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Super vertex Gracefulness of Some Special Graphs

N.Murugesan[1], R.Uma[2]
[1](*Post graduate and Research Dept. of Mathematics, Government Arts College, (Autonomous), Coimbatore- 18, India)*
[2](*Dept. of Mathematics, Sree Saraswathi Thyagaraja College, Pollachi, India)*

Abstract : *Certain classes of graphs obtained from paths are super vertex graceful. In this paper, such classes of derived graphs like twig graphs, spider graphs, regular caterpillars and fire crackers are analysed under super vertex graceful mapping.*

Keywords - *fire crackers, graceful graphs, regular caterpillars, super vertex graceful graphs, spider graphs, and. twig graphs.*

I. Introduction

Graph labelling, where the vertices are assigned values subject to certain conditions, often have been motivated by practical problems. Rosa introduced the concept of graceful labelling in 1967. Since then, many types of labelling came into existence. For an excellent survey on this topic refer to [1]. Sin Min Lee [2] introduced super vertex graceful labelling in the year 2009. N. Murugesan and R.Uma [3, 4, 5, 6] have analysed complete bipartite graphs under super vertex graceful map, amalgamation of graphs under graceful mapping and Fibonacci graceful mapping. Solairaju. A, Vimala.C, Sasikala.A, [7] studied the the odd gracefulness of fire crackers. In this paper, a discussion is made under super vertex graceful map on twig graphs, regular caterpillars, fire crackers and spider graphs.

II. Definitions

2.1Twig:

A graph G (V,E) obtained from a path by attaching exactly two pendant edges to each internal vertices of the path is called a Twig. A twig T_m with 'm' internal vertices has 3m+1 edges and 3m+2 vertices.

2.2 Spider graph:

A spider graph $\mathbf{S_{n,m}}$ is a graph with 'n' spokes in which each spoke is a path on length 'm'.

2.3 Fire cracker graphs

Fire cracker graph is a graph obtained by attaching stargraphs S_q at the each pendant vertices of a path graph P_p. Such a fire cracker graph is denoted as $P_p \Theta S_q$. It can be noted that the fire cracker graph $P_n \Theta S_m$ is of order n+2m and size n+2m-1 .

2.4 Regular caterpillar

Regular caterpillar is the graph obtained by joining rK_1 graphs at the pendant vertices of the path P_p. It is denoted as $P_n \odot rK_1$

2.5 Tensor product

The tensor product $G_1 \otimes G_2$of two simple graphs G_1 and G_2 is the graph with $V(G_1 \otimes G_2) = V_1 \times V_2$ and where $(u_1, u_2)$and (v_1, v_2) are adjacent in $G_1 \otimes G_2$ iff u_1 is adjacent to v_1 in $G_1$and u_2 is adjacent to v_2 in G_2. It is denoted as $K_{1,5}(T_p)P_2$

2.6 Super vertex graceful graph

A graph G with p vertices and q edges , vertex set V(G) and edge set E(G), is said to be super vertex graceful (SVG), if there are bijection functions *f*, from V(G) onto P, and f^+ from E(G) onto Q, such that f^+ (u,v) = *f*(u) + *f*(v) for every (u,v) $\in$ E(G), where P and Q are finite set of integers defined as follows:

$$P = \begin{cases} \pm 1, \pm 2, \dots \pm \frac{p}{2} & if\ p\ is\ even \\ 0, \pm 1, \pm 2, \dots \pm \frac{p-1}{2} & if\ p\ is\ odd \end{cases}$$

$$Q = \begin{cases} \pm 1, \pm 2, \dots \pm \frac{p}{2} & if\ n\ is\ even \\ 0, \pm 1, \pm 2, \dots \pm \frac{q-1}{2} & if\ n\ is\ odd \end{cases}$$

In the graph given in Fig 6, both the size and order is 5. Therefore P = Q = { -2,-1,0,1,2}. Here f^+ and f are defined such that f^+ (-1,1) = 0; f^+ (-1,2) = 1; f^+ (0,2) = 2; f^+ (1,-2) = -1; f^+ (-2,0) = -2.Then G is SVG. It is interesting to note that C_5 is SVG. But Golomb [2], discussed that C_5 is not graceful.

III. Results

3.1Theorem:

Let $S_{n,m}$ be a spider graph with 'n' spokes each with path of length'm'. Then the graph is super vertex graceful only if 'n' is even and m =2.

