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Analysis of Primes in Arithmetical Progressions 3*n* + *k* up to a Trillion

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Abstract : *Prime numbers are highly irregularly distributed. Every integer fits in unique form 3n+k. In this paper, distribution of primes in arithmetical progressions 3n+k is analyzed in the range of 1 to 1 trillion. As the decimal number system has the base 10, the distribution trends of the primes in the blocks of all powers of 10 are also presented.*

Keywords –*Block-wise distribution, prime density, prime numbers, prime spacing.*
Mathematics Subject Classification 2010 –*11A41, 11N05, 11N25.*

I. Introduction

A prime number is an integer greater than 1 which has only two positive divisors, viz., 1 and itself. It is well-known from the time of Euclid [1] that the number of primes is infinite. There are various proofs about infinitude of primes. For the work of this paper, huge database of prime numbers was first generated and then analyzed. For this purpose the better algorithm for prime generation could be chosen by the exhaustive comparison presented in [2].

II. Prime Distributions

Prime numbers seem highly irregularly distributed amongst the positive integers. On one side there is yet unproved but strong conjecture that there are infinitely many pairs of successive primes with spacing of 2 only; thereby occurring very close with the least spacing in-between (of course with the exception of first pair of consecutive primes of 2 and 3 with spacing of only 1!) and on the other side there is an elegantly proved property that one can find arbitrarily large spacings between many pairs of successive large primes. The number of primes less than or equal to a given positive value x is denoted by a function $\pi(x)$.

Although there have been many asymptotic formulations about prime distribution, as yet it is an unsettled question: whether there exists a regular pattern of occurrence of primes or not?

III. Prime Distributions in Arithmetical Progressions

Since excluding the first prime 2 all others are odd, it is very clear that all primes except 2 find their place in the arithmetical progression $2n + 1$. Thus this arithmetical progression $2n + 1$ contains infinitely many, in fact all (apart from first one, viz., 2) primes. At the same time it is clear that all members of this arithmetical progression $2n + 1$ are not primes. It contains infinitely many non-primes, i.e., the so called composite numbers also!

The question of 'whether there are other arithmetical progressions which contain all primes' was easy to be settled in negation. But it took genius of Dirichlet [3] to address the weaker version of this query 'whether there are other arithmetical progressions which contain infinitely many primes' in classical assertion. Dirichlet Theorem asserts that an arithmetical progression $an + b$ with $\gcd(a, b) = 1$ contains infinitely many primes and the one with $\gcd(a, b) > 1$ contains only finitely many of them.

It is since then that there have been consistent efforts to analyze the number of primes occurring in various arithmetical progressions in view of getting some direct or indirect hints about prime distribution. The number of primes less than or equal to a given positive value x and that are of the form $an + b$ is denoted by a function $\pi_{a,b}(x)$.

IV. Primes in the Arithmetical Progressions3*n* + 1 and 3*n* + 2

The basic procedure of integer division applies to give one of the numbers $0, 1, 2, \cdots, m - 1$ as remainders after dividing any positive integer by positive integer m. We consider $m = 3$ here, so that the possible values of remainders in the process of division by 3 are 0, 1, and 2. Since every positive integer after dividing by 3 has to yield as remainder one and only one amongst these values, it must be of either of the forms $3n + 0 = 3n$ or $3n + 1$ or $3n + 2$, which constitute arithmetical progressions.

First few numbers of the form $3n$ are

3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, · · ·

As can be clearly seen, each of these is perfectly divisible by 3. Except the first member, viz., 3, none of these is prime. Thus this sequence contains only one prime 3 and its all other members are composite numbers. It becomes evident also by seeing $3n$ as arithmetical progression $3n + 0$, where gcd(3, 0) = 3 > 1 and by Dirichlet Theorem, this is just not a candidate to look ahead for occurrence of many primes.

First few numbers of the form $3n + 1$ are

4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, · · ·

This does contain infinitely many primes as gcd(3, 1) is 1 as per requirement of Dirichlet Theorem.

First few numbers of the form $3n + 2$ are

5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, · · ·

This sequence also does contain infinitely many primes as gcd(3, 2) is 1 as per requirement of Dirichlet Theorem. In fact, there are independent proofs about infinitude of primes of both types $3n + 1$ and $3n + 2$ [4].

We present here a comparative analysis of the primes occurring in arithmetical progressions $3n + 1$ and $3n + 2$.

V. Prime Number Race

For a specific positive integer a and all integers b with $0 \leq b < a$, all the arithmetical progressions $an + b$ which contain infinitely many primes are compared to decide which one amongst them contains more number of primes. This is term popularly known as prime number race [5].

Here we have compared the number of primes of form $3n + 1$ and $3n + 2$ for dominance till one trillion, i.e., 1,000,000,000,000 (10^{12}). Java Programming Language, with its simple and lucid power highlighted in [6], was employed on an electronic computer to analyze prime range thoroughly.

Table 1:*Number of Primes of form 3n + k in various ranges.*

Sr. No.	Range 1-x (1 to x)	Ten Power (x)	Number of Primes of the form $3n + 1$ π3,1(x)	Number of Primes of the form $3n + 2$ π3,2(x)
1.	1-10	101	1	2
2.	1-100	102	11	13
3.	1-1,000	103	80	87
4.	1-10,000	104	611	617
5.	1-100,000	105	4,784	4,807
6.	1-1,000,000	106	39,231	39,266
7.	1-10,000,000	107	332,194	332,384
8.	1-100,000,000	108	2,880,517	2,880,937
9.	1-1,000,000,000	109	25,422,713	25,424,820
10.	1-10,000,000,000	1010	227,523,123	227,529,387
11.	1-100,000,000,000	1011	2,059,018,668	2,059,036,144
12.	1-1,000,000,000,000	1012	18,803,933,520	18,803,978,497

The dominance of number of primes of one form over the other is interesting to note.

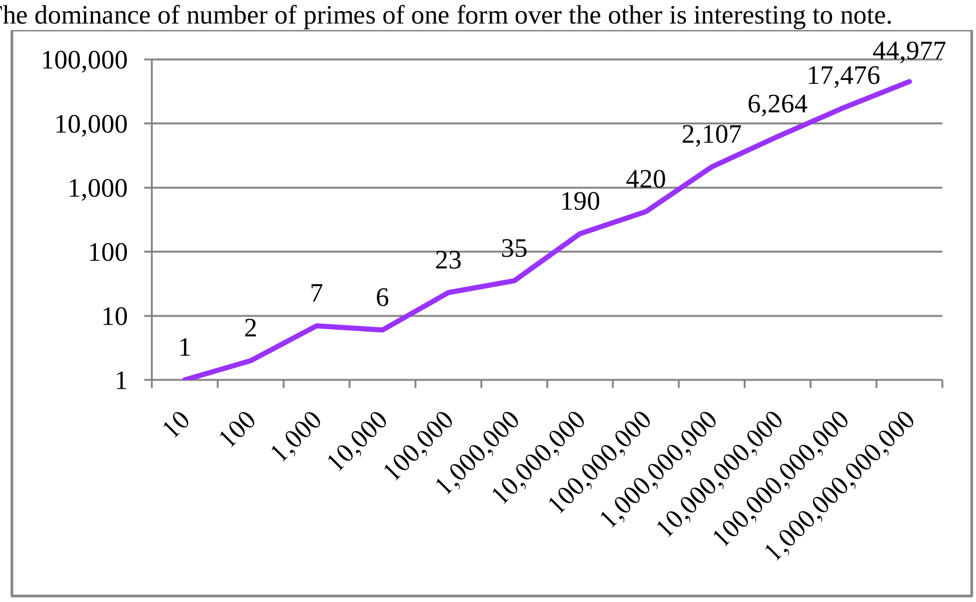


Figure 1: *Dominance of $\pi_{3,2}(x)$ over $\pi_{3,1}(x)$*

The graph is plotted with logarithmic vertical y axis. It is observed that the number of primes of the form $3n + 2$ is more than those of form $3n + 1$ in the initial ranges up to 10^{12} in discrete blocks of 10 powers. Whether this trend of $\pi_{3,2}(x) > \pi_{3,1}(x)$ continues ahead on majority is a subject matter of future explorations. This trend is only for discrete range of 10 powers and in between there is a chance of reversal in the trend.

