IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 3 Ver. III (May - Jun. 2015), PP 24-29
www.iosrjournals.org

On α-ρ-Continuity Where ρ ϵ {L, M, R, S}

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Abstract: *The authors introduced the concept of ρ-continuity between a topological space and a non empty set where ρϵ {L,M,R,S} in [ρ- continuity between a topological space and a non empty set where ρϵ {L,M,R,S} , International Journal of mathematical sciences 9(1-2)(2010), 97-104.]. In this paper, the concept of α-ρ-continuity is introduced and its properties are investigated.Recently, Navpreet singh Noorie and Rajni Bala introduced the concept of $f^{\#}$ function to characterize the closed, open and continuous functions. In this paper, α-ρ-continuity is further characterized by using $f^{\#}$ functions.*

I. Introduction

By a multifunction F: X → Y, We mean a point to set correspondence from X into Y with F(x) ≠ φ for all x ϵ X. Any function f: X → Y induces a multifunction $f^{-1}{}_{o}\, f : X \to \wp(X)$. It also induces another multifunction $f_{o}\, f^{-1}$: Y → (Y) provided f is surjective. The notions of L-Continuity, M-Continuity, R-Continuity and S-Continuity of a function f: X → Y between a topological space and a non empty set and introduced by Selvi and Priyadarshini . The purpose of this paper is to introduce α-ρ-continuity. Here we discuss their links with α–open, α–closed sets. Also we establish pasting lemmas for R-continuous and s-continuous functions and obtain some characterizations for α-ρ-continuity . Navpreet singh Noorie and Rajni Bala [2] introduced the concept of $f^{\#}$ function to characterize the closed, open and continuous functions. The authors [6] characterized ρ-continuity by using $f^{\#}$ functions. In an analog way α-ρ-continuity is characterized in this paper.

II. Preliminaries

The following definitions and results that are due to the authors [3] and Navpreet singh Noorie and Rajni Bala [2] will be useful in sequel.

Definition: 2.1(L-CONTINUOUS AND M-CONTINUOUS)

Let f: (x, τ) → Y be a function. Then f is

(i) L-Continuous if $f^{-1}(f(A))$ is open in X for every open set A in X. [3]

(ii) M-Continuous if $f^{-1}(f(A))$ is closed in X for every closed set A in X. [3]

Definition: 2.2 (R-CONTINUOUS AND S-CONTINUOUS) Let f: X → (Y, σ) be a function. Then f is

(i) R-Continuous if $f(f^{-1}(B))$ is open in Y for every open set B in Y. [3]

(ii) S-Continuous if $f(f^{-1}(B))$ is closed in Y for every closed set B in Y. [3]

Definition 2.3 : Let f : X → Y be any map and E be any subset of X. then (i) $f^{\#}(E) = \{ y \in Y : f^{-1}(y) \subseteq E \}$; (ii) $E^{\#} = f^{-1}(f^{\#}(E))$. [2]

Lemma 2.4 : Let E be a subset of X and let f : X → Y be a function . Then the following hold .

(i) $f^{\#}(E) = Y\backslash f(X\backslash E)$; (ii) $f(E) = Y\backslash f^{\#}(X\backslash E)$. [2]

Lemma 2.5 : Let E be a subset of X and let f : X → Y be a function . Then the following hold.

(i) $f^{-1}(f^{\#}(E)) = X\backslash f^{-1}(f(X\backslash E))$; (ii) $f^{-1}(f(E)) = X\backslash f^{-1}(f^{\#}(X\backslash E))$. [6]

Lemma 2.6: Let E be a subset of X and let f: X → Y be a function. Then the following hold.

(i) $f^{\#}(f^{-1}(E)) = Y\backslash f(f^{-1}(Y\backslash E))$; (ii) $f(f^{-1}(E)) = Y\backslash f^{\#}(f^{-1}(Y\backslash E))$. [6]

Definition 2.7 : Let f : X → Y , A⊆X and B⊆Y .we say that A is f-saturated if $f^{-1}(f(A))\subseteq A$ and B is f^{-1}-saturated if $f(f^{-1}(B))\supseteq B$. Equivalently A is f-saturated if and only if $f^{-1}(f(A)) = A$, and B is f^{-1}-saturated if and only if $f(f^{-1}(B)) = B$.