Proof:

Let G be spider graph with $n = 2k, k > 1$ and $m = 2$.Then the order and size of G is 'mn +1' and mn respectively. Let $V = \{x, V_1, V_2\}$ be the vertex set where $V_1 = \{v_1, v_2, v_3, \dots, v_{\frac{n}{2}}, v_{-1}, v_{-2}, v_{-3}, \dots, v_{\frac{-n}{2}}\}$ are the vertices adjacent to the apex vertex x and $V_2 = \{u_1, u_2, u_3, \dots, u_{\frac{n}{2}}, u_{-1}, u_{-2}, u_{-3}, \dots, u_{\frac{-n}{2}}\}$ are the pendant vertices. Also the edge $e_i = v_i u_i for\ all\ u_i, v_i$. By the definition of super vertex graceful labelling, the vertex label set P and edge label set Q are defined as follows:

$P = \left\{0, \pm 1,\ \pm 2, \pm 3, \dots, \pm \frac{mn}{2}\right\}$ and $P = \{0, \pm 1,\ \pm 2, \pm 3, \dots, \pm \frac{mn}{2}\}$ respectively.

The function$f: V \to P$is defined as follows:

$$f(x) = 0; f(v_i) = 2i,; f(u_i) = -(2i-1), f(v_{-i}) = -f(v_i), f(u_{-i}) = -f(u_i), i = 1, 2, \dots, \frac{n}{2}$$

Then the function $f^+(u_{i,} v_j) = f(u_i) + f(v_j)$ admits super vertex graceful labelling. Hence G is super vertex graceful.

Conversely, if "n" is odd and "m = 2" or "n" is even and "m ≠ 2" , the map $f: V \to P$ where V is the vertex set of the spider and P is the set of vertex labels does not induce the bijective map $f^+(u_{i,} v_j) = f(u_i) + f(v_j)$. Hence the proof.

3.1.1 Example:

The graph given in Fig. 8 is $S_{10,2}$ of order 21 and size 20. The apex vertex is 'x' and $f(x) = 0; f(v_i) = 2i,; f(u_i) = n - (2i-1), f(v_{-i}) = -f(v_i), f(u_{-i}) = -f(u_i), i = 1, 2, \dots, 5$. By the definition of super vertex graceful mapping $P = \{0, \pm 1,\ \pm 2, \pm 3, \dots, \pm 10\}$ and $Q = \{\pm 1,\ \pm 2, \pm 3, \dots, \pm 10\}$. The above defined "*f*" admits is a bijective map which admits super vertex graceful mapping. Hence the graph is super vertex graceful.

3.3 Theorem:

Twig T_m is super vertex graceful for all values of m.

Proof:

Let G be a twig obtained from a path of 'n' vertices. If the order of the path is 'n' then by the definition of Twig, the order and size of it are $p = 3n - 4$ and $q = 3n - 3$ respectively. Assume that G = {V, E, f} where $V = \{u_i, v_i, x_{ij}, y_{ij} / i = 1, 2, \dots, \frac{p}{2}, j = 1,2\}$ if p is even and if p is odd then V is defined as $V = \{u_i, v_i, v_0\, x_{ij}, y_{ij} / i = 1, 2, \dots, \frac{p}{2}, j = 1,2\}$. If p is even, then there will be two middle vertices, let u_i s represent the vertices of the path from the initial (left) vertex to the first mid vertex of the path, v_i's represent the vertices from the second mid vertex to the terminal (right) vertex of the path, x_{ij} represent the pendant vertices adjacent to the internal vertices u_i and y_{ij} are adjacent pendant vertices of v_i. By the definition of super vertex graceful labelling the vertex label set P and edge label set Q are defined as follows:

$$P = \begin{cases} \pm 1, \pm 2, \dots \frac{p}{2}, if\ p\ is\ even \\ 0, \pm 1, \pm 2, \dots, \frac{p-1}{2}\ if\ p\ is\ odd \end{cases}$$

$$Q = \begin{cases} \pm 1, \pm 2, \dots \frac{q}{2}, if\ q\ is\ even \\ 0, \pm 1, \pm 2, \dots, \frac{q-1}{2}\ if q\ is\ odd \end{cases}$$

The mapping *'f'* is defined as follows:

Case (i) p is odd:

$f(v_i) = -\left(\frac{3n}{2} - \left[i + \left\lfloor\frac{i}{2}\right\rfloor\right]\right)\ for\ i = 1\ mod\ 2, 1 \le i \le \frac{n}{2}$

$f(v_i) = i + \frac{i}{2}, i = 0\ mod\ 2, 1 \le i \le \frac{n}{2}$

$f(y_{ij}) = \begin{cases} i + \left\lfloor\frac{i}{2}\right\rfloor\ j = 1 \\ i + \left\lceil\frac{i}{2}\right\rceil j = 2 \end{cases} i = 1\ mod\ 2$

$f(y_{ij}) = f(v_{i-1}) + j, j = 1,2;\ i = 0\ mod\ 2$

$f(u_i) = -f(v_i); f(x_{ij}) = -f(y_{ij})\ for\ all\ i\ and\ j.$

Case (ii) p is even:

$f(v_0) = 0; f(v_1) = \frac{n}{2} - 1;$

$f(v_{i+2}) = f(v_i) - 3, i = 1, 3, 5, \dots \frac{n-1}{2}$

$f(v_i) = i + \frac{i}{2}, i = 2, 4, \dots \frac{n-1}{2}$

$f(y_{ij}) = f(v_{i-1}) + j\ ; i = 1, 3, 5 \dots \frac{n-1}{2}; j = 1, 2;$

$f(y_{ij}) = f(v_{i-1}) - j, i = 2, 4, 6, \dots \frac{n-1}{2}; j = 1, 2.$

$f(u_i) = -f(v_i); f(x_{ij}) = -f(y_{ij})\ for\ all\ i\ and\ j.$

The map $f^+ : V \times V \rightarrow Q$ defined by $f^+(s_i, s_j) = f(s_i) + f(s_j)$ is a bijective map and hence admits super vertex graceful labelling.

3.4 Example:

T_5 is given in Fig. 10 is super vertex graceful is obtained from P_5. By the definition of twig, the order and size of the graph are 11 and 10 respectively. Then by the super vertex graceful map, $P = \{0, \pm 1, \pm 2, \pm 3, \dots, \pm 5\}$ and $Q = \{\pm 1, \pm 2, \pm 3, \dots, \pm 5\}$. The labels marked in the Fig 10 admit the functions defined in the above theorem and hence the graph T_5 is super vertex graceful.

3.5 Theorem:

All fire cracker graphs $P_n \Theta S_m$ are super vertex graceful.

Proof:

Let G = {V, E, f^+} be a graph of order 'p' and size 'q'. The order of the path is "n" and the star is "m". The vertex set $V = \{v_1, v_2, v_3, \dots, v_{\lfloor\frac{n}{2}\rfloor}, u_1, u_2, u_3, \dots, u_{\lfloor\frac{n}{2}\rfloor}, x_1, x_2 \dots, x_m, y_1, y_2 \dots, y_m\}$ and $z \in V$ if 'n' is odd. Then $p = n + 2m$ and $q = n + 2m - 1$. Here $u_i's, v_i's$ are the vertices of the path and $x_i's, y_i's$ are the vertices of the star. If 'n' is even then are two middle vertices one labelled as $v_{\lfloor\frac{n}{2}\rfloor}$ and the another one is u_1. If 'n' is odd, then the middle vertex is labelled as 'z' as shown in the Fig 11 and Fig 12 .

Then by the definition of super vertex graceful map the vertex label set P is defined as follows:

$$P = \begin{cases} \pm 1, \pm 2, \pm 3, \dots \pm \frac{p}{2}\ if\ p\ is\ even \\ 0, \pm 1, \pm 2, \dots \pm \frac{p-1}{2}\ if\ p\ is\ odd \end{cases}$$

Here we have four cases depending on the value of p and the corresponding map 'f' is defined from $V \times V \to P$.

Case (i) : Let $n = 4i - 2, i \geq 1$

$$f(v_k) = \begin{cases} \left(\left(\frac{n}{2}\right) + k + -(i-1)\right), for\ k = 2i-1, i = 1,2, \ldots \left\lceil \frac{n}{4} \right\rceil, \\ \frac{n}{2} - \left(\left\lfloor \frac{n}{4} \right\rfloor + i\right) for\ i = 1,2, \ldots \left\lceil \frac{n}{4} \right\rceil, k = 2i \end{cases}$$

$f(u_i) = -(f(v_{n-i})) for\ i = 1,2, \ldots \frac{n}{2}.$

Case (ii): Let $n = 4i - 1, i \geq 1$

$f(z) = 0;$

$$f(v_k) = \begin{cases} -\left(\left\lfloor \frac{n}{4} \right\rfloor + m + i\right),\ k = 2i-1, for\ i = 1,2, \ldots, \left\lceil \frac{n}{4} \right\rceil; \\ \left\lceil \frac{n}{4} \right\rceil - i, k = 2i, for\ i = 1,2, \ldots, \left\lceil \frac{n}{4} \right\rceil \end{cases}$$

$f(u_i) = -(f\left(v_{\frac{n}{2}+1-(i-1)}\right)) for\ i = 1,2, \ldots \frac{n}{2}.$

Case (iii): Let $n = 4i, i \geq 1$

$$f(v_k) = \begin{cases} -\left\lfloor \frac{n}{3} \right\rfloor + i,\ k = 2i-1, for\ i = 1,2,3, \ldots, \frac{n}{4} \\ m + \frac{n}{4} + i,\ k = 2i\ for\ i = 1,2,3, \ldots, \frac{n}{4} \end{cases}$$