VI. Block-wise Distribution of Primes

There isno formula to consider all primes in simple go.Neither are the primes finite in number to consider them all together. So, to understand their random-looking distribution, we have adopted an approach of considering all primes up to a certain limit, viz., one trillion (10^{12}) and dividing this complete number range under consideration in blocks of powers of 10 each :

1-10, 11-20, 21-30, 31-40, · · ·
1-100, 101-200, 201-300, 301-400, · · ·
1-1000, 1001-2000, 2001-3000, 3001-4000, · · ·
⋮

A rigorous analysis has been performed on many fronts. Since our range is 1-10^{12}, it is clear that there are 10^{12-i} number of blocks of 10^i size for each $1 \le i \le 12$.

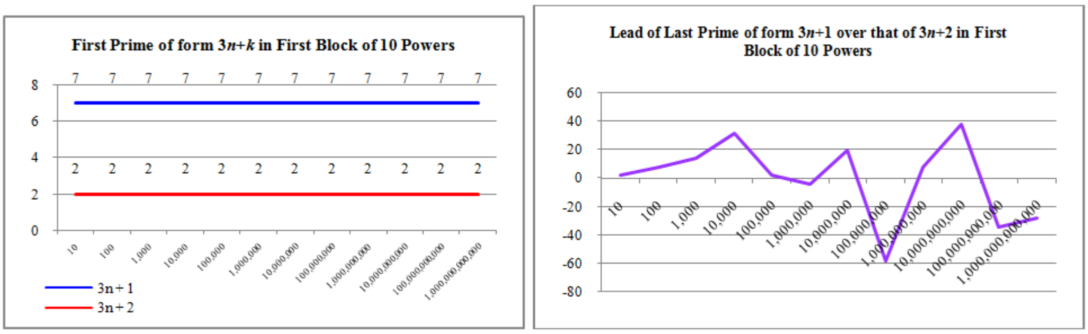
A. *The First and the Last Primes in the First Blocks of 10 Powers*

The inquiry of the first and the last prime in each first block of 10 powers till the range of 10^{12} under consideration is particularly interesting for the last primes, as the first prime of first power of 10 will naturally continue for all blocks ahead.

Table 2:*First and last primes of form 3n+k in first blocks of 10 powers.*

Sr. No.	Blocks of Size (of 10 Power)	First Prime in the First Block		Last Prime in the First Block	
		Form $3n + 1$	Form $3n + 2$	Form $3n + 1$	Form $3n + 2$
1.	10	7	2	7	5
2.	100	7	2	97	89
3.	1,000	7	2	997	983
4.	10,000	7	2	9,973	9,941
5.	100,000	7	2	99,991	99,989
6.	1,000,000	7	2	999,979	999,983
7.	10,000,000	7	2	9,999,991	9,999,971
8.	100,000,000	7	2	99,999,931	99,999,989
9.	1,000,000,000	7	2	999,999,937	999,999,929
10.	10,000,000,000	7	2	9,999,999,967	9,999,999,929
11.	100,000,000,000	7	2	99,999,999,943	99,999,999,977
12.	1,000,000,000,000	7	2	999,999,999,961	999,999,999,989

While the first primes in all the first blocks have respective fixed values, the difference in the last primes of form $3n + 1$ and $3n + 2$ in the first blocks has zigzag trend.



Figures 2:*First and last primes of form 3n+k in first blocks of 10 powers.*

B. Minimum Number of Primes in Blocks of 10 Powers

Inspecting all blocks of each 10 power ranging from 10^1 to 10^{12} till 10^{12}, the minimum number of primes found in each 10 power block has been determined rigorously for primes of both forms under consideration.

Table 3:*Minimum Number of Primes of form 3n + k in Blocks of 10 Powers*

Sr. No.	Blocks of Size (of 10 Power)	Minimum No. of Primes of form 3*n* + 1 in Block	Minimum No. of Primes of form 3*n* + 2 in Block
1.	10	0	0
2.	100	0	0
3.	1,000	1	1
4.	10,000	126	124
5.	100,000	1,653	1,646
6.	1,000,000	17,756	17,619
7.	10,000,000	180,001	180,115
8.	100,000,000	1,808,103	1,808,105
9.	1,000,000,000	18,094,690	18,093,491
10.	10,000,000,000	180,988,251	180,989,170
11.	100,000,000,000	1,812,964,422	1,812,960,010
12.	1,000,000,000,000	18,803,933,520	18,803,978,497

There is fluctuation in difference in minimum number of primes of form 3*n* + 1 and 3*n* + 2 in these blocks.