Definition 2.8: Let A be a subset of a topological space **(X,τ).** Then A is called

(i)semi-open if A⊆cl(int(A)) and semi-closed if int(cl(A))⊆A;[1] (ii) regular open if A=int(cl(A)) and regular closed if cl(int(A))=A ; [5] .

(iii) α-open if A⊆int(cl(int(A))) and α-closed if cl(int(cl(A)))⊆A;[] .

(iv) pre-open if A⊆int(cl(A)) and pre-closed if cl(int(A))⊆A;[ ]. (v)semi-pre-open if if A⊆cl(int(cl(A))) and semi-pre-closed if int(cl(int(A)))⊆A ; [ ] .

Definition: 2.9 Let f: (X, τ) → (Y, σ) be a function. Then f is α-continuous if $f^{-1}(B)$ is open in X for every α-open set B in Y.

Definition: 2.10 Let f: (X, τ) → (Y, σ) be a function. Then f is α-open (resp. α-closed) if f(A) is α-open(resp. α-closed) in Y for every α-open(resp. α-closed) set A in Y.

III. α-ρ-CONTINUITY WHERE ρ ϵ {L, M, R, S}

Definition: 3.1 (α-L CONTINUOUS AND α-M CONTINUOUS)

Let f: (x, τ) → Y be a function. Then f is

(i) α-L Continuous if f^{-1}(f (A)) is open in X for every α-open set A in X.

(ii) α-M Continuous if f^{-1}(f (A)) is closed in X for every α-closed set A in X.

Definition: 3.2 (α-R CONTINUOUS AND α-S CONTINUOUS) Let f: X → (Y, σ) be a function. Then f is

(i) α-R Continuous if f (f^{-1}(B)) is open in Y for every α-open set B in Y

(ii) α-S Continuous if f (f^{-1}(B)) is closed in Y for every α-closed set B in Y

Example: 3.3

Let X = {a, b, c, d} and Y = {1, 2, 3, 4}.

Let τ = { *Φ*, X ,{a}, {b}, {a, b}, {a, b, c} } and

τ^c ={ *Φ*, *X*,{b,c,d},{a, c, d},{c, d},{d} }

Let f : (x ,τ) → Y defined by f(a)=1, f(b)=2, f(c)=3, f(d)=4. Then f is α-L Continuous and α-M Continuous.

Example: 3.4

Let X = {a, b, c, d} and Y = {1, 2, 3, 4}.

Let σ ={ *Φ*,Y, {1},{2},{1,2},{1,2,3} }, and

σ^c ={ *Φ*,Y,{2,3,4},{1,3,4},{3,4},{4} }

Let g : X → (Y, σ) defined by g(a)=1, g(b)=2, g(c)=3, g(d)=4. Then g is α-R Continuous and α-S Continuous.

Definition: 3.5

Let f : (x ,τ) → (Y, σ) be a function , Then f is

(i) α-LR Continuous , if it is both α-L Continuous and α-R Continuous.
(ii) α-LS Continuous, if it is both α-L Continuous and α-S Continuous.
(iii) α-MR Continuous, if it is both α-M Continuous and α-R Continuous.
(iv) α-MS Continuous, if it is both α-M Continuous and α-S Continuous .

Theorem : 3.6

(i)Every injective function f: (x, τ) → (Y, σ) is α-L Continuous and α-M Continuous.

(ii)Every surjective function f: (x, τ) → (Y, σ) is α-R Continuous and α-S Continuous.

(ii)Any constant function f: (x, τ) → (Y, σ) is α-R Continuous and α-S Continuous.