$f(u_i) == -(f\left(v_{\frac{n}{2}-(i-1)}\right)) for\ i = 1,2, \ldots \frac{n}{2}$

Case (iv): Le t $n = 4i + 1, i \geq 1$

$f(z) = 0;$

$$f(v_k) = \begin{cases} \left\lfloor \frac{n}{3} \right\rfloor - i, k = 2i-1, for\ i = 1,2, \ldots, \left\lceil \frac{p}{4} \right\rceil \\ \left(\frac{n}{2} + (i+1)\right), k = 2i, for\ i = 1,2, \ldots, \left\lceil \frac{p}{4} \right\rceil \end{cases}$$

$f(u_i) == -(f\left(v_{\frac{n}{2}-(i-1)}\right)) for\ i = 1,2, \ldots \frac{n}{2}$

$f(y_j) = -(f(v_n) - j)\ for\ j = 1,2, \ldots, m$ and $f(z_j) = -\left(f(y_j)\right)\ j = 1,2, \ldots, m.$

The above map admits super vertex graceful map defined by $f^+(u,v) = \{f(u) + f(v) where\ u, v \in V\}.$

3.6 Example:

The graph given in Fig. 13 is SVG and the order is 13 and the size is 12.Then, the centre vertex of the path is labeled zero and the other vertices are labeled alternatively with positive and negative values defined in P where $P = \{0, \pm1, \pm2, \pm3, \pm4, \pm5, \pm6\}$. The resulting edge labels 2-5 = -3, 3-5 =-2, 4-5 = -1… 5-2 =3, 5-3 =2, 5-4 =1. Hence $Q = \{\pm1, \pm2, \pm3, \pm4, \pm5, \pm6\}$. Hence the graph is super vertex graceful.

Similarly, the graphs in Fig. 14, Fig 15 and Fig. 16 are examples respectively for n =8, 9 10. The labeling in the graph satisfies the function map defined in the above theorem.

3.7 Theorem: Regular caterpillars $P_n \odot rK_1$ are super vertex graceful only if the order of the path is even for all r.

Proof:

Let us assume that the regular caterpillars of the path of odd order. Let (2r+1) K_1 graphs be adjacent to all the vertices of the path. Then the size of the graph is odd. By the definition of the super vertex gracefulness the edge label set is $Q = \{0, \pm1, \pm2, \pm3, \ldots\}$. But the function $f: V(G) \to P$ cannot induce the edge label 0 where $P = \{0, \pm1, \pm2, \pm3, \ldots\}$. Hence the graphs are not super vertex graceful.

Let us now consider the graphs of paths with even order.

Let G = {V, E, f^+} be a regular caterpillar of order p and size q = p-1, where $V = \{V_1, V_2\}$ be the vertex set. Here $V_1 = \{u_{-\frac{n}{2}}, \ldots u_{-3}, u_{-2}, u_{-1}, u_1, u_2, u_3, \ldots, u_{\frac{n}{2}}\}$ be the vertices of path of length n and $V_2 = \{x_{ij}, y_{ij} / i = 1, 2, \ldots, \frac{n}{2}, j = 1, 2, \ldots, r\}$ x_{ij}'s and y_{ij}'s be the pendant vertices adjacent to u_i's and u_{-i}'s respectively (Fig 17). By the definition of super vertex graceful labeling the vertex label set P and edge label set Q are given below:

$P = \{\pm 1, \pm 2, \ldots, \pm \frac{p}{2}\}$ and $Q = \{0, \pm 1, \pm 2, \ldots, \pm \frac{p-1}{2}\}$

Then the function $f: V(G) \to P$ is defined as follows:

$f(u_1) = \frac{p}{2}; f(u_{i+1}) = f(u_i) - (r+1), i = 1\ (mod)2,\ 1 \le i \le \frac{n}{2};$

$f(u_2) = -(r+1); f(u_{i+2}) = f(u_i) - (r+1), i = 0\ (mod)2, 1 \le i \le \frac{n}{2};$

$f(x_{1j}) = -j, j = 1, 2, \ldots, r;$

$f(x_{i+1j}) = f(x_{ij}) - (r+1), i = 1\ (mod)2, 1 \le i \le \frac{n}{2}, j = 1, 2, \ldots, r;$

$f(x_{ij}) = f(u_{i-1j}) - j, i = 0\ (mod)2, 1 \le i \le \frac{n}{2}, j = 1, 2, \ldots, r;$

$f(u_{-i}) = -f(u_i)\ \forall\, i\, ; f(y_{ij}) = -f(x_{ij}), \forall\, i, j.$

f^+ is a mapping from VxV to Q where Q = $\left\{0, \pm 1, \pm 2, \pm 3, \ldots \pm \frac{n+1}{2}\right\}$. The above defined maps admit super vertex graceful labeling.