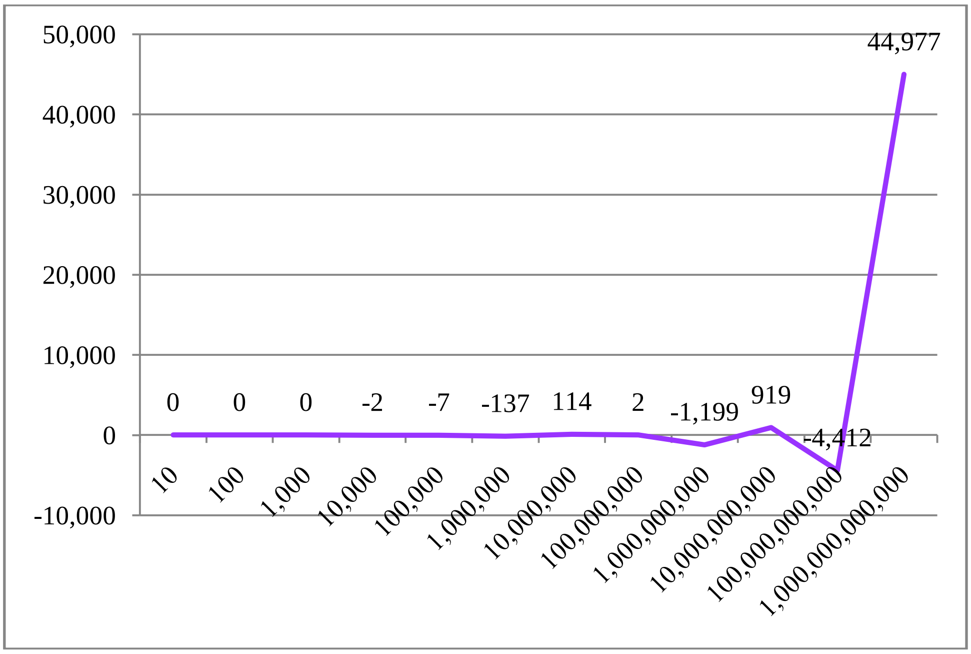


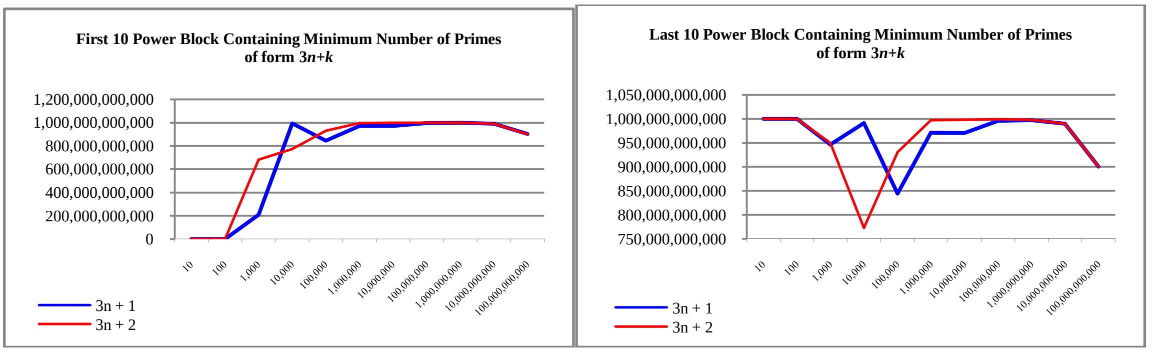
Figure 3 :*Minimality Lead of Number of Primes of form 3n+1 over 3n+2 in 10 Power Blocks.*

The first and last blocks in our range of one trillion with minimum number of primes of forms 3*n* + 1 and 3*n* + 2 in them are also determined. Here block 0 means first block and consequent numbers are for higher blocks. Like for 10, block 0 is 0-9, block 10 is 10 - 19 and so on.

Table 4 :*First and last blocks of 10 powers with minimum number of primes of form 3n + k.*

Sr. No.	Blocks of Size (of 10 Power)	First Block with Minimum Number of Primes		Last Block with Minimum Number of Primes	
		Form 3*n* + 1	Form 3*n* + 2	Form 3*n* + 1	Form 3*n* + 2
1.	10	20	30	999,999,999,990	999,999,999,990
2.	100	69,500	103,100	999,999,999,700	999,999,999,000
3.	1,000	208,627,276,000	682,833,699,000	946,441,029,000	949,672,786,000
4.	10,000	991,093,580,000	772,787,800,000	991,093,580,000	772,787,800,000
5.	100,000	844,002,100,000	930,488,800,000	844,002,100,000	930,488,800,000
6.	1,000,000	970,693,000,000	997,040,000,000	970,693,000,000	997,040,000,000
7.	10,000,000	970,280,000,000	998,020,000,000	970,280,000,000	998,020,000,000
8.	100,000,000	995,400,000,000	999,300,000,000	995,400,000,000	999,300,000,000
9.	1,000,000,000	997,000,000,000	998,000,000,000	997,000,000,000	998,000,000,000
10.	10,000,000,000	990,000,000,000	990,000,000,000	990,000,000,000	990,000,000,000
11.	100,000,000,000	900,000,000,000	900,000,000,000	900,000,000,000	900,000,000,000

The comparative trend deserves graphical representation.



Figures4 :*First and last blocks of 10 powers with minimum number of primes of form 3n + k.*

The determination of frequency of blocks with minimum number of primes of form 3*n* + 1 and 3*n* + 2 becomes due.

Table 5 : *Number of 10 power blocks with minimum number of primes of form 3n + k.*

Sr. No.	Blocks of Size (of 10 Power)	Number of Times the Minimum Number of Primes of form 3*n* + 1 Occurring in Blocks	Number of Times the Minimum Number of Primes of form 3*n* + 2 Occurring in Blocks
1.	10	82,443,117,633	82,443,091,281
2.	100	1,227,978,147	1,228,005,131
3.	1,000	8	5

For rest 10 powers blocks for both forms of primes, the number of blocks containing minimum number of primes become 1. The percentage decrease in occurrences of blocks with minimum number of primes in them follows.

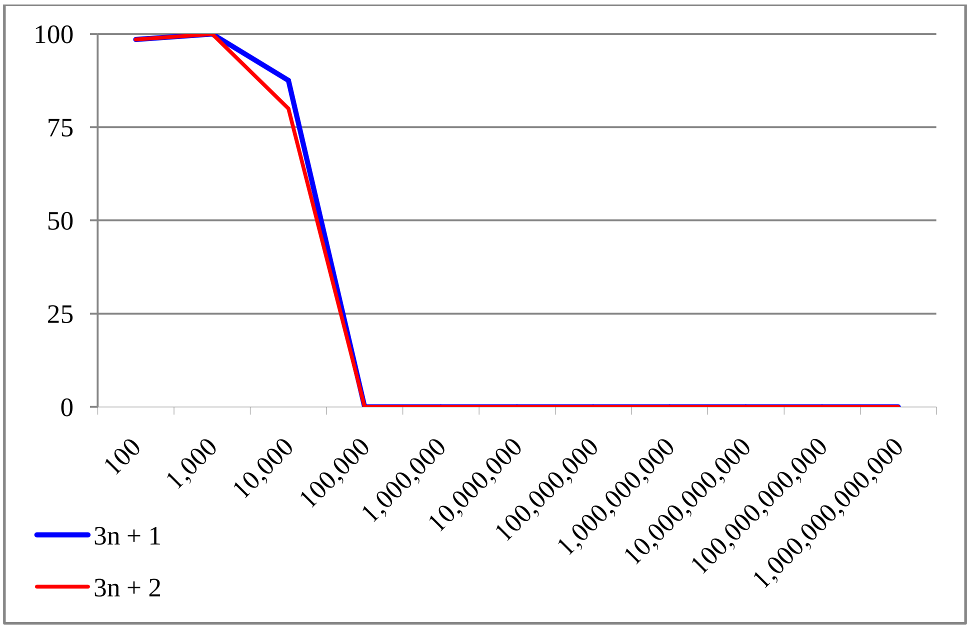


Figure 5 :*% Decrease in Occurrences of Minimum Number of Primes of form 3n+k in Blocks of 10 Powers.*

C. Maximum Number of Primes in Blocks of 10 Powers

All blocks of each 10 power ranging from 10^1 to 10^{12} till 10^{12} have also been analyzed for the maximum number of primes found in each of them.

Table 6 :*Maximum Number of Primes of form 3n + k in Blocks of 10 Powers.*

Sr. No.	Blocks of Size (of 10 Power)	Maximum No. of Primes of form 3*n* + 1 in Block	Maximum No. of Primes of form 3*n* + 2 in Block
1.	10	2	2
2.	100	11	13
3.	1,000	80	87
4.	10,000	611	617
5.	100,000	4,784	4,807

Sr. No.	Blocks of Size (of 10 Power)	Maximum No. of Primes of form 3*n* + 1 in Block	Maximum No. of Primes of form 3*n* + 2 in Block
6.	1,000,000	39,231	39,266
7.	10,000,000	332,194	332,384
8.	100,000,000	2,880,517	2,880,937
9.	1,000,000,000	25,422,713	25,424,820
10.	10,000,000,000	227,523,123	227,529,387
11.	100,000,000,000	2,059,018,668	2,059,036,144
12.	1,000,000,000,000	18,803,933,520	18,803,978,497

Here primes of form 3*n* + 2 dictate in all blocks except the first block size of 10.