Proof:

(i) Let f: (x, τ) → (Y, σ) be injective function. Then

α-L Continuity and α-M Continuity follow from the fact that f^{-1}(f (A)) =A. This proves

(ii) Let f: (x, τ) → (Y, σ) be surjective function. Since f is surjective, f (f^{-1}(B)) =B for every subset B of Y. Then f is both α-R Continuous and α-S Continuous. This proves (ii).

(iii) Suppose f(x) = y_o for every x in X. Then f (f^{-1}(B)) =Y if y_o ϵ B and f(f^{-1}(B))=Φ if y_o ϵ Y\B. This proves (iii).

Corollary : 3.7

If f : (x ,τ) → (Y, σ) be bijective function then f is

α-L Continuous , α-M Continuous , α-R Continuous and α-S Continuous .

Theorem : 3.8

Let f : (x ,τ) → (Y, σ) . (i) If f is L-Continuous (resp. M-Continuous) then it is α-L Continuous (resp. α-M Continuous).

(ii) If f is R-Continuous (resp. S-Continuous) then it is α-R Continuous (resp. α-S Continuous).

Proof :

(i) Let A ⊆ X be α-open (resp.α-closed) in X. since every α-open (resp.α-closed) set is open (resp. closed) and since f is L-continuous (resp. M-continuous), f^{-1}(f(A)) is open (resp. closed) in X. Therefore f is α-L Continuous (resp. α-M Continuous).

(ii) Let B ⊆ Y be α-open (resp.α-closed) in Y. since every α-open (resp.α-closed) set is open (resp. closed) and since f is R-continuous (resp. S-continuous) , f(f^{-1}(B)) is open (resp. closed) in Y. Therefore f is α-R Continuous (resp. α-S Continuous).

Theorem : 3.9

Let f : (x ,τ) → Y be α-L Continuous. Then int(cl(int(A))) is f-saturated whenever A is f-saturated and semi-pre-closed.

Proof:

Let A ⊆ X be f-saturated. Since f is α-L Continuous, int(cl(int (A))) ⊆f^{-1}(f(int(cl(int(A))))). And since A is semi- pre-closed f^{-1}(f(int(cl(int(A))))) ⊆ f^{-1}(f(A)) .

Therefore int(cl(int(A))) $\subseteq$ f^{-1}(f(int(cl(int(A))))) $\subseteq$ f^{-1}(f(A)). since A is f-saturated, f^{-1}(f(A)) = A so that int(cl(int(A))) $\subseteq$ f^{-1}(f(int(cl(int(A))))) $\subseteq$ int(cl(int(A))) .That implies int(cl(int(A))) = f^{-1}(f(int(cl(int(A))))) . Therefore Hence int(cl(int(A))) is f-saturated whenever A is f-saturated and semi- pre-closed.

Corollary : 3.10

Let f : (x ,τ) → Y be α-L Continuous. Then int(cl(int(f^{-1}(B)))) is f-saturated for every subset B of Y.

Proof :

Let B $\subseteq$ Y. we know that f(f^{-1}(B)) $\subseteq$ B, Then f^{-1}(f(f^{-1}(B))) $\subseteq$ f^{-1}(B). Also f^{-1}(B) $\subseteq$ f^{-1}(f(f^{-1}(B))) $\subseteq$ f^{-1}(B) . So that f^{-1}(f(f^{-1}(B))) = f^{-1}(B).

This proves that f^{-1}(B) is f-saturated , and hence by using theorem :3.9 , int(cl(int(f^{-1}(B)))) is f-saturated .

Theorem : 3.11

Let f : x → (Y, σ) be α-S Continuous Then cl(int(cl(B))) is f^{-1} – saturated whenever B is f^{-1} –saturated and semi-pre-open .