3.8 Example:

In the graph given, in Fig. 18, the regular cater pillar G is obtained by joining r = 3 pendant edges to the path oif order p = 4. Hence the order and size of G are $p = n + nr = 4 + 12 = 16$ and $m = n + nr - 1 = 15$. Hence $P = \{\pm 1, \pm 2, \ldots, \pm 8\}$ and $Q = \{0, \pm 1, \pm 2, \ldots, \pm 7\}$. The vertices of the path are $\{u_{-2}, u_{-1}, u_1, u_2\}$. The vertices of the pendant edges are $\{x_{ij}, y_{ij}\ i = 1, 2, j = 1, 2, 3\}$. $f(u_1) = -8, f(u_2) = f(u_1) + (r+1) = -8 + 4 = -4, f(x_{11}) = 1, f(x_{12}) = 2, f(x_{13}) = 3, f(x_{21}) = f(u_1) - j = -7, f(x_{22}) = -6, f(x_{23}) = -5.$

Now, $f^+(v_i, v_j) = f(v_i) + f(v_j)$ *for all i and j* induces the edge label set Q so that the regular cater pillar is super vertex graceful. But the regular caterpillar in Fig. 19 is formed from the path of order 3 and it is not super vertex graceful.

3.9 Theorem:

The tensor product of P_2 and star $K_{1,n}$ $(K_{1,n}(T_p)P_2)$ is super vertex graceful.

Proof:

Let G be the graph obtained by the tensor product of P_2 and star $K_{1,n}$. By the definition of tensor product given in 2.3 the graph obtained is disconnected, the order and size is $2(n+1)$ and $2n$ respectively. Let $u_1, u_2, \ldots u_n, u_{n+1}$ be the vertices of the star $K_{1,n}$, where u_1 is the apex vertex. Let v_1, v_2 be the vertices of P_2. Then the vertices of $K_{1,n}(T_p)P_2$ are partitioned into two sets $V_1 = \{x_i = (u_i, v_1) / i = 1, 2, \ldots n+1\}$ and $V_2 = \{y_i = (u_i, v_2) / i = 1, 2, \ldots n+1\}$. Also the degree (and y) of two vertices in G is two and others are one. Let $d(x) = 2\ \&\ d(y) = 2$. The vertices adjacent to x are x_i's and to y are y_i's. Then by the definition of super vertex graceful map the vertex label set P and edge label set Q are defined as follows: $P = \{\pm 1, \pm 2, \ldots \pm (n+1)\}$ and $Q = \{\pm 1, \pm 2, \ldots \pm n\}$. Define $f: V(G) \to P$ as follows:

$f(x) = n + 1; f(y) = -(n+1); f(x_i) = -i$ *and* $f(y_i) = i$ *for* $i = 1, 2, \ldots, n$. Then the map $f^+: V \times V \to Q$ defined by $f^+(x_i, y_j) = f(x_i) + f(y_i)$, *for* $x_i, y_j \in V$ admits super vertex graceful mapping. That is, $K_{1,n}(T_p)P_2$ is a super vertex graceful graph.

3.10 Example:

The graph G in Fig. 20 is $(K_{1,5}(T_p)P_2)$. The order and size are 12 and 10 respectively. Then by super vertex graceful map the vertex label set P and edge label set Q are defined as $P = \{\pm 1, \pm 2, \dots \pm 6\}$ and $Q = \{\pm 1, \pm 2, \dots \pm 5\}$. Then the vertex set are labeled as followed by the map $f: V \to P$ as follows:

$f(x) = 6; f(y) = -6; f(x_1) = -1; f(x_2) = -2; f(x_3) = -3; f(x_4) = -4; f(x_5) = -5; f(y_1) = 1; f(y_2) = 2; f(y_3) = 3; f(y_4) = 4; f(y_5) = 5$

. Then the map $f^+: V \times V \to Q$ induces the set Q as follows:

$f^+(6, +(-1)) = 6 - 1 = 5; f^+(6, +(-2)) = 6 - 2 = 4; f^+(6, +(-3)) = 6 - 3 = 3; f^+(6, +(-4)) = 6 - 4 = 2; f^+(6, +(-5)) = 6 - 5 = 1; f^+(-6, +1) = -6 + 1 = -5; f^+(-6, +2) = -6 + 2 = -4; f^+(-6, +3) = -6 + 3 = -3; f^+(-6, +4) = -6 + 4 = -2; f^+(-6, +5) = -6 + 5 = -1$

It is a super vertex graceful map. Hence G is super vertex graceful.