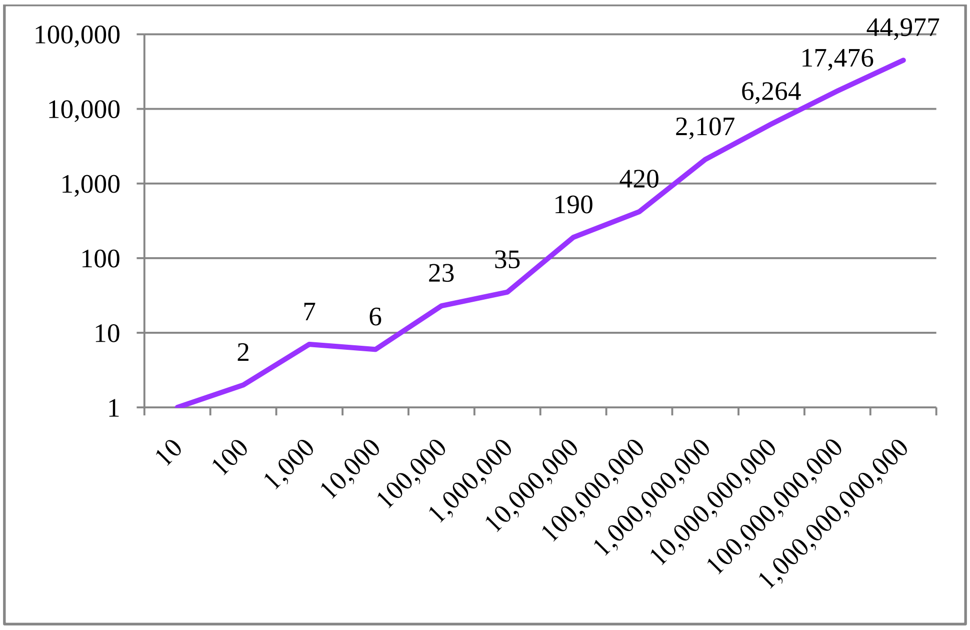


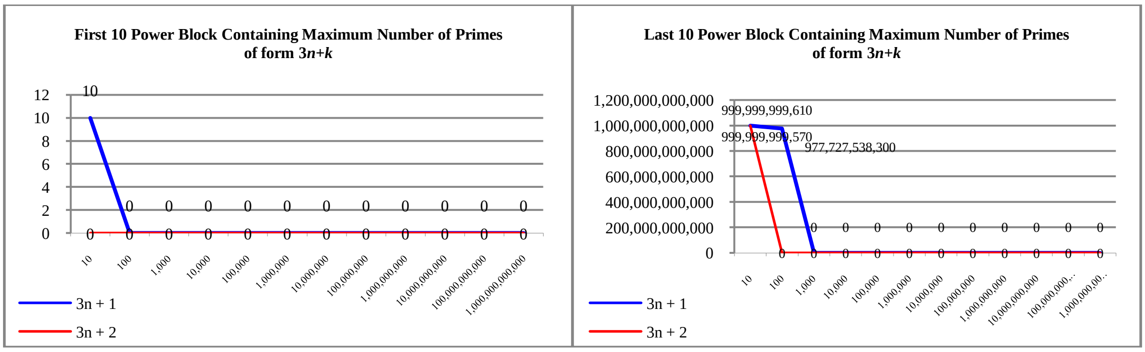
Figure 6 :*Maximality Lead of Number of Primes of form 3n+2 over 3n+1 in 10 Power Blocks.*

The first and last blocks in our range of one trillion with maximum number of primes of forms 3*n* + 1 and 3*n* + 2 in them are also determined.

Table 7 :*First and last blocks of 10 powers with maximum number of primes of form 3n + k.*

Sr. No.	Blocks of Size (of 10 Power)	First Block with Maximum Number of Primes		Last Block with Maximum Number of Primes	
		Form 3*n* + 1	Form 3*n* + 2	Form 3*n* + 1	Form 3*n* + 2
1.	10	10	0	999,999,999,570	999,999,999,610
2.	100	0	0	977,727,538,300	0
3.	1,000	0	0	0	0
4.	10,000	0	0	0	0
5.	100,000	0	0	0	0
6.	1,000,000	0	0	0	0
7.	10,000,000	0	0	0	0
8.	100,000,000	0	0	0	0
9.	1,000,000,000	0	0	0	0
10.	10,000,000,000	0	0	0	0
11.	100,000,000,000	0	0	0	0

Since in general, the prime density shows a decreasing trend with higher range of numbers, it is natural that for larger block sizes, the first as well as the last occurrences of maximum number of primes in them starts in the block of 0, i.e., the very first block.



Figures 7 :*First and last blocks of 10 powers with maximum number of primes of form 3n + k.*

Decrease in the prime density asserts that the maximum number of primes cannot occur frequently, at least for higher ranges.

Table 8 :*Number of 10 power blocks with maximum number of primes of form 3n + k.*

Sr. No.	Blocks of Size (of 10 Power)	Number of Times the Maximum Number of Primes of form 3*n* + 1 Occurring in Blocks	Number of Times the Maximum Number of Primes of form 3*n* + 2 Occurring in Blocks
1.	10	1,247,051,153	1,247,069,778
2.	100	40	1
3.	1,000	1	1
4.	10,000	1	1
5.	100,000	1	1
6.	1,000,000	1	1
7.	10,000,000	1	1
8.	100,000,000	1	1
9.	1,000,000,000	1	1
10.	10,000,000,000	1	1
11.	100,000,000,000	1	1
12.	1,000,000,000,000	1	1

Here too, initially frequency of maximum primes of form 3*n* + 2 has surpassed that of form 3*n* + 1, then 3*n* + 1 has taken a marginal lead over earlier and the figures for both have settled down to 1. The percentage decrease in occurrences of blocks with maximum number of primes in them follows.

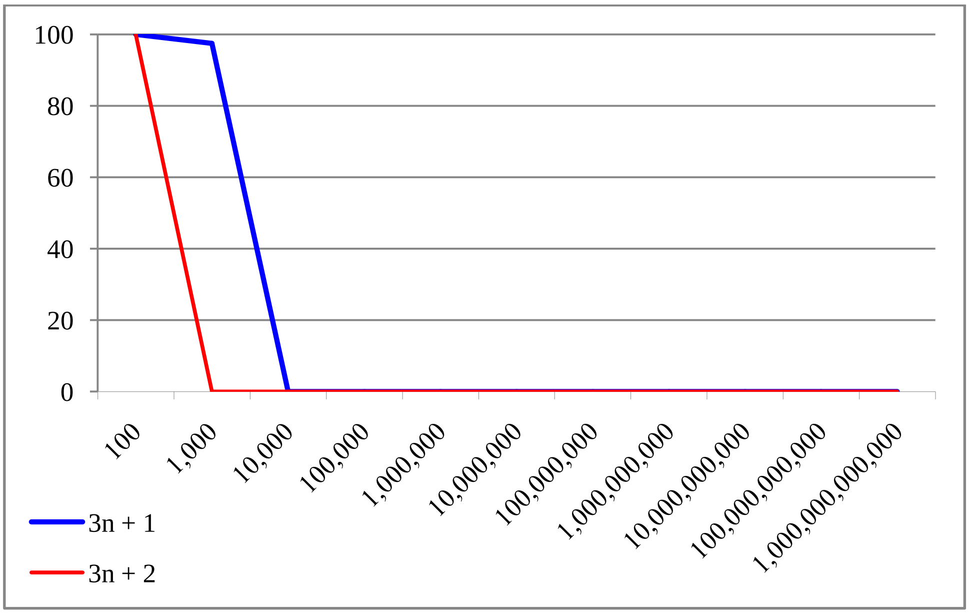


Figure 8 :*% Decrease in Occurrences of Maximum Number of Primes of form 3n+k in Blocks of 10 Powers.*

VII. Spacings between Primes of Form 3*n* + *k* in Blocks of 10 Powers

A. Minimum Spacings between Primes of Form 3n + k in Blocks of 10 Powers

Exempting prime-empty blocks, the minimum spacing between primes of form 3*n* + 1 and 3*n* + 2 in blocks of 10 powers are determined to be 6 and 3, respectively, beginning with the first power block $10^1 = 10$. Since for larger block sizes, the minimum spacing value cannot increase, it remains same ahead for all blocks of all higher powers of 10 in all ranges, even beyond our range of a trillion, virtually till infinity!