Proof :

Let B $\subseteq$ Y be f^{-1} –saturated. Since f^{-1} is α-S Continuous ,cl(int(cl(B))) $\supseteq$ f(f^{-1}(cl(int(cl(B))))), and since B is semi-pre-open , f(f^{-1}(cl(int(cl(B))))) $\supseteq$ f(f^{-1}(B)), Therefore cl(int(cl(B))) $\supseteq$ f(f^{-1}(cl(int(cl(B))))) $\supseteq$ f(f^{-1}(B)), since B is f^{-1} –saturated , f(f^{-1}(B)) = B . So that cl(int(cl(B))) $\supseteq$ f(f^{-1}(cl(int(cl(B))))) $\supseteq$ cl(int(cl(B))), which implies that cl(int(cl(B))) = f(f^{-1}(cl(int(cl(B))))), Therefore hence cl(int(cl(B))) is f^{-1} –saturated .

Corollary : 3.12

Let f : x → (Y, σ) be α-S Continuous Then cl(int(cl(f(A)))) is f^{-1} – saturated for every subset A of X.

Proof :

Let A $\subseteq$ X . We know that f^{-1}(f(A)) $\supseteq$ A ,Then f(f^{-1}(f(A))) $\supseteq$ f(A) Also f(A) $\supseteq$ f(f^{-1}(f(A))) $\supseteq$ f(A) , So that f(f^{-1}(f(A))) = f(A) . This proves that hence by using (theorem 2.11) cl(int(cl(f(A)))) is f^{-1} – saturated .

IV. Properties

In this section we prove certain theorems related with α-open and α-closed functions .

Theorem : 4.1

(i) Let f : (x ,τ) → (Y, σ) be α-open and α- Continuous , Then f is α-L Continuous.
(ii) Let f : (x ,τ) → (Y, σ) be open and α- Continuous , Then f is α-R Continuous.

Proof :

(i) Let A $\subseteq$ X be α-open in X . Let f : (x ,τ) → (Y, σ) be α-open and α- Continuous, since f is α-open f(A) is α-open in Y , and since f is α-continuous f^{-1}(f(A)) is open in X . Therefore f is α-L Continuous , This proves (i) .

(ii) Let B $\subseteq$ Y be α-open in Y . Let f : (x ,τ) → (Y, σ) be open and α-Continuous , since f is α-continuous f^{-1}(B) is open in X, and since f is open f(f^{-1}(B)) is open in Y, Therefore f is α-R Continuous, This proves (ii) .

Theorem : 4.2

(i) Let f : (x ,τ) → (Y, σ) be α-closed and α- Continuous, Then f is α-M Continuous.
(ii) Let f : (x ,τ) → (Y, σ) be closed and α- Continuous , Then f is α-S Continuous.

Proof :

(i) Let A $\subseteq$ X be α-closed in X . Let f : (x ,τ) → (Y, σ) be α-closed and α- Continuous , since f is α-closed f(A) is α-closed in Y, and since f is α-continuous $\Longrightarrow$ f^{-1}(f(A)) is closed in X . Therefore f is α-M Continuous . This proves (i) .

(ii) Let B $\subseteq$ Y be α-closed in Y . Let f : (x ,τ) → (Y, σ) be closed and α- Continuous , since f is α-continuous f^{-1}(B) is closed in X, and since f is closed $\Longrightarrow$ f(f^{-1}(B)) is closed in Y . Therefore f is α-S Continuous , This proves (ii) .

Theorem : 4.3

Let X be a topological space .

(i) If A is an α-open subspace of X , the inclusion function j : A → X is α-L-continuous and α-R-continuous .
(ii) If A is an α-closed subspace of X , the inclusion function j : A → X is α-M-continuous and α-S-continuous .

Proof :

(i) Suppose A is an α-open subspace of X . Let j : A → X be an inclusion function . Let UcX be α-open in X then j(j^{-1}(U))=j(UnA)=UnA Which is open in X .Hence j is α-R-continuous .Now , let U $\subseteq$ A be α-open in A. Then j^{-1}(j(U)) = j^{-1}(U) =U which is open in A .

Hence j is α-L-continuous , this proves (i)

(ii) Suppose A is an α-closed subspace of X .