IV. Figures and Tables

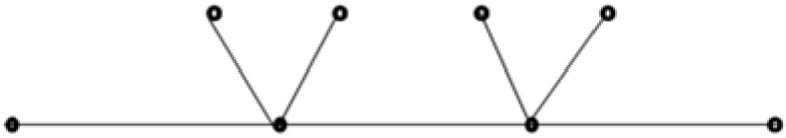

Figure1. A Twig graph T_2 formed from P_4

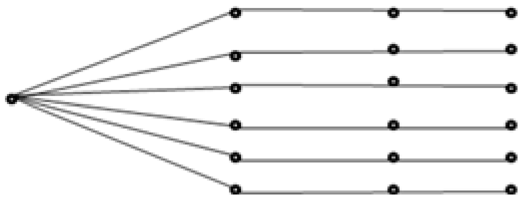

Figure 2. A spider graph $S_{6,3}$

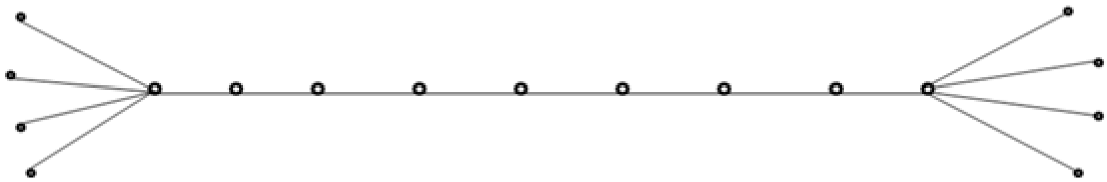

Figure 3. A Fire Cracker graph $P_9 \Theta S_4$

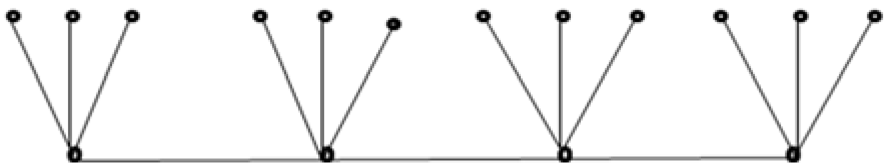

Figure 4. A Regular Caterpillar graph $P_4 \odot 3K_1$

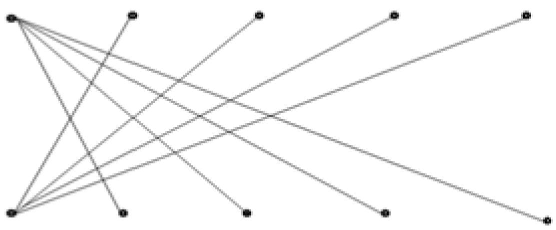

Figure 5. The tensor product of P_2 and star $K_{1,4}$ $(K_{1,4}(T_p)P_2)$.

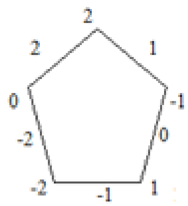

Figure 6. A super vertex graceful graph with order and size 5.