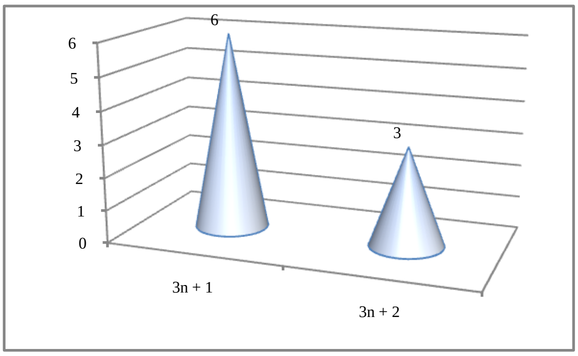
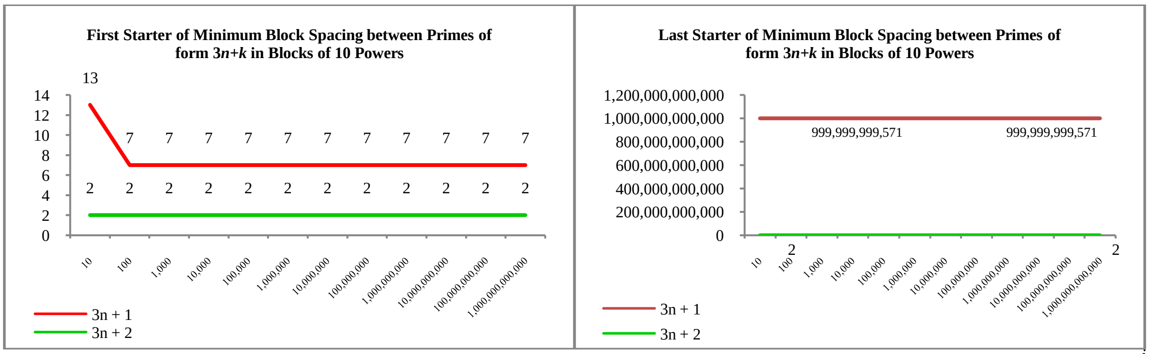


Figure 9 :*Minimum Block Spacing between Primes of form 3n+k*

This minimum block spacing of 6 occurs for primes of form 3*n* + 1 first at 13 for blocks of 10 and for higher power blocks at 7. For blocks of 10, it is not in first block at 7 as the next prime for this form 13 with spacing of 6 occurs in next block. For primes of form 3*n* + 2, its minimum block spacing of 3 occurs first at 2 for all blocks of 10 powers.

The minimum block spacing of 6 for primes of form 3*n* + 1 occurs last in our range at 999,999,999,571 for all 10 power blocks. While the same 3 for primes of form 3*n* + 2 occurs last also at 2. In fact, as except 2 all primes are odd, there cannot be a spacing of odd number between any two primes until one of them is 2.



Figures 10 :*First & Last Starters of Minimum Block Spacing between Primes of form 3n+k in Blocks of 10^i.*

It is worthwhile to determine the number of times this minimum block spacing occurs between primes of form 3*n* + 1 and 3*n* + 2.

Table 9 :*Frequency of minimum block spacings between primes of form 3n + k.*

Sr. No.	Blocks of Size (of 10 Power)	Number of Times the Minimum Block Spacing Occurring for Primes of form 3*n* + 1	Number of Times the Minimum Block Spacing Occurring for Primes of form 3*n* + 2
1.	10	1,247,051,153	1
2.	100	1,808,234,686	1
3.	1,000	1,864,352,043	1
4.	10,000	1,869,963,048	1
5.	100,000	1,870,524,725	1
6.	1,000,000	1,870,580,790	1
7.	10,000,000	1,870,586,258	1
8.	100,000,000	1,870,586,799	1
9.	1,000,000,000	1,870,586,847	1
10.	10,000,000,000	1,870,586,853	1
11.	100,000,000,000	1,870,586,855	1
12.	1,000,000,000,000	1,870,586,855	1

With increase in the block-size, there is increase in the number of times the minimum spacing occurs between primes of form 3*n* + 1. This is because whenever we increase block size, some primes with desired spacing occurring at the crossing of earlier blocks find themselves in same larger blocks raising the count. Of course, this rate of increase gradually decreases as shown in graph as we reach the block size of our limit.

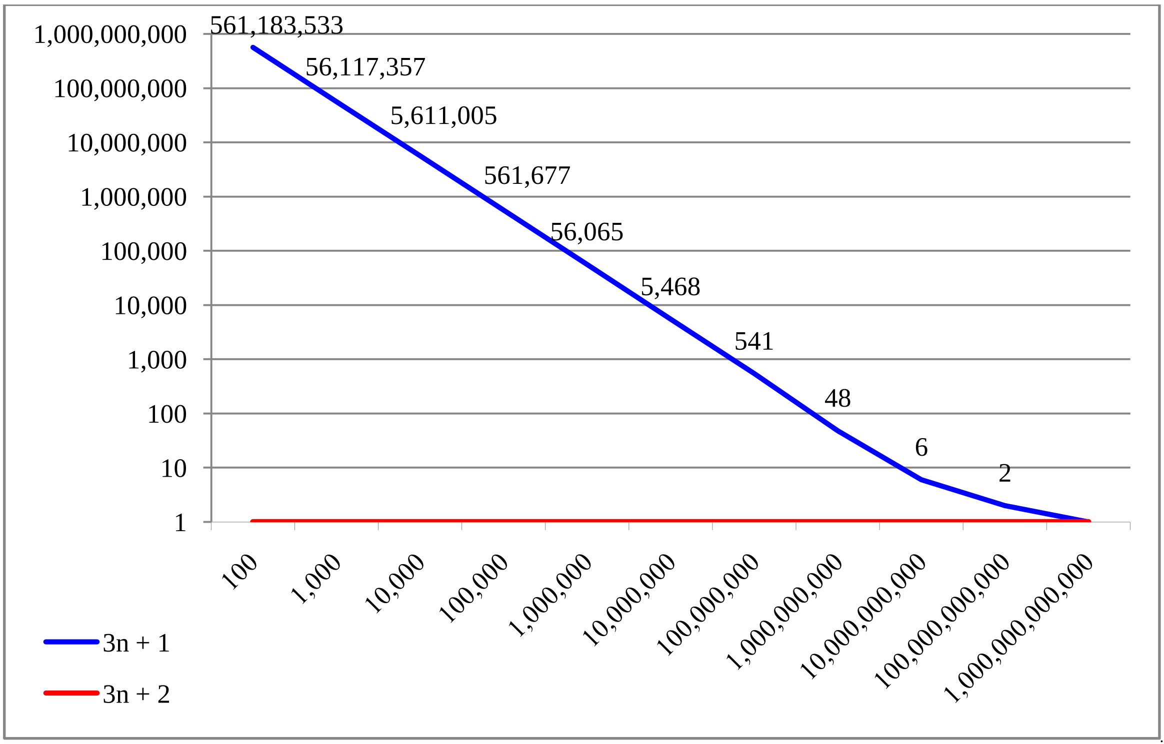


Figure 11 :*Increase in Occurences of Minimum Block Spacing between Primes of form 3n+k in Blocks of* 10^i.