Let j : A → X be an inclusion function . Let UcX be α-closed in X then $j(j^{-1}(U))=j(UnA)=UnA$,Which is closed in X .Hence j is α-S-continuous .Now , let U $\subseteq$ A be α-closed in A. Then $j^{-1}(j(U)) = j^{-1}(U) =U$ which is closed in A. Hence j is α-M-continuous , this proves (ii) .

Theorem : 4.4

Let g : Y → Z and f : X → Y be any two functions. Then the following hold.

(i) If g : Y → Z is α-L-continuous (resp. α-M-continuous) and f : X → Y is α-open (resp . α-closed) and continuous, then g o f : X → Z is α-L-continuous (resp. α-M-continuous) .

(ii) If g : Y → Z is open (resp.closed) and α-continuous and f : X → Y is R-continuous (resp. S-continuous) ,then g o f is α- R-continuous (resp. α-S-continuous).

Proof :

Suppose g is α-L-continuous (resp. α-M continuous) and f is α-open (resp. α-closed) and continuous .Let A be α-open (resp. α-closed) in X .Then $(g \circ f)^{-1}.(g \circ f)(A)=f^{-1}(g^{-1}(g(f(A))))$. Since f is α-open (resp. α-closed) f (A) is α-open (resp. α-closed) in Y. since g is α-L-continuous (resp. α-M-continuous), $g^{-1}(g(f(A)))$is open (resp. closed)in Y since f is continuous $f^{-1}(g^{-1}(g(f(A))))$ is open (resp. α-closed) in X .Therefore, g ○ f is α-L-continuous (resp. α-M-continuous) . This proves (i) .

(ii) Let f : X → Y be R-continuous (resp . S-continuous) and g : Y → Z be open (resp. closed) and α-continuous . Let B be α-open (resp. α-closed) in Z . Then $(g \circ f)(g \circ f)^{-1}(B) = (g \circ f)(f^{-1}(g^{-1}(B))) = g(f(f^{-1}(g^{-1}(B))))$. since g is α-continuous⇒ $g^{-1}(B)$ is open (resp. closed) in Y. since f is R-continuous (resp. S-continuous)⇒ $f(f^{-1}(g^{-1}(B)))$ is open (resp. closed) in Y. since g is open (resp. closed)⇒ g(f(f-1(g-1(B)))) is open (resp. closed) in Z . Therefore, g ○ f is α-R-continuous (resp. α-R-continuous). This proves (ii).

Theorem : 4.5

If f : X → Y is α-L-continuous and if A is an open subspace of X , then the restriction of f to A is α-L-continuous .

Proof :

Let $h = f/_A$. Then h = f ○ j , where j is the inclusion map j:A→X since j is open and continuous and since f : X → Y is α-L-continuous , using theorem (4.4 (i)) ,

Therefore , hence h is α-L-continuous .

Theorem : 4.6

If f : X → Y is α-M-continuous and if A is a closed subspace of X , then the restriction of f to A is α-M-continuous .

Proof :

Let $h = f/_A$. Then h = f ○ j , where j is the inclusion map j:A→X since j is closed and continuous and since f : X → Y is α-M-continuous , using theorem (4.4 (i)) ,

Therefore , hence h is α-M-continuous .

Theorem : 4.7

Let f: X → Y be α-R-continuous . Let f(x) $\subseteq$ Z $\subseteq$ Y and f(X) be open in Z . Let h: X → Z be obtained by from f by restricting the co-domain of f to Z . Then h is α-R-continuous.

Proof :

Clearly h = j ○ f where j : f(x) → Z is an inclusion map . since f(X) is open in Z , the inclusion map j is both open and α-continuous . Then by applying theorem 4.4 (ii) . Hence h is α-R-continuous .

Theorem : 4.8

Let f : X → Y be α-S-continuous . Let f(x) $\subseteq$ Z $\subseteq$ Y and f(X) be closed in Z . Let h : X → Z be obtained by from f by restricting the co-domain of f to Z. Then h is α-S-continuous.