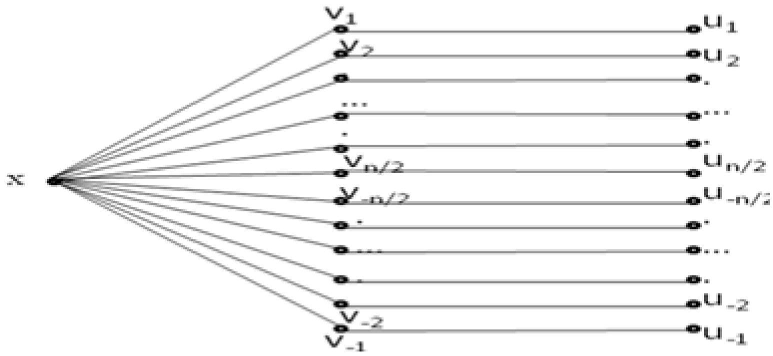

Figure 7 Super vertex graceful spider graph $S_{n,2}$

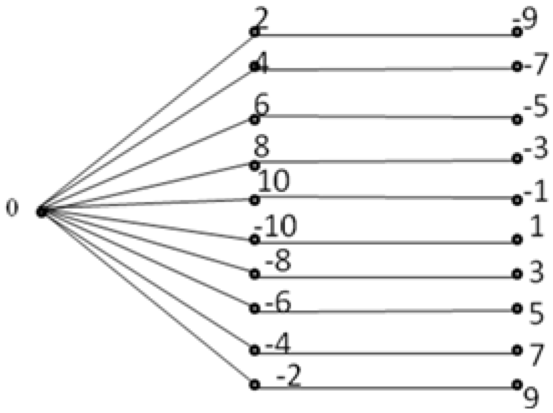

Figure 8 Super vertex graceful spider graph $S_{10,2}$

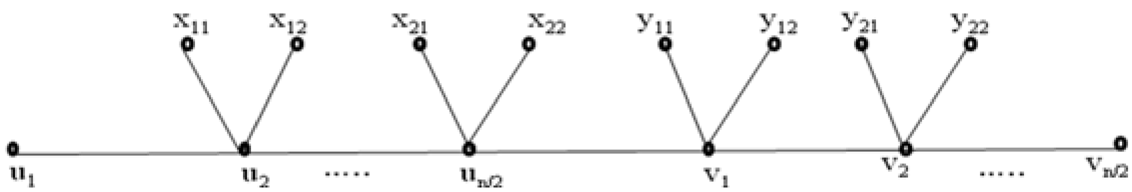

Figure 9 Twig formed from the path with n vertices T_m

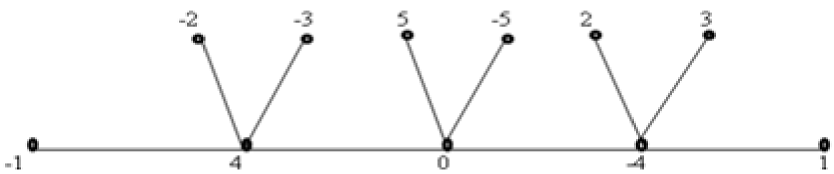

Figure 10 Twig formed from the path with 5 vertices T_3

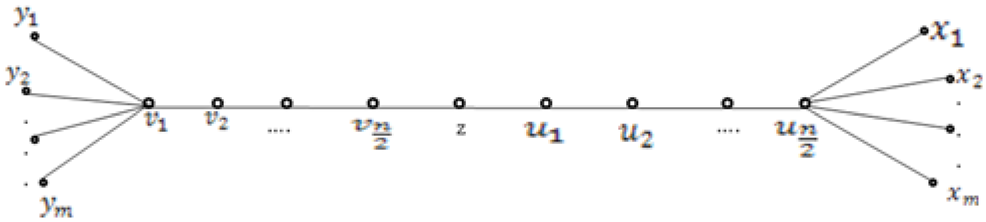

Figure 11 A super vertex graceful graph $P_n \Theta S_m$ (n is odd)

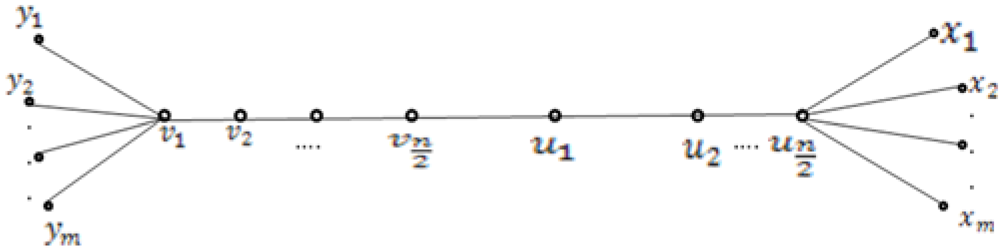

Figure 12 A super vertex graceful graph $P_n \Theta S_m$ (n is even)

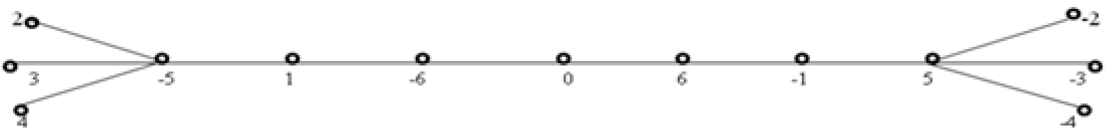

Figure 13 A super vertex graceful graph $P_7 \Theta S_3$ (n=4i-1)

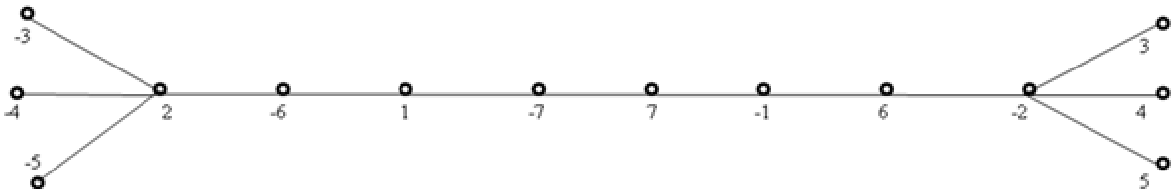

Figure 14 A super vertex graceful graph $P_8 \Theta S_3$ (n=4i)