As mentioned earlier, the occurrence of minimum block spacing in case of primes of form $3n + 2$ is unique for all blocks, keeping its increase 0.

B. Maximum Spacings between Primes of Form 3n + k in Blocks of 10 Powers

Unlike the minimum spacing between primes in blocks of 10 powers, the maximum spacing in these blocks goes on increasing with increase in the block size. Till our ceiling of one trillion, the following trend of increase and settling is seen.

Table 10 :*Maximum Block Spacing between Primes of form 3n + k.*

Sr. No.	Blocks of Size (of 10 Power)	Maximum Block Spacing Occurring for Primes of form $3n + 1$	Maximum Block Spacing Occurring for Primes of form $3n + 2$
1.	10	6	6
2.	100	96	96
3.	1,000	960	942
4.	10,000	1,068	1,068
5.	100,000	1,068	1,068
6.	1,000,000	1,068	1,068
7.	10,000,000	1,068	1,068
8.	100,000,000	1,068	1,068
9.	1,000,000,000	1,068	1,068
10.	10,000,000,000	1,068	1,068
11.	100,000,000,000	1,068	1,068
12.	1,000,000,000,000	1,068	1,068

In our range of inspection of 1 trillion, except for the block of 1000, in-block maximum spacing for primes of both forms is same.

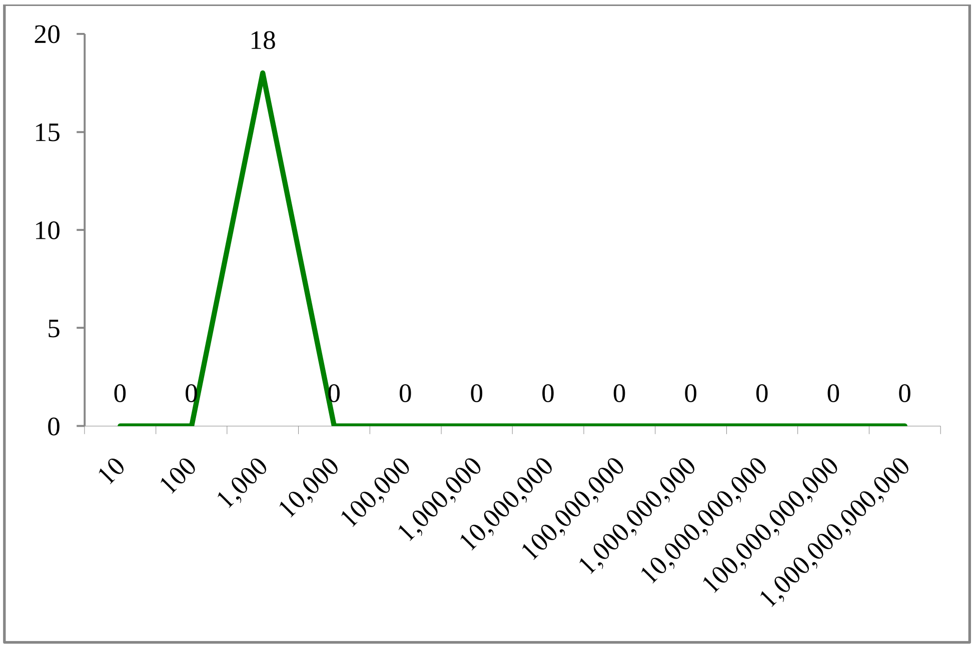


Figure 12 :*Dominance of Maximum Block Spacing between Primes of form 3n+1 over 3n+2.*

The first and last primes of forms $3n + 1$ and $3n + 2$ with these maximum in-block spacings for various blocks are also determined.

Table 11 :*First & last primes with maximum block spacings.*

Sr. No.	Blocks of Size (of 10 Power)	First Prime with Respective Maximum Block Spacing		Last Prime with Respective Maximum Block Spacing	
		Form $3n + 1$	Form $3n + 2$	Form $3n + 1$	Form $3n + 2$
1.	10	13	11	999,999,999,571	999,999,999,611
2.	100	93,001	144,203	999,999,994,801	999,999,981,503
3.	1,000	653,064,334,009	596,580,025,049	653,064,334,009	596,580,025,049
4.	10,000	759,345,224,761	423,034,793,273	759,345,224,761	423,034,793,273
5.	100,000	759,345,224,761	423,034,793,273	759,345,224,761	423,034,793,273
6.	1,000,000	759,345,224,761	423,034,793,273	759,345,224,761	423,034,793,273
7.	10,000,000	759,345,224,761	423,034,793,273	759,345,224,761	423,034,793,273
8.	100,000,000	759,345,224,761	423,034,793,273	759,345,224,761	423,034,793,273
9.	1,000,000,000	759,345,224,761	423,034,793,273	759,345,224,761	423,034,793,273
10.	10,000,000,000	759,345,224,761	423,034,793,273	759,345,224,761	423,034,793,273
11.	100,000,000,000	759,345,224,761	423,034,793,273	759,345,224,761	423,034,793,273
12.	1,000,000,000,000	759,345,224,761	423,034,793,273	759,345,224,761	423,034,793,273

The comparative trend is clear from graphical representation.

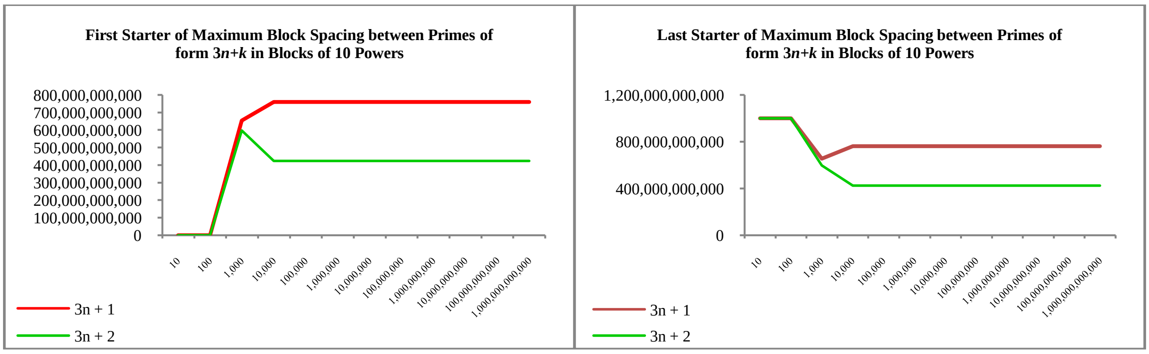


Figure 13 :*First & last primes with maximum block spacings.*

The determination of frequency of maximum block spacing occurrence of primes of form $3n + 1$ and $3n + 2$ is done.

Table 12 :*Frequency of maximum block spacings between primes of form 3n + k.*

Sr. No.	Blocks of Size (of 10 Power)	Number of Times the Maximum Block Spacing Occurs for Primes of form 3*n* + 1	Number of Times the Maximum Block Spacing Occurs for Primes of form 3*n* + 2
1.	10	1,247,051,153	1,247,069,777
2.	100	21,217,945	21,205,830

For all rest 10 powers blocks for both prime forms, the maximum block spacing occurs only once yielding following pattern for their respective % decrease over preceding values.