Proof :

Clearly h = j ○ f where j : f(x) → Z is an inclusion map . since f(X) is closed in Z , the inclusion map j is both closed and α-continuous . Then by applying theorem 4.4 (ii) . Hence h is α-S-continuous

V. Characterizations

Theorem : 5.1

A function f : X → Y is α-L-continuous if and only if $f^{-1}(f^{\#}(A))$ is closed in X for every α-closed subset A of X .

Proof :

Suppose f is α-L-continuous . Let A be α- closed in X . Then G = X /A is α-open in X . since f is α-L-continuous and since G is α-open in X $f^{-1}(f(G))$ is open in X . By applying lemma((2.5)-(i)) $f^{-1}(f^{\#}(A)) = X \setminus f^{-1}(f(X\setminus A))$ $= X \setminus f^{-1}(f(G))$. That implies $f^{-1}(f^{\#}(A))$ is closed in X.

Conversely , we assume that $f^{-1}(f^{\#}(A))$ is closed in X for every α-closed subset A of X . Let G be a α-open in X . By our assumption, $f^{-1}(f^{\#}(A))$ is closed in X , where A = X\G .By using lemma ((2.5)-(ii)) $f^{-1}(f(G)) = X \setminus f^{-1}(f^{\#}(X\setminus G)) = X \setminus f^{-1}(f^{\#}(A))$. That implies $f^{-1}(f(G))$ is open in X .Therefore , hence f is α-L-continuous .

Theorem : 5.2

A function f : X → Y is α-M-continuous if and only if $f^{-1}(f^{\#}(G))$ is open in X for every α-open subset G of X .

Proof :

Suppose f is α-M-continuous . Let G be α- open in X. Then A = X \G is α-closed in X . since f is α-M-continuous and since A is α-closed in X ⟹ $f^{-1}(f(A))$ is closed in X . By applying lemma((2.5)-(i)) $f^{-1}(f^{\#}(G)) = X \setminus f^{-1}(f(X\setminus G)) = X \setminus f^{-1}(f(A))$. ⟹ That implies $f^{-1}(f^{\#}(G))$ is open in X.

Conversely , we assume that $f^{-1}(f^{\#}(G))$ is open in X for every α-open subset G of X . Let A be a α-closed in X . By our assumption, $f^{-1}(f^{\#}(G))$ is open in X , where G = X\A .By using lemma ((2.5)-(ii)) $f^{-1}(f(A)) = X \setminus f^{-1}(f^{\#}(X\setminus A)) = X \setminus f^{-1}(f^{\#}(G))$. That implies $f^{-1}(f(A))$ is open in X .Therefore , hence f is α-M-continuous .

Theorem : 5.3

The function f : X → Y is α-R-continuous if and only if $f^{\#}(f^{-1}(B))$ is closed in Y for every α-closed subset B of Y.

Proof :

Suppose f is α-R-continuous . Let B be α-closed in Y . ⟹ Then G=Y\B is α-open in Y . since f is α-R-continuous and since G is α-open in Y ⟹ $f(f^{-1}(G))$ is open in Y . Now by using lemma ((2.6)(i)) $f^{\#}(f^{-1}(B)) = Y \setminus f(f^{-1}(Y\setminus B)) = Y\setminus f(f^{-1}(G))$. ⟹ That implies $f^{\#}(f^{-1}(B))$ is closed in Y .

Conversely , we assume that $f^{\#}(f^{-1}(B))$ is closed in Y for every α-closed subset B of Y . Let G be α-open in Y. Let B = Y\G . By our assumption , $f^{\#}(f^{-1}(B))$ is closed in Y . By lemma((2.6)(ii)) $f(f^{-1}(G)) = Y \setminus (f^{\#}(f^{-1}(Y\setminus G))) = Y \setminus f^{\#}(f^{-1}(B))$, ⟹ this proves that $f(f^{-1}(G))$ is open inY . Therefore , hence f is α -R-continuous .

Theorem : 5.4

The function f : X → Y is α-S-continuous if and only if $^{\#}(f^{-1}(G))$ is open in Y for every α-open subset G of Y.

