\right\}$$

$$\frac{\partial P}{\partial q_3}=\frac{1}{6}\left\{2\left(H_{s_3}+C_{d_3}d_3\right)\left[\frac{D_3}{P_2}\left(1-\frac{N}{2}\right)+\frac{1}{2}(N-1)\right]-\frac{2}{q_3^2}\left(\frac{D_2C_2}{N}\right)\right\}$$

$$\frac{\partial P}{\partial q_4}=\frac{1}{6}\left\{\left(H_{s_4}+C_{d_4}d_4\right)\left[\frac{D_4}{P_1}\left(1-\frac{N}{2}\right)+\frac{1}{2}(N-1)\right]-\frac{1}{q_4^2}\left(\frac{D_1C_1}{N}\right)\right\}$$

Let all the above partial derivatives equal to zero and solve q_1, q_2, q_3, q_4

$$q_1=\sqrt{\frac{2D_4C_4}{N\left\{\left(H_{s_1}+C_{d_1}d_1\right)\left[\frac{(2-N)D_1}{P_4}+N-1\right]\right\}}}$$

$$q_2=\sqrt{\frac{4D_3C_3}{2N\left\{\left(H_{s_2}+C_{d_2}d_2\right)\left[\frac{(2-N)D_2}{P_3}+N-1\right]\right\}}}$$

$$q_3=\sqrt{\frac{4D_2C_2}{2N\left\{\left(H_{s_3}+C_{d_3}d_3\right)\left[\frac{(2-N)D_3}{P_2}+N-1\right]\right\}}}$$

$$q_4=\sqrt{\frac{2D_1C_1}{N\left\{\left(H_{s_4}+C_{d_4}d_4\right)\left[\frac{(2-N)D_4}{P_1}+N-1\right]\right\}}}$$

Because the above show that $q_1>q_2>q_3>q_4$ Therefore set K=1 and go to step 2.

Step 2: Convert the inequality constraint $q_2-q_1\geq 0$ into equality constraint $q_2-q_1=0$ and optimize $P\left(\widetilde{TC}(q,N)\right)$ subject to $q_2-q_1=0$ by the Lagrangean method.

$$L(q_1,q_2,q_3,q_4,\lambda)=P\left(\widetilde{TC}(q,N)\right)-\lambda(q_2-q_1)$$

Taking the partial derivatives of $L(q_1,q_2,q_3,q_4,\lambda)$ with respect to q_1,q_2,q_3,q_4 and λ to find the minimization of $L(q_1,q_2,q_3,q_4,\lambda)$

Let all the above partial derivatives $\frac{\partial L}{\partial q_1},\frac{\partial L}{\partial q_2},\frac{\partial L}{\partial q_3},\frac{\partial L}{\partial q_4},\frac{\partial L}{\partial \lambda}$ equal to zero and solve to q_1, q_2, q_3, q_4. Then we get

$$q_1=q_2=\sqrt{\frac{2\left(\frac{D_4C_4}{N}+\frac{2D_3C_3}{N}\right)}{\left(H_{s_1}+C_{d_1}d_1\right)\left[\frac{D_1}{P_4}(2-N)+N-1\right]+2\left(H_{s_2}+C_{d_2}d_2\right)\left[\frac{D_2}{P_3}(2-N)+N-1\right]}}$$

$$q_3=\sqrt{\frac{4\frac{D_2C_2}{N}}{2\left\{\left(H_{s_3}+C_{d_3}d_3\right)\left[\frac{D_3}{P_2}(2-N)+N-1\right]\right\}}}$$

$$q_4=\sqrt{\frac{\frac{2D_1C_1}{N}}{\left(H_{s_4}+C_{d_4}d_4\right)\left[\left(1-\frac{D}{P_1}\right)\left(2-N\right)+N-1\right]}}$$

Because the above show that $q_3>q_4$ it does not satisfy the constraint $0<q_1\le q_2\le q_3\le q_4$. Therefore it is not a local optimum. Set K=2 and go to step 3.

Step 3: Convert the inequality constraint $q_2 - q_1 \ge 0$, $q_3 - q_2 \ge 0$ into equality constraints $q_2-q_1=0$ and $q_3-q_2=0$. We optimize $P\left(\tilde{T}C\left(q,N\right)\right)$ subject to $q_2-q_1=0$ and $q_3-q_2=0$ by the Lagrangean method. Then the Lagrangean method is

$$L\left(q_1,q_2,q_3,q_4,\lambda_1,\lambda_2\right)=P\left(\tilde{T}C\left(q,N\right)\right)-\lambda_1\left(q_2-q_1\right)-\lambda_2\left(q_3-q_2\right)$$

In order to find the minimization of $L(q_1, q_2, q_3, q_4, \lambda_1, \lambda_2)$. We take the partial derivatives of $L(q_1, q_2, q_3, q_4, \lambda_1, \lambda_2)$ with respect to $q_1, q_2, q_3, q_4, \lambda_1, \lambda_2$ and let all the partial derivatives.