VIII. Units Place & Tens Place Digits in Primes of form 3*n* + *k*

Prime numbers have only six different possible digits in units place. Exhaustive analysis shows the number of primes of form 3*n* + *k* with different digits in unit's place to be as follows :

Table 13 :*Number of primes of form 3n + k with different units place digits till one trillion.*

Sr. No.	Digit in Units Place	Number of Primes of form	
		3*n* + 1	3*n* + 2
1.	1	4,700,968,833	4,700,992,147
2.	2	0	1
3.	3	4,700,984,929	4,700,994,974
4.	5	0	1
5.	7	4,701,002,681	4,700,994,319
6.	9	4,700,977,077	4,700,997,055

2 is only even prime and 5 in only prime with its unit place digit. Following analysis has neglected 2 and 5 in units places as they have exceptional nature.

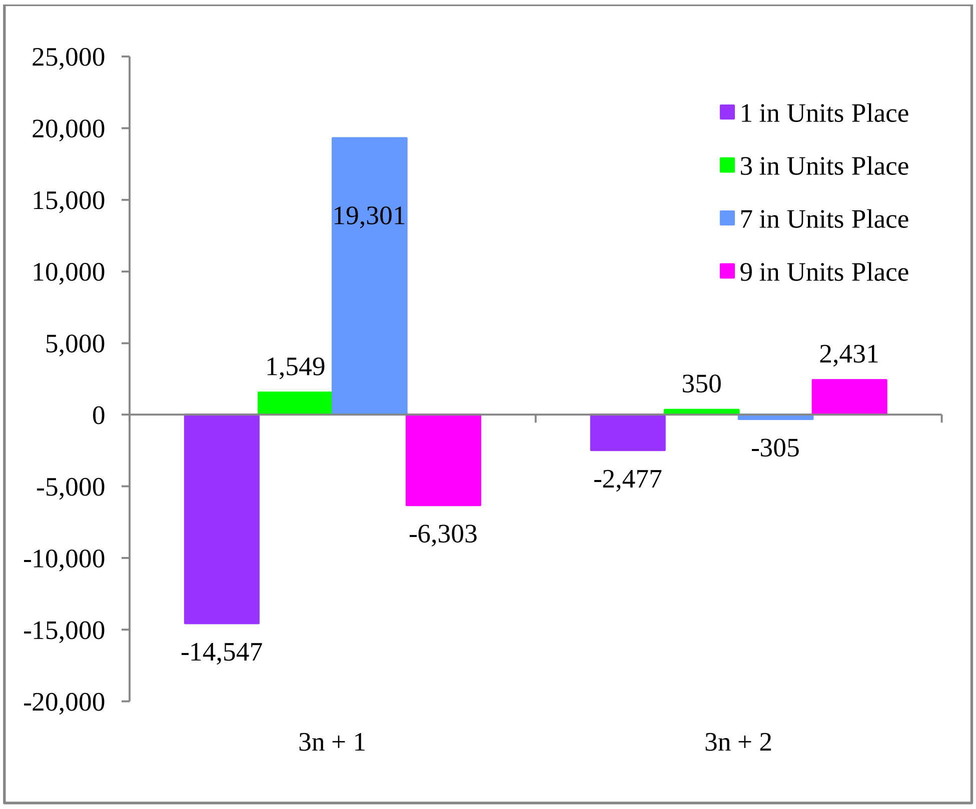


Figure 14 : Deviation of Units Place Digits of Primes of form 3n+k from Average.

Now follows the figures for both tens place and units place digits together.

Table 14 :*Number of primes of form 3n + k with different tens and units place digits till one trillion.*

Sr. No.	Digits in Tens & Units Place	Number of Primes of form	
		3*n* + 1	3*n* + 2
1.	01	470,091,333	470,109,891
2.	02	0	1
3.	03	470,094,770	470,104,271
4.	05	0	1
5.	07	470,097,248	470,104,276
6.	09	470,094,613	470,103,424
7.	11	470,102,397	470,089,234
8.	13	470,100,789	470,099,915
9.	17	470,091,448	470,097,857
10.	19	470,106,050	470,118,517
11.	21	470,098,988	470,108,463
12.	23	470,102,820	470,102,293
13.	27	470,103,643	470,103,729
14.	29	470,102,782	470,094,647
15.	31	470,103,390	470,097,906
16.	33	470,101,752	470,095,882
17.	37	470,104,142	470,094,694
18.	39	470,101,627	470,093,736
19.	41	470,093,947	470,096,059
20.	43	470,095,523	470,102,070
21.	47	470,095,217	470,102,515
22.	49	470,105,420	470,095,356
23.	51	470,107,468	470,097,412
24.	53	470,094,341	470,101,246
25.	57	470,099,603	470,093,392
26.	59	470,103,290	470,096,232
27.	61	470,093,061	470,103,049
28.	63	470,102,739	470,092,627
29.	67	470,104,073	470,099,284
30.	69	470,085,723	470,086,721
31.	71	470,100,170	470,096,319
32.	73	470,094,450	470,102,497
33.	77	470,097,789	470,098,854
34.	79	470,091,636	470,097,190
35.	81	470,087,306	470,092,697
36.	83	470,090,913	470,100,987
37.	87	470,105,175	470,093,879
38.	89	470,093,415	470,108,593
39.	91	470,090,773	470,101,117
40.	93	470,106,832	470,093,186
41.	97	470,104,343	470,105,839
42.	99	470,092,521	470,102,639

Neglecting the cases 02 and 05, following deviation from average is seen for occurrence of other possibilities of last two digits in range of $1\text{-}10^{12}$ for primes of form 3*n* + *k*.

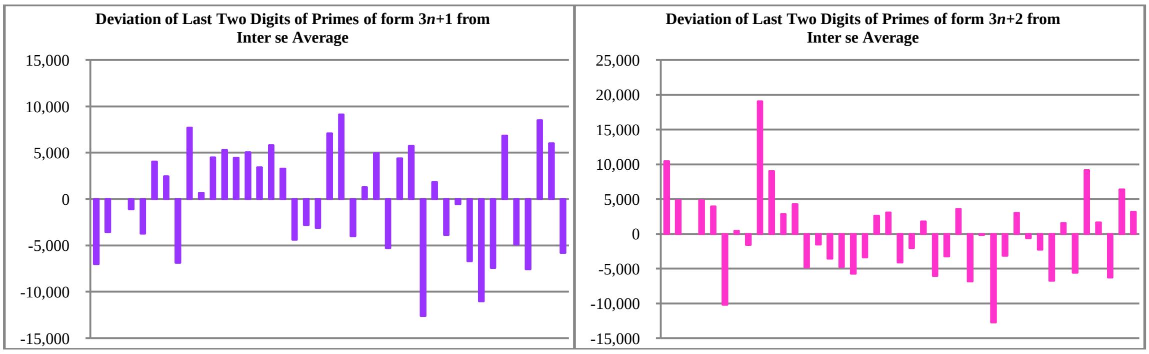
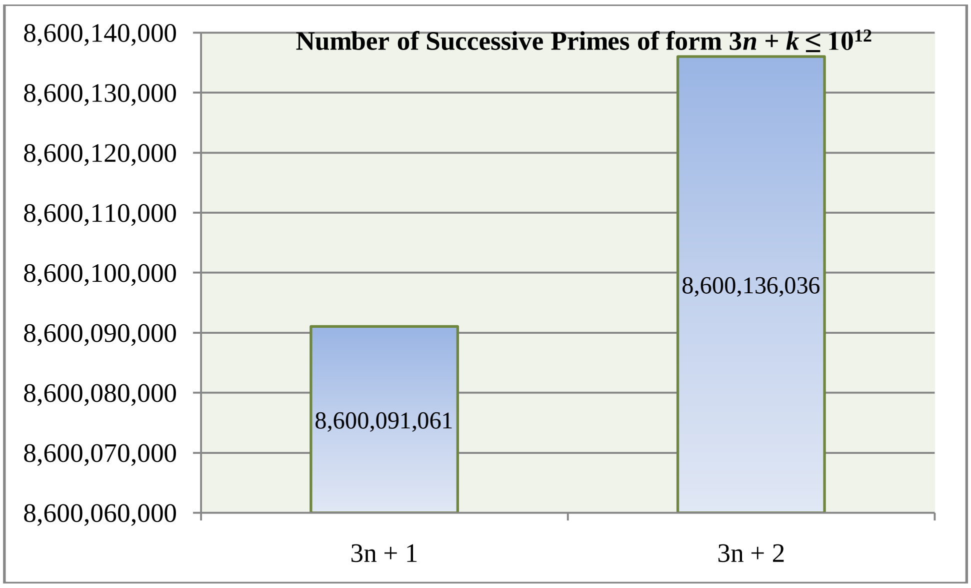