Proof :

Suppose f is α-S-continuous . Let G be α-open in Y . Then B=Y\G is α-closed in Y . since f is α-S-continuous and since B is α-closed in Y ⟹ $f(f^{-1}(B))$ is open in Y . Now by using lemma ((2.6)(i)) $f^{\#}(f^{-1}(G)) = Y \setminus f(f^{-1}(Y\setminus G)) = Y\setminus f(f^{-1}(B))$. ⟹ That implies $f^{\#}(f^{-1}(G))$ is open in Y .

Conversely , we assume that $f^{\#}(f^{-1}(G))$ is open in Y for every α-open subset G of Y . Let B be α-closed in Y. Let G = Y\B . By our assumption , $f^{\#}(f^{-1}(G))$ is open in Y . By lemma ((2.6)(ii)) $f(f^{-1}(B)) = Y \setminus (f^{\#}(f^{-1}(Y\setminus B))) = Y \setminus f^{\#}(f^{-1}(G))$ ⟹ This proves that $f(f^{-1}(B))$ is closed inY . Therefore , hence f is α -S-continuous .

Theorem : 5.5

Let f : (X,τ)→ Y be a function . Then the following are equivalent.

(i) f is α-L-continuous ,

(ii) for every α-closed subset A of X , $f^{-1}(f^{\#}(A)$ is closed in X ,

(iii) (iii)for every x ϵ X and for every α-open set U in X with f(x) ϵ f(U) there is an open set G in X with x ϵ G and f(G) ⊆ f(U),

(iv) $f^{-1}(f(int(cl(int(A))))) \supseteq int(f^{-1}(f(A)))$ for everysemi-pre-closed subset A of X.

(v) $cl(f^{-1}(f^{\#}(A))) \supseteq f^{-1}(f^{\#}(cl(int(cl(A)))))$ for every semi-pre- open subset A of X .

Proof : (i)↔ (ii) : follows from theorem 5.1 . (i)↔ (iii): Suppose f is α-L-continuous .Let U be α-open set in X such that f(x) ϵ f(U). since f is α-L-continuous, $f^{-1}(f(U))$ is open in X . since x ϵ $f^{-1}(f(U))$ there is an open set G in X such that x ϵ G ⊆ $f^{-1}(f(U))$ ⇒f(G) – $f(f^{-1}(f(U)))$ ⊆ f(U). This proves (iii). conversely, suppose(iii) holds . Let U be α-open set in X and x ϵ $f^{-1}(f(U))$. Then f(x) ϵ f(U). By using (iii), there is an open set G in X containing x such that f (G) ⊆ f (U). Therefore x ϵ G ⊆ $f^{-1}(f(G))$ ⊆ $f^{-1}(f(U))$ $f^{-1}(f(U))$ is open set in X This completes the proof for (i)↔(iii) .

(i)↔(iv) :

Suppose f is α-L-continuous . Let A be a semi-closed subset of X. Then int(cl(int(A))) is α-open set in X . By the α-L-continuity of f we see that $f^{-1}(f(int(cl(int(A)))))$ is open in X . since A is semi-pre-closed in X , We have $f^{-1}(f(int(cl(int(A))))) \supseteq f^{-1}(f(A))$ and since $f^{-1}(f(int(A)))$ is open in X . It follows that $f^{-1}(f(int(A)) \supseteq int(f^{-1}(f(A)))$. This proves (iv). conversely , we assume that (iv) holds . Let U be α-open set in X . since U is semi-pre-closed by applying (iv) we get $f^{-1}(f(int(cl(int(U))))) \supseteq int(f^{-1}(f(U)))$, Therefore $f^{-1}(f(U)) \supseteq int(f^{-1}(f(U)))$ and hence $f^{-1}(f(U))$ is open in X . This proves that f is α-L-continuous . (ii)↔(v) : Suppose (ii) holds . Let A be a semi-pre-open subset of X . By using (ii) $f^{-1}(f^{\#}(cl(int(cl(A))))$ is closed in X . since A is semi-pre-open $f^{-1}(f(^{\#}(A))) \subseteq f^{-1}(f^{\#}(cl(int(cl(A)))))$ This proves (v),