$\frac{\partial L}{\partial q_1},\frac{\partial L}{\partial q_2},\frac{\partial L}{\partial q_3},\frac{\partial L}{\partial q_4},\frac{\partial L}{\partial \lambda_1},\frac{\partial L}{\partial \lambda_2}$ equal to zero and to solve q_1, q_2, q_3, q_4

$$q_1=q_2=q_3=\sqrt{\frac{2\left(\frac{D_1C_1}{N}+\frac{2D_2C_2}{N}+\frac{2D_3C_3}{N}\right)}{\begin{array}{l}\left(H_{s_1}+C_{d_1}d_1\right)\left[\left(1-\frac{D}{P_4}\right)\left(2-N\right)+\left(N-1\right)\right]+2\left(H_{s_2}+C_{d_2}d_2\right)\left[\left(1-\frac{D}{P_3}\right)\left(2-N\right)+\left(N-1\right)\right]+\\2\left(H_{s_3}+C_{d_3}d_3\right)\left[\left(1-\frac{D}{P_1}\right)\left(2-N\right)+\left(N-1\right)\right]\end{array}}}$$

$$q_4=\sqrt{\frac{\frac{2D_1C_1}{N}}{\left(H_{s_4}+C_{d_4}d_4\right)\left[\left(1-\frac{D}{P_4}\right)\left(2-N\right)+N-1\right]}}$$

The above result $q_1>q_4$ does not satisfy the constraint $o\le q_1\le q_2\le q_3\le q_4$. There fore set K=3 and go to step 4.

Step 4: Convert the inequality constraint $q_2-q_1\ge0$, $q_3-q_2\ge0$ & $q_4-q_3\ge0$ into equality constraints $q_2-q_1=0$, $q_3-q_2=0$, $q_4-q_3=0$. We optimize $P\left(\tilde{T}C\left(q,N\right)\right)$ subject to $q_2-q_1=0$, $q_3-q_2=0$, $q_4-q_3=0$ by the Lagrangean method. The Lagrangean function is given by

$$L\left(q_1,q_2,q_3,q_4,\lambda_1,\lambda_2,\lambda_3\right)=P\left(\tilde{T}C\left(q,N\right)\right)-\lambda_1\left(q_2-q_1\right)-\lambda_2\left(q_3-q_2\right)-\lambda_3\left(q_4-q_3\right)$$

In order to find the minimization of $L\left(q_1,q_2,q_3,q_4,\lambda_1,\lambda_2,\lambda_3,\lambda_4\right)$. We take the partial derivative of $L\left(q_1,q_2,q_3,q_4,\lambda_1,\lambda_2,\lambda_3\right)$ with respect to $q_1, q_2, q_3, q_4, \lambda_1, \lambda_2, \lambda_3$. Let all the partial derivatives.

$\frac{\partial L}{\partial q_1},\frac{\partial L}{\partial q_2},\frac{\partial L}{\partial q_3},\frac{\partial L}{\partial q_4},\frac{\partial L}{\partial \lambda_1},\frac{\partial L}{\partial \lambda_2},\frac{\partial L}{\partial \lambda_3}$ equal to zero.

$$q^{c}=q_1=q_2=q_3=q_4=\sqrt{\frac{2\left(\frac{D_4C_4}{N}+\frac{2D_3C_3}{N}+\frac{2D_2C_2}{N}+\frac{D_1C_1}{N}\right)}{\left(H_{s_1}+C_{d_1}d_1\right)\left(\frac{D_1}{P_4}\left(2-N\right)+N-1\right)+2\left(H_{s_1}+C_{d_2}d_2\right)\left(\frac{D_2}{P_3}\left(2-N\right)+\left(N-1\right)\right)+2\left(H_{s_3}+C_{d_3}d_3\right)\left(\frac{D_3}{P_2}\left(2-N\right)+\left(N-1\right)\right)+\left(H_{s_4}+C_{d_4}d_4\right)\left(\frac{D_4}{P_1}\left(2-N\right)+N-1\right)}}$$

III. Conclusion

This paper presents fuzzy optimal lotsize integrated production model for a deteriorating inventory item and minimizing the total cost inventory function. In this model N → the number of deliveries is treated as a fixed constant and other notations are represented by fuzzy numbers. For this fuzzy model, a method of defuzzification, graded mean integration representation is applied to find estimate of total cost function for the integrated production model for a deteriorating inventory item.

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