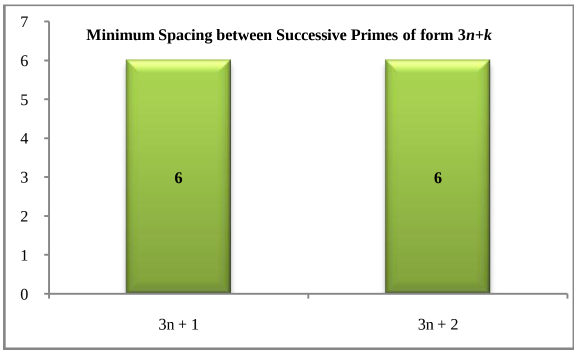
Figure 15 :*Deviation of last 2 digits of primes of form 3n+k from inter se average*

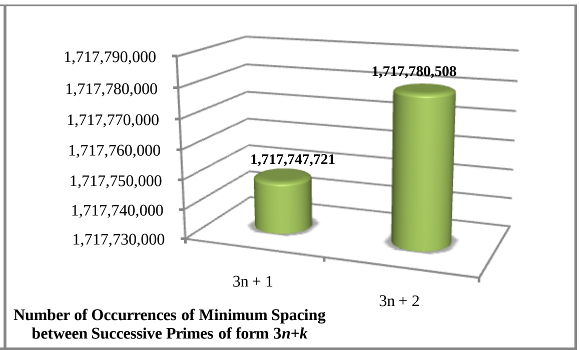
IX. Analysis of Successive Primes of form 3*n* + 1 and 3*n* + 2

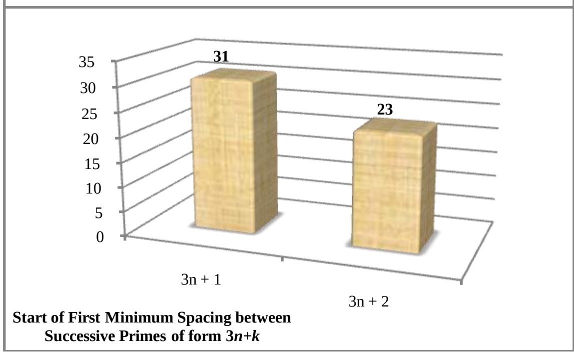
The situation when two successive primes are of same form; either 3*n* + 1 or 3*n* + 2; is interesting. The number of successive primes of desired forms is as follows.

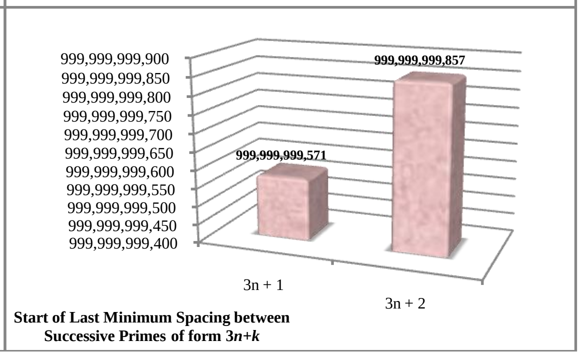


We have exhaustively analyzed these cases. The minimum spacing between successive primes of forms $3n + k$ has following properties.

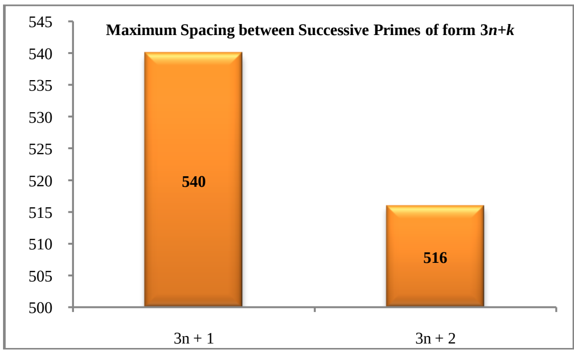


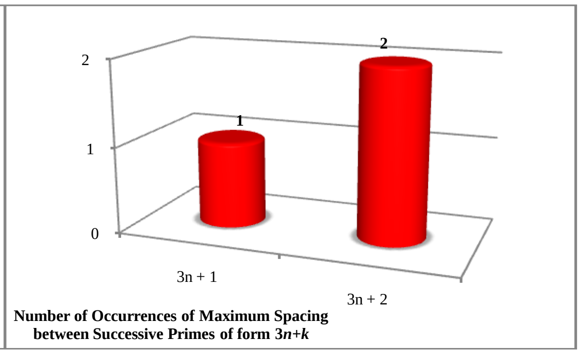


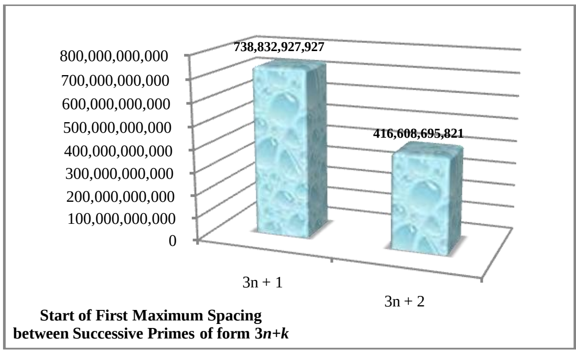


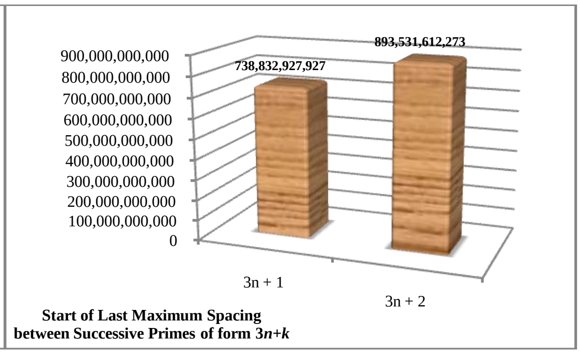


The maximum spacing between successive primes of forms $3n + k$ has following trends.









There have been consistent efforts to study random distribution of primes. The work presented here is an addition to that with respect to a specific linear pattern of $3n + k$. The author is sure that the availability of rigorous analysis like this will help give a deeper insight into prime distribution.

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