Conversely , let us assume that (v) holds . Let A be a α-closed subset of X , since A is semi-pre-open by (v) , we see that $cl(f^{-1}(f^{\#}(A))) \supset f^{-1}(f^{\#}(cl(int(cl(A))))) = f^{-1}(f^{\#}(A))$, Therefore $f^{-1}(f^{\#}(A))$ is closed in X . This proves (ii)

Theorem : 5.6

Let f : (X,τ)→ Y be a function . Then the following are equivalent.

(i) f is α-M-continuous ,

(ii) for every α-open subset G of X , $f^{-1}(f^{\#}(G)$ is open in X , (iii) $cl(f^{-1}(f(A))) \supseteq f^{-1}(f(cl(int(cl(A)))))$ for every semi-pre-open subset A of X . (iv) $f^{-1}(f^{\#}(int(cl(int(A))))) \supseteq int(f^{-1}(f^{\#}(A)))$ for every semi-pre-closed subset A of X .

Proof : (i) ↔ (ii) : follows from theorem 5.2 . (i) ↔ (iii) :Suppose f is α-M-continuous . Let A be a semi-pre-open set in X .Since f is α-M-continuous , $f^{-1}(f(cl(int(cl(A)))))$ is closed in X . Since A is semi-pre-open in X we see that $f^{-1}(f(A)) \supseteq f^{-1}(f(cl(int(cl(A)))))$, It follows that $cl(f^{-1}(f(A))) \supseteq cl(f^{-1}(f(cl(int(cl(A)))))) = f^{-1}(f(cl(int(cl(A)))))$. This proves (iii) . conversely , suppose (iii) holds . Let A be α-closed subset in X Since A is semi-pre-open by applying (iii) , $cl(f^{-1}(f(A))) \supseteq f^{-1}(f(cl(A))) = f^{-1}(f(cl(int(cl(A)))))$ That implies $f^{-1}(f(A))$ is closed set in X . This completes the proof for (i)↔(iii) .

(ii)↔(iv) :

Suppose (ii) holds . Let A be a semi-pre-closed subset of X . Then int(cl(int(A))) is α-open in X .

By (ii) , $f^{-1}(f^{\#}(int(cl(int(A))))) \supseteq f^{-1}(f^{\#}(A))$. Since , $f^{-1}(f^{\#}(int(cl(int(A)))))$ is open in X . we see that $f^{-1}(f^{\#}(int(cl(int(A))))) \supseteq int(f^{-1}(f^{\#}(A)))$. This proves (iv) . Suppose (iv) holds . Let G be α-open in X . since G is semi-pre-closed in X , by using (iv) we see that $f^{-1}(f^{\#}(G)) = f^{-1}(f^{\#}(int(cl(int(A))))) \supseteq int(f^{-1}(f^{\#}(G)))$. Then it follows that $f^{-1}(f^{\#}(G))$ is open in X . This proves (ii) .

Theorem : 5.7

Let f : (X,τ)→ Y be a function and y be a space with a base consisting of f^{-1} saturated open sets . Then the following are equivalent .

(i) f is α-R-continuous ,

ii) for every α-closed subset B of X , $f^{\#}(f^{-1}(B)$ is closed in Y , (iii)for every x ϵ X and for every α-open set V in Y with x ϵ $f^{-1}(V)$ there is an open set G in Y with f(x) ϵ G and $f^{-1}(G) \subseteq f^{-1}(V)$,

(iv) $f(f^{-1}(int(cl(int(B))))) \supseteq int(f(f^{-1}(B)))$ for every semi-pre-closed subset B of Y .

(v) $cl(f^{\#}(f^{-1}(B))) \supseteq f^{\#}(f^{-1}(cl(int(cl(B)))))$ for every semi-pre- open subset B of Y .