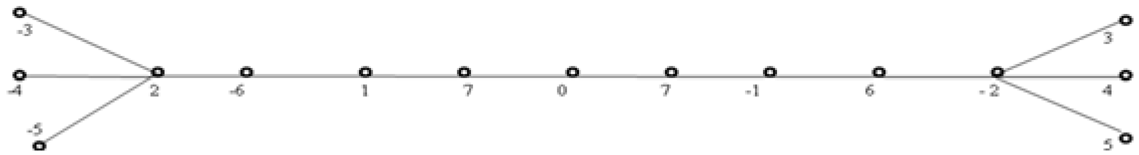

Figure 15 A super vertex graceful graph $P_9 \Theta S_3$ (n=4i+1)

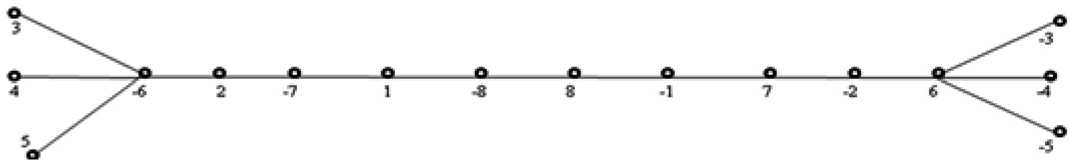

Figure 16 A super vertex graceful graph $P_{10} \Theta S_3$ (n=4i-2)

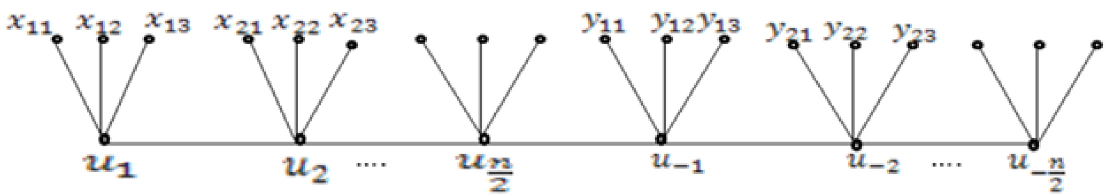

Fig 17 A super vertex regular caterpillar $P_n \odot 3K_1$

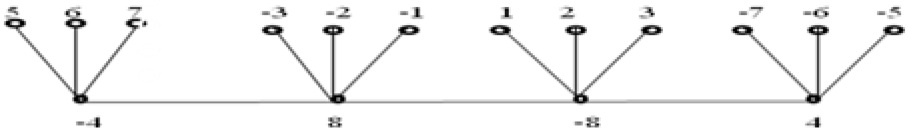

Fig 18 A super vertex regular caterpillar $P_4 \odot 3K_1$

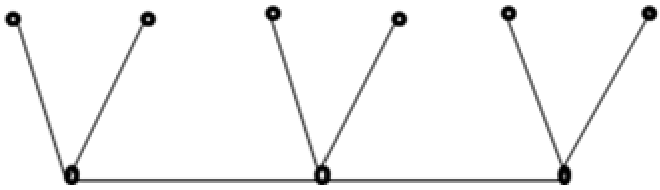

Fig. 19 A non super vertex graceful regular caterpillar $P_3 \odot 2K_1$

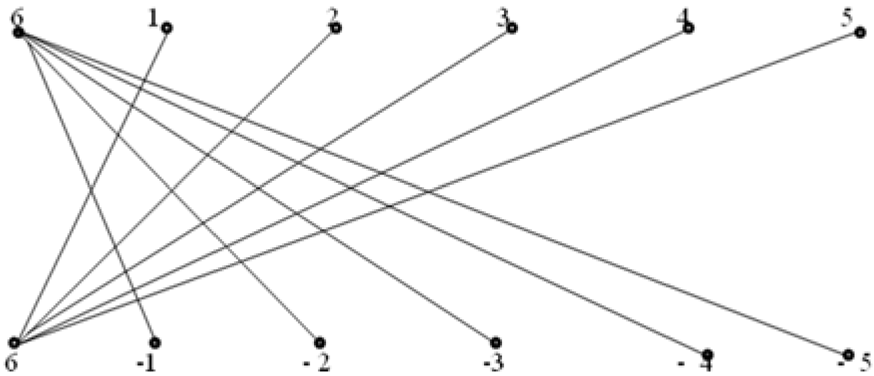

Figure 20 A Super vertex graceful $K_{1,5}(T_p)P_2$

V. Conclusion

The following results are proved:

(i) Firecrackers are super vertex graceful

(ii) Regular caterpillars are super vertex graceful for defined order.

(iii) Tensor product of path and complete bipartite graph is super vertex graceful for specific order for the path.

Thus, it is interesting to observe that the family of graphs obtained from the paths under super vertex graceful mapping . The same type of analysis can be done under different mappings.

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